# Revision Geometry

## [1] X - axis

① $A \in X\text{-axis} \longrightarrow A = (x, 0, 0)$

② eq. of X-axis $\longrightarrow y = 0,\ z = 0$

③ the $\perp$ distance between the point $(x_1, y_1, z_1)$ with X-axis

$$= \sqrt{y_1^2 + z_1^2}$$

## [2] y - axis

① $A \in y\text{-axis} \longrightarrow A = (0, y, 0)$

② eq. of y-axis $\longrightarrow x = 0,\ z = 0$

③ the $\perp$ distance between the point $(x_1, y_1, z_1)$ with y-axis

$$= \sqrt{x_1^2 + z_1^2}$$

## [3] z - axis

① $A \in Z\text{-axis} \longrightarrow A = (0, 0, Z)$

② eq. of Z-axis $\longrightarrow x = 0,\ y = 0$

③ the $\perp$ distance between the point $(x_1, y_1, z_1)$ with Z-axis

$$= \sqrt{x_1^2 + y_1^2}$$

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## [1] xy plane

① $A \in xy$ plane $\longrightarrow (x, y, 0)$

② eq. of xy-plane $\longrightarrow Z = 0$

eq. of a plane parallel to xy plane is $Z = \text{constant}$

③ the $\perp$ distance between the point $(x_1, y_1, z_1)$ with xy plane

$$= \sqrt{z_1^2} = |z_1|$$

## 2 yz plane

① $A \in$ yz plane $\rightarrow (0, y, z)$

② eq. of yz-plane $\rightarrow x = 0$

eq. of a plane parallel to yz plane is $x =$ constant

③ the $\perp$ distance between the point $(x_1, y_1, z_1)$ with yz plane

$= \sqrt{x_1^2} = |x_1|$

## 3 xz plane

① $A \in$ xz plane $\rightarrow (x, 0, z)$

② eq. of xz-plane $\rightarrow y = 0$

eq. of a plane parallel to xz plane is $y =$ constant

③ the $\perp$ distance between the point $(x_1, y_1, z_1)$ with xz plane

$= \sqrt{y_1^2} = |y_1|$

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the image of the point $(a, b, c)$ by reflection in

① x_axis is the point $(a, -b, -c)$
② y_axis is the point $(-a, b, -c)$
③ z_axis is the point $(-a, -b, c)$
④ xy plane is the point $(a, b, -c)$
⑤ yz plane is the point $(-a, b, c)$
⑥ xz plane is the point $(a, -b, c)$

the image of the point $(a, b, c)$ by rotation with angle $\pm 180$ about the origin point is $(-a, -b, -c)$

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**Choose the correct answer from the given ones :**

**1** The point (0 , 0 , – 3) lies on ..............

x y z

(a) y-axis. (b) z-axis. (c) $X$ y-plane. (d) $X$ z-plane.

**2** If A (m , 2 m – n , m + n + 3) ∈ $X$-axis , then m = –1 ...........

$2m - n = 0$

$m + n = -3$

(a) 0 (b) – 2 (c) 3 (d) – 1

**3** The point (2 , 0 , – 3) lies in the coordinates plane whose equation is ..............

x y z xz

(a) z = 0 (b) y = 0 (c) $X$ = 0 (d) $X$ + y = – 1

**4** All points in space in the form ($X$ , 5 , z) lies in the plane whose equation is ..............

// xz plane

(a) $X$ = 5 (b) y = 5 (c) z = 0 (d) y = 0

**5** The point A (1 – k , 2 k , 3 + k) lies on the plane $X$ y , then A = ..............

$3 + K = 0$

$K = -3$

(a) (– 2 , 6 , 0) (b) (0 , 2 , 4) (c) (1 , 0 , 3) (d) (4 , – 6 , 0)

**6** The point A (n – 1 , n + 4 , 2 n) lies on the plane y = 6 , then n = ..............

$n = 2$

(a) 7 (b) 2 (c) 3 (d) any real number ≠ 0

**7** If the point ($X$ , y , z) lies in the $X$ z-plane , then ..............

y = 0

(a) $X$ = 0 (b) y = 0 (c) z = 0 (d) $X$ + y = 0

**8** The point A (2 , – 3 , 0) lies ..............

x y z

(a) ~~on the z-axis.~~ (b) in the y z-plane. (c) in the $X$ y-plane. (d) ~~on the $X$-axis.~~

**9** If the point (2 a , a + 3 , 5) lies in the cartesian plane $X$ z , then its distance from y z-plane equals .............. unit length.

$a = -3$

y = 0

(a) 3 (b) 5 (c) 6 (d) zero

(– 6 , 0 , 5)

$\sqrt{(x)^2} = |x|$

5 $a=7$ $a-4=3 \rightarrow a=7$

**10** If the point $(a-2, 5, a-4)$ at a distance 5 units from the y z-plane and at a distance 3 units from the $X$ y-plane , then a = ............

(a) 2 (b) 4 (c) 7 (d) 7 or − 3

$|a-2|=5$

$a-2=5$, $a-2=-5$

$a=7$ $a=-3$

$|a-4|=3$

$a-4=3$ | $a-4=-3$

$a=7$ | $a=1$

**11** The distance between the point $(a, b, c)$ and y-axis equals ............

(a) $\sqrt{a^2+c^2}$ (b) $\sqrt{a^2+b^2}$ (c) $\sqrt{b^2+c^2}$ (d) $\sqrt{a^2+b^2+c^2}$

**12** The perpendicular distance from the point $(-5, -3, 4)$ to the $X$-axis = ............ length unit.

(a) 3 (b) 5 (c) 4 (d) 10

**13** If the point A $(\ell+5, 2\ell, \ell)$ is at a distance $2\sqrt{5}$ length unit from the $X$-axis , then A = ...............

3 , −4

$\sqrt{(2\ell)^2+\ell^2} = \sqrt{5\ell^2} = \sqrt{5}|\ell|$

$\ell=2$ $\ell=-2$

(a) (3 , 4 , 2) (b) (3 , −4 , −2)

(c) (7 , 4 , 2) (d) b , c together.

**14** If the point (3 , k , −2) equidistant from the two axes y and z , then k = ...............

(a) ± 3 (b) ± 2 (c) $\pm\sqrt{13}$ (d) ± 5

**15** The smallest distance between the point (− 4 , 5 , − 2) and the plane z = 0 equals ............... length unit.

(a) − 2 (b) 5 (c) 4 (d) 2

**16** The distance between the point (− 2 , − 4 , 5) and the y z-plane equals ............... length units.

(a) 2 (b) 4 (c) 5 (d) $\sqrt{41}$

**17** The point A (3 , − 5 , 1) in the space , then the sum of its dimensions from the three coordinate planes = ............... length units.

(a) − 1 (b) 1 (c) 9 (d) 35

with xy plane → $|z| = 1$

with xz plane → $|y| = |-5| = 5$

with yz plane → $|x| = 3$

14 If the point (3 , k , – 2) equidistant from the two axes y and z , then k = ...............

ⓐ ± 3 ⓑ ± 2 ⓒ $\pm\sqrt{13}$ ⓓ ± 5

$$\sqrt{3^2+(-2)^2} = \sqrt{3^2+K^2}$$

$$13 = 9+K^2$$

$$K = \pm 2$$

**18** The equation of z-axis in the space is ..............

(a) $x = 0 , y = 0$ (b) $x = 0 , z = 0$

(c) $y = 0 , z = 0$ (d) $x = 0$

**19** The two coordinate planes $z = 0$ , $x = 0$ are intersecting at ..............

(a) origin point (b) $x$-axis (c) y-axis (d) z-axis

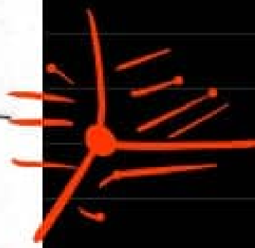
xy plane

**20** The $x$-axis and the y-axis belong to the plane with equation ..............

(a) $x = 0$ (b) $y = 0$ (c) $z = 0$ (d) $x + y = 0$

**21** $x$y-plane and yz-planes intersect at ..............

(a) origin point. (b) $x$-axis. (c) y-axis. (d) z-axis.



**22** The coordinate planes $x$y , $x$z , zy intersecting at ..............

(a) the origin. (b) $x$-axis. (c) y-axis. (d) z-axis.

**23** The straight lines $\overleftrightarrow{XX'}$ , $\overleftrightarrow{ZZ'}$ form the coordinate plane whose equation is ..............

(a) $x = 0$ (b) $y = 0$ (c) $z = 0$ (d) $y = 2$

**24** The coordinates of the midpoint of the line segment its end points are $(-3 , 2 , 4)$ , $(5 , 1 , 8)$ is ..............

(a) $\left(1 , \frac{3}{2} , 6\right)$ (b) $(2 , -1 , 4)$ (c) $(8 , -1 , 4)$ (d) $\left(1 , \frac{3}{2} , 2\right)$

**25** If A (a , b , c) is the midpoint of $(-4 , 0 , 5)$ , $(-2 , 4 , -13)$ , then $a + b + c =$ ..............

(a) $-5$ (b) $-6$ (c) $3$ (d) $4$

**26** If C $(-1 , 6 , -5)$ is the midpoint of $\overline{AB}$ where A $(k - 2 , -1 , m + 3)$ , B $(2 , n - 7 , -2)$ , then the value of $k + m - n =$ ..............

$-2 + -11 - 20$

(a) $33$ (b) $23$ (c) $-27$ (d) $-33$

$\times 2$

**27** If $(5 , 6 , -3)$ is the midpoint of $\overline{AB}$ where A $(3 , -1 , 5)$ , then B = ..............

(a) $\left(4 , \frac{5}{2} , 1\right)$ (b) $(7 , 13 , -11)$ (c) $(-2 , -7 , 8)$ (d) $(3 , 2 , 13)$

$-1 = \frac{k - 2 + 2}{2}$ $k = -2$

$6 = \frac{-1 + n - 7}{2}$ $n = 20$

$-5 = \frac{m + 3 + -2}{2}$ $m = -11$

(28) If the midpoint of $\overline{AB} \in X$-axis where A (2 , 12 + k , k) , B (4 , m , 8 − m) , then k − 3 m = ............

$(x, 0, 0) = \left(3, \frac{12+k+m}{2}, \frac{k+8-m}{2}\right)$

$-10 - 3 \times -2$

$k = -10$

$m = -2$

$y = 0$

(a) 4 (b) − 4 (c) − 2 (d) − 10

$\frac{12+k+m}{2} = 0$

$\boxed{k + m = -12}$

$\frac{k-m+8}{2} = 0$

$\boxed{k - m = -8}$

(29) If the midpoint of $\overline{AB}$ lies in the cartesian plane $X$ z and A (− 3 , 12 + k , 5) , B (1 , 3 k , − 2) , then k = ............

(a) 5 (b) − 3 (c) − 2 (d) 1

(38) If the points A (6 , 0 , 3) , B (7 , 1 , 7) , C (9 , 3 , 15) lie on the same straight line , then A divides $\overline{BC}$ in the ratio ............

(a) 3 : 1 internally. (b) 1 : 3 externally. (c) 2 : 3 internally. (d) 1 : 2 externally.

$\text{ratio} = \frac{BA}{AC} = \frac{7-6}{6-9} = \frac{1}{-3}$

externally

B (7, 1, 7) A (6, 0, 3) C (9, 3, 15)

$\frac{1-0}{0-3} = \frac{-1}{3}$

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# Distance between 2 points

If $A=(x_1, y_1, z_1)$, $B=(x_2, y_2, z_2)$ then

$$AB=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

To prove that $A$, $B$, $C$ are collinear.

1. $AB = AC + BC$, $AB$ is the greatest.
2. $\overline{AB} \parallel \overline{BC}$
3. find equation of $\overleftrightarrow{AB}$. Prove that $C \in \overleftrightarrow{AB}$

To prove that $\Delta ABC$ is right angled $\Delta$

$$(AB)^2 = (AC)^2 + (BC)^2$$

if $(AB)^2 > (AC)^2 + (BC)^2 \longrightarrow \Delta ABC$ obtuse at $C$

if $(AB)^2 < (AC)^2 + (BC)^2 \longrightarrow \Delta ABC$ is acute angled

To prove that $ABCD$ is a square

$AB = BC = CD = DA$, $AC = BD$

To prove that $ABCD$ is rhombus

$AB = BC = CD = DA$, $AC \neq BD$

To prove that $ABCD$ is rectangle

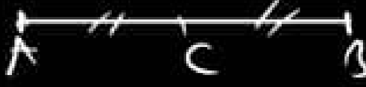
$AB = DC$, $BC = AD$, $AC = BD$

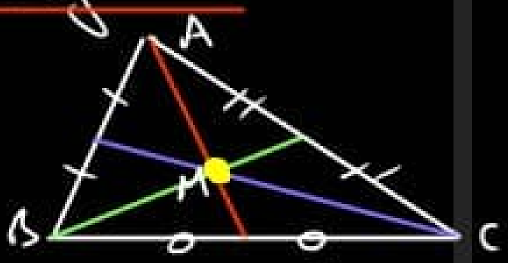
To prove that $ABCD$ is a parallelogram

$AB = DC$, $BC = AD$, $AC \neq BD$

## the midpoint of a line segment

① $C = \frac{A+B}{2}$



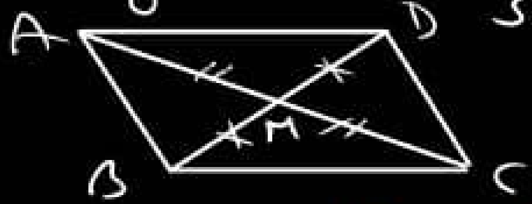

② $B = 2C - A$

$A = 2C - B$

③ point of concurrence of medians of $\triangle ABC = \frac{A+B+C}{3}$

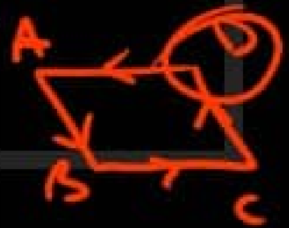
④ If ABCD is a ▱

then $D = A + C - B$




46 If A (6 , −2 , 4) , B (2 , 4 , −8) , C (−2 , 2 , 4) are three consecutive vertices of a parallelogram ABCD , then D = ...... $A + C - B$

(a) $(2 , \frac{4}{3} , 0)$ (b) (−2 , 4 , −16) (c) (2 , −4 , 16) (d) (1 , −2 , 8)




47 *(Trial 2021)* If A (3 , −4 , 0) , B (15 , 0 , 2) , C (0 , −8 , 4) are three points in the space and they form triangle ABC , then the distance between its centroid and the $x$ z-plane is ...4...

$\frac{A+B+C}{3}$

(6 , −4 , 2)

(a) greater than the distance from the $x$ y-plane.

(b) smaller than or equal to the distance from the $x$ y-plane.

(c) greater than the distance from the y z-plane.

(d) greater than or equal to the distance from the y z-plane.

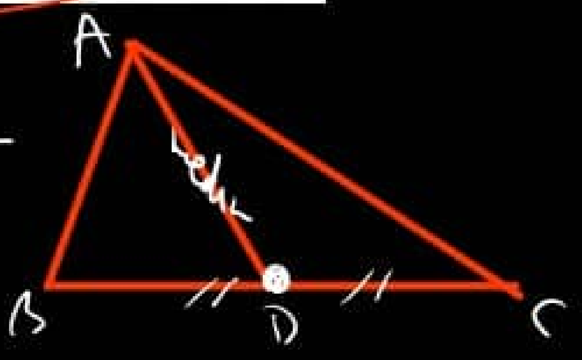
44 *(2nd Session 2021)* If ABC is a triangle in which D is the midpoint of $\overline{BC}$ , A (3 , 1 , 5) , B (2 , 3 , 7) , C (0 , 3 , 1) , then the length of $\overline{AD}$ = ............ length unit

(a) 9 (b) 2 (c) 7 (d) 3

$D = \frac{B+C}{2} = (1, 3, 4)$

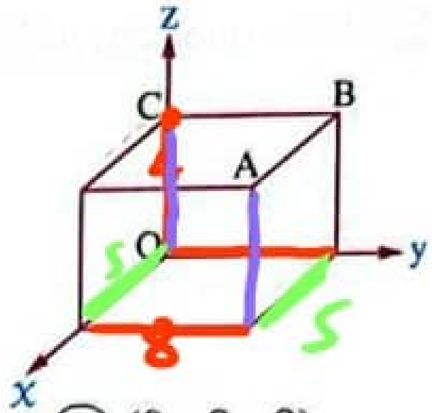
$AD = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$

$= \sqrt{4+4+1} = 3$



48 The opposite figure is a cuboid : A (5 , 8 , 4) , then :

x y z




**First :** The coordinates of the point B are ............

(a) (4 , 8 , 0) (b) (0 , 8 , 4)

(c) (5 , 8 , 0) (d) (5 , 0 , 4)

**Second :** The coordinates of point C are ............

(a) (0 , 0 , 0) (b) (0 , 0 , 4) (c) (5 , 0 , 4) (d) (0 , 8 , 0)

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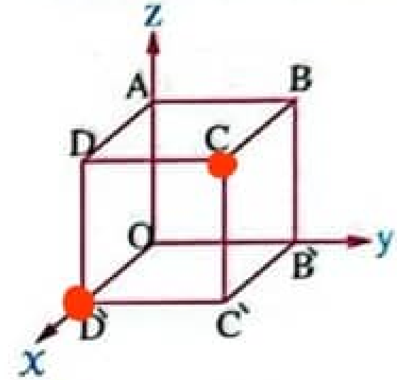
49 ABCDOB̀C̀D̀ is a cube of edge length 5 length unit , then :




**First :** The coordinates of the point C are ............

(a) (5 , 5 , 0) (b) (5 , 5 , 5)

(c) (0 , 0 , 5) (d) (0 , 5 , 0)

**Second :** The coordinates of D̀ are ............

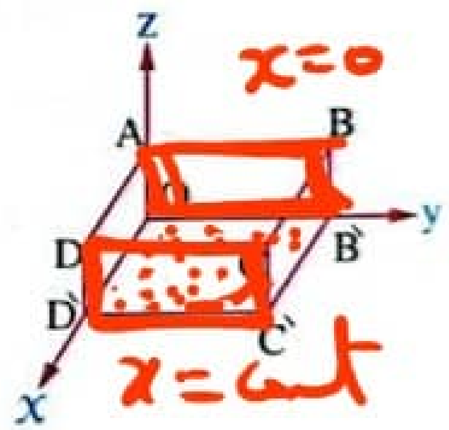
(a) (0 , 0 , 5) (b) (5 , 5 , 0) (c) (0 , 5 , 5) (d) (5 , 0 , 0)

**Third :** The diagonal length of the cube = ............ length units.

(a) $5\sqrt{2}$ (b) $5\sqrt{3}$ $\ell\sqrt{3}$ (c) 5 (d) $5\sqrt{6}$

$$\sqrt{x^2+y^2+z^2} = \sqrt{5^2+5^2+5^2} = 5\sqrt{3}$$

**50 In the opposite figure :**




A cuboid ; $C'(5, 8, 0)$ , $D(5, 0, 3)$ , then :

**First :** The coordinates of C ............

ⓐ $(5, 8, 0)$ ⓑ $(5, 3, 8)$

ⓒ $(5, 8, 3)$ ⓓ $(5, 8, 8)$

$5 \times 8 \times 3$

**Second :** The volume of the cuboid ............ cubic units.

ⓐ 64 ⓑ 120 ⓒ 144 ⓓ 150

**Third :** The equation of the plane $OB'C'D'$ is ......x y plane

ⓐ $x = 0$ ⓑ $y = 0$ ⓒ $z = 0$ ⓓ $z = 3$

**Fourth :** The equation of the plane $DD'C'C$ is ............

ⓐ $x = 0$ ⓑ $y = 0$ ⓒ $x = 5$ ⓓ $z = 3$

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# Equation of sphere

1. standard form: $(x-\ell)^2+(y-K)^2+(z-n)^2=r^2$
   the center $(\ell, K, n)$

2. general form: $x^2+y^2+z^2-2\ell x-2Ky-2nz+d=0$
   نغير الاشارة, ونقسم ÷2
   the center $(\ell, K, n)$
   $d=\ell^2+K^2+n^2-r^2$
   $r=\sqrt{\ell^2+K^2+n^2-d}>0$ لازم

## How to find the center

① $(x-2)^2+(y+4)^2+(z-5)^2=64 \rightarrow$ the center is .....

② $x^2+y^2+z^2+8x-12y+2z+4=0 \rightarrow$ the center is ...

③ the center on X-axis is $(\ell, 0, 0)$
   " " " y-axis is $(0, K, 0)$
   " " on Z axis is $(0, 0, n)$
   " "

④ the sphere touches the 3 coordinate planes and its radius r is $(\pm r, \pm r, \pm r)$

⑤ the sphere touches the 3 axis and its radius r is $\left(\frac{\pm r}{\sqrt{2}}, \frac{\pm r}{\sqrt{2}}, \frac{\pm r}{\sqrt{2}}\right)$, $r=|a|\sqrt{2}$
   8 spheres

   if the center $(a, b, c)$
   dist with x-axis = dist with y-axis
   = dist with z-axis
   $\sqrt{b^2+c^2}=\sqrt{a^2+c^2}=\sqrt{a^2+b^2}$
   $\therefore |a|=|b|=|c| \rightarrow r=|a|\sqrt{2}$

⑥ the smallest sphere passes through non collinear points
   $A(a,0,0)$, $B(0,a,0)$, $C(0,0,a)$
   or $A(a,a,0)$, $B(a,0,a)$, $C(0,0,a)$

* its center is $M=\frac{A+B+C}{3}$, $r=\frac{|a|}{3}\sqrt{6}$
  centroid

⑥ the smallest sphere passes through non collinear points

$A(a,0,0)$ , $B(0,a,0)$ , $C(0,0,a)$

or $A(a,a,0)$ , $B(a,0,a)$ , $C(0,0,a)$

* its center is $M = \frac{A+B+C}{3}$ centroid , $r = \frac{|a|}{3}\sqrt{6}$
* ABC is equilateral $\Delta$ on the greatest circle which divides the sphere into 2 halves and the side length of the triangle $= \sqrt{2}\,|a|$

## How to find the radius

① circle its center $(x, y, z)$

* touches xy plane $\rightarrow r = \sqrt{z^2} = |z|$
* touches yz plane $\rightarrow r = \sqrt{x^2} = |x|$
* touches xz plane $\rightarrow r = \sqrt{y^2} = |y|$

* touches x-axis $\rightarrow r = \sqrt{y^2+z^2}$
* touches y-axis $\rightarrow r = \sqrt{x^2+z^2}$
* touches z-axis $\rightarrow r = \sqrt{x^2+y^2}$

* touches the plane $x=2$ , its center $(5,4,0)$

$\therefore r = |5-2| = 3$

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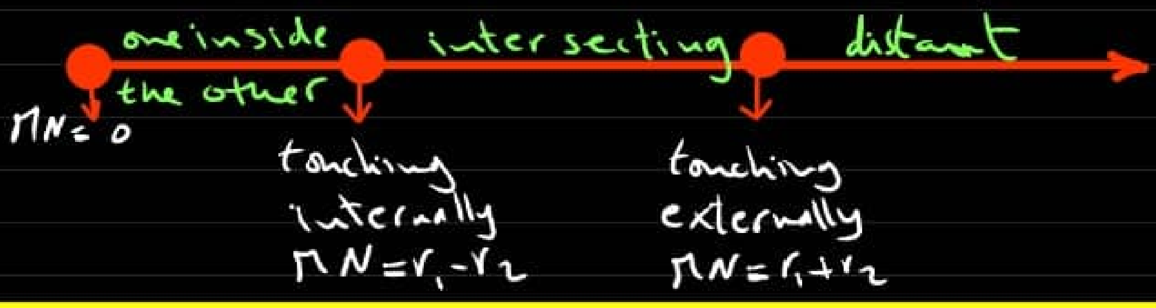


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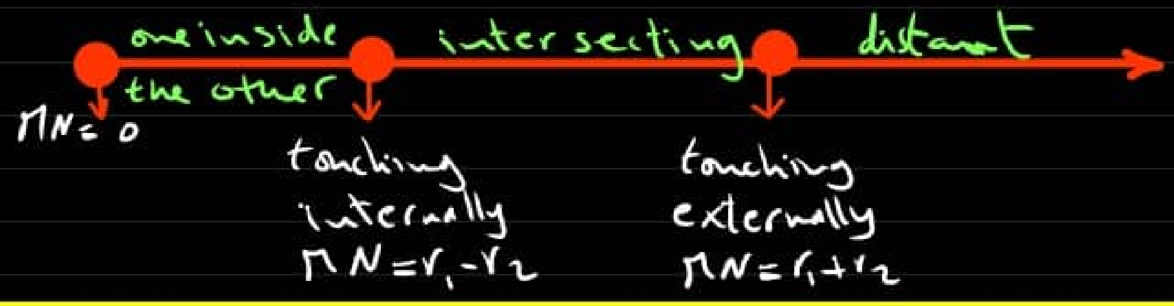
V of sphere $= \frac{4}{3} r^3$ , Area of sphere $= 4\pi r^2$




$V$ of sphere $= \frac{4}{3} r^3$ , Area of sphere $= 4\pi r^2$

find the equation of sphere lies in $xz$-plane and passes through the points
$A(0,8,0)$ , $B(4,6,2)$ , $C(0,12,4)$

## Solution

$\because$ the center lies on $xz$-plane then the center $(\ell, 0, n)$

eq: $x^2+y^2+z^2-2\ell x-\cancel{2ky}-2nz+d=0$

$\because A(0,8,0) \in$ sphere $\longrightarrow 8^2+d=0 \rightarrow d=-64$

$B\ 4\ 6\ 2 \qquad (12)^2+4-2n(4)-64=0 \rightarrow n=12$

$C\ 0\ 12\ 4 \qquad (4)^2+6^2+2^2-8\ell-4n-64=0 \rightarrow \ell=-7$

the equation is: $x^2+y^2+z^2+14x-24z-64=0$

## Choose the correct answer from the given ones :

1. The equation of the sphere whose centre $(2 , -3 , 5)$ and its radius length $2\sqrt{5}$ length unit is ............

$(x-2)^2+(y+3)^2+(z-5)^2 = 20$

(a) $(x+2)^2+(y-3)^2+(z+5)^2 = 2\sqrt{5}$
(b) $x^2+y^2+z^2 = 20$
(c) $(x-2)^2+(y+3)^2+(z-5)^2 = 20$
(d) $(x-2)^2+(y+3)^2+(z-5)^2 = 2\sqrt{5}$

2. The equation of the sphere whose centre is the origin and its radius length is 3 units is ............

(a) $x^2+y^2+z^2 = 3$
(b) $x^2+y^2+z^2 = 9$
(c) $(x-2)^2+(y-3)^2+(z-3)^2 = 9$
(d) $x^2+y^2+z^2+9 = 0$

3. The equation of the sphere whose centre is the origin and cuts 5 units from the positive part of the $x$-axis is ............

(a) $x^2+y^2+z^2-25 = 0$
(b) $x^2+y^2+z^2+25 = 0$
(c) $x^2+y^2+z^2-100 = 0$
(d) $x^2+y^2+z^2-\sqrt{5} = 0$

4. The equation of the sphere whose centre is the origin and passes through $(3 , -1 , 2)$ is ............

$x^2+y^2+z^2$

(a) $x^2+y^2+z^2 = 4$
(b) $(x-3)^2+(y+1)^2+(z-2)^2 = 14$
(c) $(x-3)^2+(y+1)^2+(z-2)^2 = \sqrt{14}$
(d) $x^2+y^2+z^2 = 14$

$d=0$

**5** If the origin lies on a sphere whose centre $(-1, 2, 2)$, then its equation is ............

ⓐ $x^2+y^2+z^2+2x+4y+4z=\cancel{-3}$

ⓑ $x^2+y^2+z^2+2x-4y-4z=\cancel{3}$

ⓒ $(x+1)^2+(y-2)^2+(z-2)^2+\cancel{9}=0$

ⓓ $x^2+y^2+z^2+2x-4y-4z=0$

---

**6** The equation of the sphere whose centre is the origin and passes through vertices of a cube whose edge length 12 length unit is ............ $x^2+y^2+z^2=(6\sqrt{3})^2=108$

ⓐ $x^2+y^2+z^2=144$

ⓑ $x^2+y^2+z^2=108$

ⓒ $x^2+y^2+z^2=36$

ⓓ $x^2+y^2+z^2+108=0$


diagonal $=l\sqrt{3}$ $=12\sqrt{3}$

$r=6\sqrt{3}$

---

$r=d=|z|=4$

The equation of a sphere with centre $(2, -3, 4)$ and touches $xy$-plane is ............

ⓐ $(x-2)^2+(y+3)^2+(z-4)^2=4$

ⓑ $(x-2)^2+(y+3)^2+(z-4)^2=9$

ⓒ $(x-2)^2+(y+3)^2+(z-4)^2=16$

ⓓ $(x+2)^2+(y-3)^2+(z+4)^2=16$

The equation of the sphere whose centre $(-3, -3, 5)$ and touches the two planes $x z$ and $y z$ is ............

$r = |-3| = 3$

$r = |-3| = 3$

ⓐ $(x+3)^2 + (y+3)^2 + (z-5)^2 = 3$

ⓑ $(x+3)^2 + (y+3)^2 + (z-5)^2 = 9$

ⓒ $(x-3)^2 + (y-3)^2 + (z+5)^2 = 34$

ⓓ $x + y^2 + z^2 + 6x + 6y - 10z = 9$

9 The equation of the sphere with centre $M(1, -3, -1)$ and passes through the point $A(-2, -1, -1)$ is ............

ⓐ $(x+2)^2 + (y+1)^2 + (z+1)^2 = 13$

ⓑ $x^2 + y^2 + z^2 - 2x + 6y + 2z - 13 = 0$

ⓒ $x^2 + y^2 + z^2 - 2x + 6y + 2z = 2$

ⓓ $(x-1)^2 + (y+3)^2 + (z+1)^2 = \sqrt{13}$

$r = MA = \sqrt{13}$

$d = 1^2 + (-3)^2 + (-1)^2 - r^2$

$= 11 - 13 = -2$

Center $= \frac{A+B}{2}$

10 The equation of the sphere whose diameter is $\overline{AB}$ where $A(7, 1, -4)$ , $B(3, -1, 2)$ is ............

ⓐ $(x-7)^2 + (y-1)^2 + (z+4)^2 = 28$

ⓑ $(x-3)^2 + (y+1)^2 + (z-2)^2 = 14$

ⓒ $x^2 + y^2 + z^2 - 10x + 2z - 14 = 0$

ⓓ $(x-5)^2 + (z+1)^2 + y^2 = 14$

**11** The $\mathcal{X}$-axis touches the sphere whose centre (2 , 3 , 4) then the equation of the sphere is ............

$r = \sqrt{3^2 + 4^2} = 5$   $x - 2$

(a) $(\mathcal{X}-2)^2 + (y-3)^2 + (z-4)^2 = 15$
(b) $(\mathcal{X}-2)^2 + (y-3)^2 + (z-4)^2 = 5$
(c) $\mathcal{X}^2 + y^2 + z^2 - 4\mathcal{X} - 6y - 8z + 4 = 0$
(d) $\mathcal{X}^2 + y^2 + z^2 - 25 = 0$

$d = \cancel{4 + 9 + 16 - 25}$

**12** ***(2nd Session 2021)*** The equation of the sphere whose centre is (–1 , 0 , 5) and its volume $36\,\pi$ volume unit is ............

$V = \frac{4}{3}\pi r \rightarrow r = 3$

(a) $(\mathcal{X}+1)^2 + y^2 + (z-5)^2 = 36$
(b) $(\mathcal{X}-1)^2 + y^2 + (z+5)^2 = 6$
(c) $(\mathcal{X}+1)^2 + y^2 + (z-5)^2 = 27$
(d) $(\mathcal{X}+1)^2 + y^2 + (z-5)^2 = 9$

$4\pi r^2 \rightarrow r = 5$

**13** Equation of the sphere has centre (2 , – 1 , 4) and its area $100\,\pi$ square units is ............

(a) $(\mathcal{X}+2)^2 + (y-1)^2 + (z+4)^2 = 25$
(b) $(\mathcal{X}-2)^2 + (y+1)^2 + (z-4)^2 = 25$
(c) $(\mathcal{X}-2)^2 + (y+1)^2 + (z-4)^2 = 100$
(d) $(\mathcal{X}-2)^2 + (y+1)^2 + (z-4)^2 = 0$

14 If the points (0 , 0 , 0) , (4 , 0 , 0) , (0 , 4 , 0) , (0 , 0 , 4) are 4 vertices of a cube , then the equation of the sphere touches its faces from inside is ...............

(a) $(x-4)^2+(y-4)^2+(z-4)^2=4$

(b) $(x-2)^2+(y-2)^2+(z-2)^2=4$

(c) $(x-2)^2+y^2+z^2=4$

(d) $x^2+(y-2)^2+(z-2)^2=4$

$r = \frac{4}{2} = 2$


15 The centre of the sphere whose equation $x^2+y^2+z^2+2x-4y+6z-16=0$ is ............

$(-1, 2, -3)$

(a) (1 , 1 , 1)

(b) (2 , – 4 , 6)

(c) (– 1 , 2 , – 3)

(d) (– 2 , 4 , – 6)

17 If $\overline{AB}$ is a diameter of the sphere which its equation : $x^2+y^2+z^2-x+2y+3z-44=0$ and : A = (2 , 4 , – 6) , then B = ...............

(a) (1 , – 2 , – 3)

(b) (– 1 , – 6 , 3)

(c) (0 , 4 , 1)

(d) (2 , 3 , – 5)

Centre $= \left(\frac{1}{2}, -1, \frac{-3}{2}\right)$

$B = 2M - A$

$= (1, -2, -3) - (2, 4, -6) = -1$

x y z

**25** If point $(-2, 4, m)$ lies on the sphere $\cancel{(x+2)^2} + (y-1)^2 + (z-3)^2 = 25$ , then $m = \cdots\cdots\cdots$

$9 + (m-3)^2 = 25$

(a) 7 (b) 4 (c) 7 or – 1 (d) 4 or – 1

---

x y z

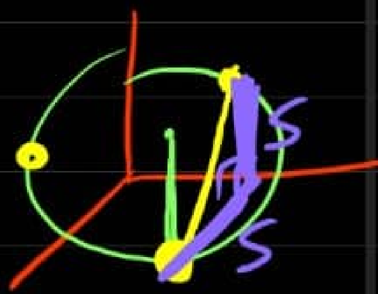
**26** *(1st Session 2021)* If the point $(7, -2, 2)$ lies on the surface of a sphere with equation : $(x-4)^2 + (y-1)^2 + (z+1)^2 = K^2$ , then $|K| = \cdots\cdots\cdots$

(a) 3 (b) $3\sqrt{3}$ (c) 27 (d) $\sqrt{3}$

$(5, 5, 5)$

---

**27** A sphere with radius 5 length units , touches the coordinates planes. If the coordinates of its centre are positive , then the distance between the point where it touches the $x$ y-plane and where it touches the y z-plane equals $\cdots\cdots\cdots$ length units.




(a) 5 (b) $5\sqrt{2}$ (c) $5\sqrt{3}$ (d) $5\sqrt{6}$

28 The centre of the sphere which touches the positive cartesian planes and its radius length is 5 units is ............

(a) (0 , 0 , 0)    (b) (5 , 5 , 5)    (c) (5 , 0 , 0)    (d) (0 , 5 , 5)

(0, 0, 1)

29 The centre of the sphere $x^2 + y^2 + z^2 - 2z = 0$ lies on the ............

(a) $x$-axis    (b) plane $z = 0$    (c) y-axis    (d) z-axis

30 If $x^2 + y^2 + z^2 - 2x + 4y - 4z + k = 0$ is the equation of a sphere , then the value of k could be ............

center (1, -2, 2)

(a) 9    (b) 18    (c) 5    (d) 10

$$r = \sqrt{1+4+4-K} > 0$$

$$9 - K > 0$$

$$K < 9$$

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$(0,5,0)$

Center $(0,y,0) \xrightarrow{r} (1,3,2)$

$\sqrt{1^2+(y-3)^2+4} = \sqrt{4+(y-4)^2+4}$

$\searrow r \; (-2,4,2)$

$y=5$

Center $(0,5,0) \to r = \sqrt{1+4+4}$

$r=3$

32. The sphere whose centre lies on y-axis and passes through the two points (1 , 3 , 2) , (− 2 , 4 , 2) its radius = ............ length unit.

(a) 6 (b) 8 (c) 3 (d) 9

$3$ $(0,-1,1)$ $r=2$

33. The point A (3 , 2 , 1) lies ............ the sphere whose equation $x^2 + (y + 1)^2 + (z - 1)^2 = 4$

(a) on (b) inside (c) outside (d) on the centre of

$r=2$

34. The sphere whose equation : $(x - 2)^2 + (y + 4)^2 + (z + 3)^2 = 4$ touches ............

(a) $x$-axis (b) y z-plane. (c) $x$ y-plane. (d) y-axis.

35. Which of the following represents a sphere , its centre lies on z-axis and touches the $x$ y-plane ?

$r = |z|$ $(0,0,2)$

(a) $x^2 + y^2 + z^2 = 25$ (b) $x^2 - 10x + y^2 + z^2 = 0$

(c) $x^2 + y^2 + z^2 - 10z = 25$ (d) $x^2 + y^2 + z^2 - 10z = 0$

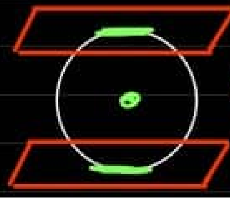
36. The sphere lies between the two planes z = − 1 and z = 5 and has centre (2 , − 1 , k) , then k = ...............

(a) zero (b) 1 (c) 2 (d) 3

$x^2 + y^2 \cancel{-2lx - 2ky} - 2nz + d = 0$

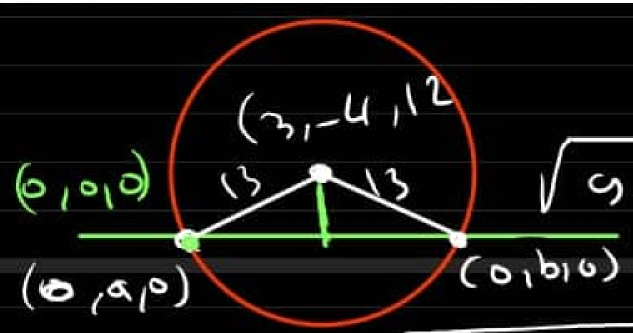
$d = z^2 - r^2 = 0$

$k = \frac{5 + -1}{2} = 2$



**37** If y-axis intersects the sphere whose centre is (3 , – 4 , 12) and its radius length 13 cm. at the two points A , B , then AB = ............ length units.

(a) 8 (b) 10 (c) 13 (d) 26




$\sqrt{9+(b+4)^2+144} = 13 \to b = 0$

$b = -8$

$a = 0$

$b = -8$

$M = (1, 1, -1)$ , $r_1 = \sqrt{1+1+1--1} = 2$

$N = (5, -2, 0)$ , $r_2 = 2$

$MN = \sqrt{16+9+1}$

$MN = \sqrt{26}$

$MN > r_1 + r_2$

**39** The two spheres with equations : $x^2 + y^2 + z^2 - 2x - 2y + 2z - 1 = 0$ , $(x - 5)^2 + (y + 2)^2 + z^2 = 4$ are ...............

(a) touching externally. (b) touching internally.

(c) intersecting. (d) distant.

$MN$

$r_1 =$

$r_2$

**40** If $(x + 3)^2 + (y - 2)^2 + (z - 4)^2 = 1$ , $(x + 4)^2 + (y - 4)^2 + (z - 2)^2 = 4$ are the equations of two spheres , then the two spheres are ...............

(a) intersecting. (b) touching externally.

(c) touching internally. (d) distant.

**41** If M and N are two spheres of radii $r_1$ and $r_2$ respectively where $r_1 > r_2$ , if the two spheres are tangential , then MN = ...............

(a) $r_1 + r_2$ (b) zero (c) $r_1 - r_2$ (d) a or c

43 *(1st Session 2021)* If $M_1$ , $M_2$ are two spheres touching internally and $M_1\left(-3 , 2 , -6\sqrt{2}\right)$ , $r_1 = 8$ length unit , $M_2\left(-2 , 1 , -5\sqrt{2}\right)$ , then $r_2 =$ ………… length units where $r_1 > r_2$

(a) 5 (b) 2 (c) 7 (d) 6

---

44 The shortest distance between the point (5 , – 1 , 7) and the surface of the sphere : $(x-2)^2 + (y+5)^2 + (z+5)^2 = 25$ equals …………

(a) 8 (b) 9 (c) 10 (d) 13

---

45 *(Trial 2021)* If the shortest distance between A (3 , 5 , 1) and the surface of a sphere with centre M (1 , 2 , – 5) equals 2 length units where A lies outside the shpere , then the radius of the sphere = ………… length units.

(a) 5 (b) 2 (c) 7 (d) 12

46 A sphere touches the $x$ y-plane , y z-plane and $x$ z-plane and passes through the point $(1, -4, 5)$ , then its radius could be ............ length units

(a) 6 (b) 7 (c) 8 (d) 9

47 The equation of the sphere whose centre $(3, m-1, 5)$ and touches the coordinate axes $x$ and y is ............

(a) $x^2 + y^2 + z^2 - 3x - 3y + 5z = 34$
(b) $(x-3)^2 + (y \pm 3)^2 + (z-5)^2 = 34$
(c) $(x+3)^2 + (y \pm 3)^2 + (z+5)^2 = 34$
(d) $(x \pm 3)^2 + (y \pm 3)^2 + (z \pm 5)^2 = 34$

48 The radius of the smallest sphere that the points $(0, 0, 5)$ , $(5, 0, 0)$ , $(0, 5, 0)$ lie on it is ............ length units.

(a) 15 (b) $\frac{5\sqrt{6}}{3}$ (c) $5\sqrt{3}$ (d) 5

49 The radius of the smallest sphere passes through $(5, 5, 0)$ , $(0, 5, 5)$ , $(5, 0, 5)$ is ............ length units.

(a) 5 (b) 10 (c) $\frac{5\sqrt{6}}{3}$ (d) $5\sqrt{2}$

50 The number of spheres touch the coordinate axes and its diameter length is 16 units is ............

(a) 1 (b) 2 (c) 4 (d) 8

51 If $\overrightarrow{AB} \perp \overrightarrow{BC}$ where $A(1, 2, 3)$ , $B(1, 8, 3)$ , $C(9, 8, 3)$ , then the equation of the smallest sphere passes through A , B and C is ............

(a) $(x-5)^2 + (y-5)^2 + (z-3)^2 = 25$
(b) $\left(x - \frac{11}{3}\right)^2 + (y-6)^2 + (z-2)^2 = 25$
(c) $(x-2)^2 + (y+3)^2 + (z-4)^2 = 16$
(d) $(x+2)^2 + (y-3)^2 + (z+4)^2 = 16$

# Vectors in space

$A = (2,3)$

$\vec{OA} = \vec{A}$


① Position vector

* the position vector of point $A = \vec{OA} = (A_x, A_y, A_z)$
* length of $\vec{A} = \|\vec{A}\| = \sqrt{A_x^2 + A_y^2 + A_z^2}$
* Unit vector in the direction of $\vec{A} = \vec{U}_A = \dfrac{\vec{A}}{\|\vec{A}\|}$

$$= \left( \frac{A_x}{\|\vec{A}\|}, \frac{A_y}{\|\vec{A}\|}, \frac{A_z}{\|\vec{A}\|} \right)$$

$$= (\cos\theta_x, \cos\theta_y, \cos\theta_z)$$

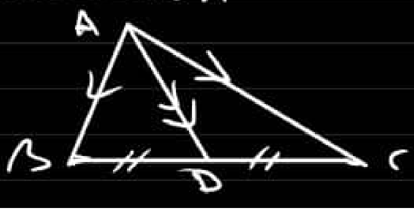
* $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$
* $\sin^2\theta_x + \sin^2\theta_y + \sin^2\theta_z = 2$
* $\cos 2\theta_x + \cos 2\theta_y + \cos 2\theta_z = -1$
* $\vec{A} = \|\vec{A}\| \cdot \vec{U}_A = \|\vec{A}\| (\cos\theta_x, \cos\theta_y, \cos\theta_z)$

Don't forget

① $\vec{AB} = B - A$ , $\vec{AB} = -\vec{BA}$

② $\vec{AB} + \vec{BC} = \vec{AC}$

③ $\vec{AB} + \vec{AC} = 2\vec{AD}$



Don't forget

① $\overrightarrow{AB} = B - A$ , $\overrightarrow{AB} = -\overrightarrow{BA}$

② $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

③ $\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$


④ $\|\vec{A}\| + \|\vec{B}\| \geq \|\vec{A} + \vec{B}\|$ triangle inequality

⑤ $\vec{F} = \|\vec{F}\| \times \dfrac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|}$

⑥ $\|K\vec{A}\| = |K|\,\|\vec{A}\|$

⑦* $A_x$ is the component of $\vec{A}$ in the direction of X-axis

$A_x = \|\vec{A}\| \cos\theta_x \Longrightarrow A_{yz} = \|\vec{A}\| \sin\theta_x$

because $\vec{A}$ makes angle $90-\theta_x$ with yz-plane

* $A_y = \|\vec{A}\| \cos\theta_y \Longrightarrow A_{xz} = \|A\| \sin\theta_y$

because $\vec{A}$ makes angle $90-\theta_y$ with xz-plane

⑧ the cosine direction angles of the x, y, z axis

unit vector are $(1,0,0)$ , $(0,1,0)$ , $(0,0,1)$

the direction angles: $(0,90,90)$ , $(90,0,90)$ , $(90,90,0)$

(9) If a vector makes equal angles with the coordinate axis then

$$\theta_x = \theta_y = \theta_z \rightarrow \cos\theta_x = \pm \frac{1}{\sqrt{3}}$$

$$\therefore \theta_x = \theta_y = \theta_z = 54^\circ\, 44'\, 8'' \ \boxed{\text{or}}\ 125^\circ\, 15'\, 52''$$

$$\vec{U}_A = \left( \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}} \right)$$

(10) $\theta_x + \theta_y \geq 90$ دائما

if $\theta_x + \theta_y = 90^\circ$ $\boxed{\text{or}}$ $270^\circ$ then $\theta_z = 90^\circ$

(11) If $\vec{A}$ makes direction angles $(\theta_x, \theta_y, \theta_z)$

then $K\vec{A}$ makes angles

- $(\theta_x, \theta_y, \theta_z)$ if $K > 0$
- $(180 - \theta_x, 180 - \theta_y, 180 - \theta_z)$ if $K < 0$

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# Vectors Multiplication

① $\vec{A} \cdot \vec{B} \longrightarrow$ scalar product
② $\vec{A} \times \vec{B} \longrightarrow$ vector product
③ $\vec{A} \cdot (\vec{B} \times \vec{C}) \longrightarrow$ scalar triple product

## I) Scalar Product

① $\vec{A} \cdot \vec{B} = \|\vec{A}\| \, \|\vec{B}\| \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2 \longrightarrow \vec{A} \cdot \vec{A} = \|\vec{A}\|^2$

② $\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{\|\vec{A}\| \, \|\vec{B}\|}$ , $0 \leq \theta \leq 180$

$\cos\theta = U_A \cdot U_B$


* if $\theta = 0 \longrightarrow \vec{A} // \vec{B}$ and in the same direction
* if $\theta = 180 \longrightarrow \vec{A} // \vec{B}$ and in opposite directions
* if $\theta = 90 \longrightarrow \vec{A} \perp \vec{B}$

③ If $\vec{A} \cdot \vec{B} = 0$ then $\vec{A} \perp \vec{B}$

④ the Alg. Component (Projection) of $\vec{A}$ in the direction of $\vec{B} = \|\vec{A}\| \cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{\|\vec{B}\|}$

⑤ Vector projection of $\vec{A}$ in the direction of $\vec{B} = \dfrac{\vec{A} \cdot \vec{B}}{\|\vec{B}\|} \times \dfrac{\vec{B}}{\|\vec{B}\|}$

⑥ Area of rectangle space whose dimensions are $\vec{B}$ and the Alg. comp of $\vec{A}$ in the direction of $\vec{B}$

$$= |\vec{A}\cdot\vec{B}|$$

⑦ Work done $= w = \vec{F}\cdot\vec{S} = \|\vec{F}\|\,\|\vec{S}\|\cos\theta$.

⑧ $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A} \longrightarrow \vec{A}\cdot\vec{A} = \|\vec{A}\|^2 \longrightarrow \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\vec{A}\cdot\vec{B} = 0 \longrightarrow \vec{A} \perp \vec{B}$

$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{i}\cdot\hat{k} = 0$

⑨ If the required is to find

$\|\vec{A}+\vec{B}\|$ or $\|\vec{A}-\vec{B}\|$ or $\|\vec{A}+\vec{B}+\vec{C}\|$

we have to get $\|\vec{A}+\vec{B}\|^2$, $\|\vec{A}-\vec{B}\|^2$, $\|\vec{A}+\vec{B}+\vec{C}\|^2$

$$\|\vec{A}+\vec{B}\|^2 = (\vec{A}+\vec{B})\cdot(\vec{A}+\vec{B}) = \|\vec{A}\|^2 + 2(\vec{A}\cdot\vec{B}) + \|\vec{B}\|^2$$

$$\|\vec{A}-\vec{B}\|^2 = (\vec{A}-\vec{B})\cdot(\vec{A}-\vec{B}) = \|\vec{A}\|^2 - 2(\vec{A}\cdot\vec{B}) + \|\vec{B}\|^2$$

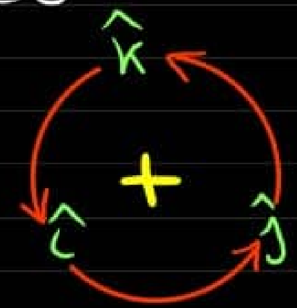
$$\|\vec{A}+\vec{B}+\vec{C}\|^2 = (\vec{A}+\vec{B}+\vec{C})\cdot(\vec{A}+\vec{B}+\vec{C})$$

$$= \|\vec{A}\|^2 + \|\vec{B}\|^2 + \|\vec{C}\|^2 + 2(\vec{A}\cdot\vec{B}) + 2(\vec{A}\cdot\vec{C}) + 2(\vec{B}\cdot\vec{C})$$

## $\boxed{2}$ the vector product

① $\vec{A} \times \vec{B} = (\|\vec{A}\| \, \|\vec{B}\| \sin\theta)\,\hat{k} \longrightarrow \vec{A} \times \vec{A} = 0$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$




② $\sin\theta = \dfrac{\|\vec{A} \times \vec{B}\|}{\|\vec{A}\| \, \|\vec{B}\|}$

③ If $\vec{A} \times \vec{B} = \vec{0}$ then $\vec{A} \parallel \vec{B}$

④ If $\vec{A}, \vec{B}, \vec{C}$ are 3 points on the space and $\vec{AB} \times \vec{BC} = \vec{0}$ then $A, B, C$ are collinear

⑤ If $\vec{A}, \vec{B}$ are 2 adjacent sides of

- a triangle then area of $\Delta = \frac{1}{2} \|\vec{A} \times \vec{B}\|$
- a parallelogram then area of $\square = \|\vec{A} \times \vec{B}\|$

⑥ Unit vector $\perp$ to the plane of $\vec{A}, \vec{B} = \pm \dfrac{\vec{A} \times \vec{B}}{\|\vec{A} \times \vec{B}\|}$

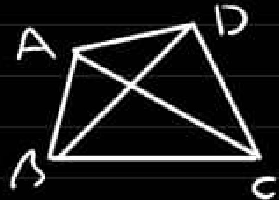
⑦ If $\|\vec{A} \times \vec{B}\| = \vec{A} \cdot \vec{B}$ then the angle between the 2 vectors $\vec{A}, \vec{B}$ equals $45^\circ$


⑧ Area of parallelogram $= \|\vec{AB} \times \vec{AD}\|$

area of $\triangle ABC = \frac{1}{2}\|\vec{AB} \times \vec{AC}\|$

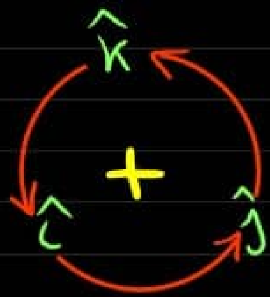
Area of quadrilateral $= \frac{1}{2} \times d_1 \times d_2 \sin\theta = \frac{1}{2}\|\vec{AC} \times \vec{BD}\|$

[illegible]




⑨ $\hat{i} \times \hat{j} = \hat{k}$ , $\hat{j} \times \hat{k} = \hat{i}$

$\hat{j} \times \hat{i} = -\hat{k}$



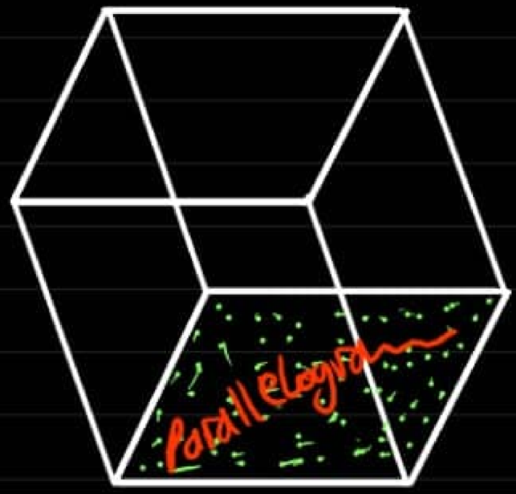

⑩ If $\vec{A} + \vec{B} = \vec{A} \times \vec{B}$ then $\vec{A}, \vec{B}$ one of them additive inverse to the other.

⑪ $\vec{A} \perp \vec{B}$ if $\vec{A} \cdot \vec{B} = 0$

$\vec{A} \parallel \vec{B}$ if
- $\rightarrow \frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2}$
- $\rightarrow \vec{A} \times \vec{B} = \vec{0}$
- $\rightarrow \vec{A} = K\vec{B}$

## 3 Scalar triple Product

$$\vec{A}\cdot\vec{B}\times\vec{C} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

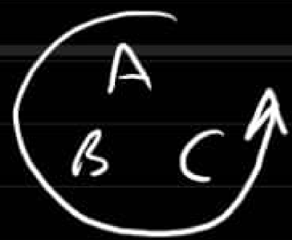



1. volume of parallelpiped $= |\vec{A}\cdot\vec{B}\times\vec{C}|$

$$h = \frac{\text{Volume}}{\text{Base Area}} = \frac{|\vec{A}\cdot\vec{B}\times\vec{C}|}{\|\vec{A}\times\vec{B}\|}$$

2. volume of quadratic Pyramid $= \frac{1}{3}|\vec{A}\cdot\vec{B}\times\vec{C}|$
3. Volume of triangular Pyramid $= \frac{1}{6}|\vec{A}\cdot\vec{B}\times\vec{C}|$

### V.I Remarks




1) $\vec{A}\cdot\vec{B}\times\vec{C} = \vec{B}\cdot\vec{C}\times\vec{A} = \vec{C}\cdot\vec{A}\times\vec{B}$

2) points A, B, C are collinear if $\overline{AB} \parallel \overline{BC}$, B is Common

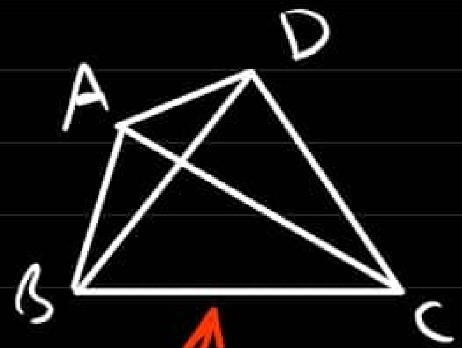
2) points A, B, C are collinear if $\overline{AB} \parallel \overline{BC}$, B is common

3) vectors $\vec{A}, \vec{B}, \vec{C}$ are coplanar if $\vec{A} \cdot \vec{B} \times \vec{C} = 0$

4) $|\vec{A} \cdot \vec{B}|$ = Area of rectangle whose dimensions are $\vec{B}$ and the Alg. Comp of $\vec{A}$ in the direction of $\vec{B}$

5) $\|\vec{A} \times \vec{B}\|$ = area of parallelogram
= 2 area of triangle

$\frac{1}{2}\|\vec{A} \times \vec{B}\|$ = area of triangle




6) $\frac{1}{2}\|\vec{d}_1 \times \vec{d}_2\| = \frac{1}{2}\|\vec{AC} \times \vec{BD}\|$ = area of quadrilateral ABCD

7) $\vec{A} + \vec{B} + \vec{C} = \vec{0}$ if → $\vec{A}, \vec{B}, \vec{C}$ are 3 sides in a triangle in the same cyclic order

Area of $\triangle ABC = \frac{1}{2}\|\vec{A} \times \vec{B}\|$

→ $\vec{A} \times \vec{B} = \vec{C} \times \vec{A}$

# chapter 2 straight lines and planes in space

## Equation of a st. line

1) Vector equation $\boxed{\vec{r} = A + t\vec{d}}$

$(x, y, z) = (x_1, y_1, z_1) + t(a, b, c)$

$\vec{d} = \vec{AB}$ or $\vec{d} = \vec{BA}$

2) Parametric equations

$$\boxed{\begin{aligned} x &= x_1 + at \\ y &= y_1 + bt \\ z &= z_1 + ct \end{aligned}}$$

3) Cartesian equation $\frac{x - x_1}{a} = t$, $\frac{y - y_1}{b} = t$, $\frac{z - z_1}{c} = t$

$$\boxed{\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}}$$

if $a = 0 \longrightarrow x = x_1$, $\frac{y - y_1}{b} = \frac{z - z_1}{c}$

if $b = 0 \longrightarrow y = y_1$, $\frac{x - x_1}{a} = \frac{z - z_1}{c}$

if $c = 0 \longrightarrow z = z_1$, $\frac{x - x_1}{a} = \frac{y - y_1}{b}$

## Special cases

① equation of X-axis is $y = 0$, $z = 0$

equation of st. line // X-axis and passes through the point $(x_1, y_1, z_1)$ is $y = y_1$, $z = z_1$

## Special cases

① equation of X-axis is $y=0,\ z=0$

equation of st. line // X-axis and passes through the point $(x_1, y_1, z_1)$ is $y=y_1,\ z=z_1$

② equation of y-axis is $x=0,\ z=0$

equation of st. line // y-axis and passes through the point $(x_1, y_1, z_1)$ is $x_1=x_2,\ z=z_1$

③ equation of Z-axis is $x=0,\ y=0$

equation of st. line // Z-axis and passes through the point $(x_1, y_1, z_1)$ is $x=x_1,\ y=y_1$

V.I

① st. line parallel to x-axis $\longrightarrow \vec{d}=(1,0,0)$

② st. line parallel to y-axis $\longrightarrow \vec{d}=(0,1,0)$

③ st. line parallel to z-axis $\longrightarrow \vec{d}=(0,0,1)$

④ st. line parallel to xy-plane $\longrightarrow \vec{d}=(a,b,0)$

⑤ st. line parallel to yz-plane $\longrightarrow \vec{d}=(0,b,c)$

⑥ st. line parallel to xz-plane $\longrightarrow d=(a,0,c)$

* st. line makes equal angles with the positive directions of the coordinate axis $\longrightarrow d=(1,1,1)$

* st. line parallel to st. line its direction $\vec{d_1}$ then $\vec{d_2}=K\ \vec{d_1}$

* st. line makes equal angles with the positive directions of the coordinate axis $\longrightarrow \vec{d} = (1,1,1)$

* st. line parallel to st. line its direction $\vec{d_1}$ then $\vec{d_2} = K\,\vec{d_1}$

## Examples

$y-5 \quad z-4.5$

1. $\frac{x-3}{2} = \frac{5-y}{3} = \frac{2z-9}{6} \longrightarrow$ point $(3, 5, 4.5)$, $\vec{d} = (2, -3, 3)$

$x \quad \overset{-3}{y} \quad \overset{3}{z-1}$

2. $\frac{3x}{1} = \frac{4y}{1} = \frac{1-z}{5} \longrightarrow$ point $(0, 0, 1)$, $\vec{d} = (\frac{1}{3}, \frac{1}{4}, -5)$

$1/3 \quad 1/4 \quad -5$

$z + \frac{1}{2} = 0$

3. $y = 5,\ \frac{x-4}{2} = \frac{2z+1}{8} \longrightarrow$ point $(4, 5, -\frac{1}{2})$, $\vec{d} = (2, 0, 4)$

$4$

4. $x = y = z \longrightarrow$ point $(0, 0, 0)$, $\vec{d} = (1, 1, 1)$

5. $x = 3 - 2t,\ y = t,\ z = 5 \longrightarrow$ point $(3, 0, 5)$, $\vec{d} = (-2, 1, 0)$

## The measure of the angle between 2 st. lines in space


① $\cos\theta = \dfrac{|\vec{d_1} \cdot \vec{d_2}|}{\|\vec{d_1}\| \, \|\vec{d_2}\|}$ , $0 \leq \theta \leq 90$ , $\theta$ is acute

② If given is unit vectors $\longrightarrow$ $\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$

③ If $\overleftrightarrow{L_1} \parallel \overleftrightarrow{L_2}$ then
- $\rightarrow \vec{d_1} = K \vec{d_2}$
- $\rightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$
- $\rightarrow \vec{d_1} \times \vec{d_2} = \vec{0}$

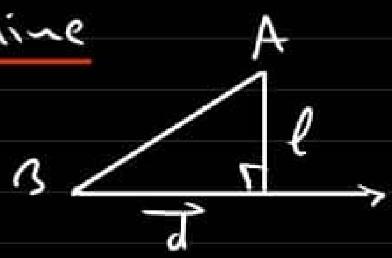
④ $\overleftrightarrow{L_1} \perp \overleftrightarrow{L_2}$ then $\vec{d_1} \cdot \vec{d_2} = 0$

⑤ If $\overleftrightarrow{L_1}$ not parallel to $\overleftrightarrow{L_2}$ then the 2 lines are
- $\rightarrow$ intersecting or
- $\rightarrow$ skew

⑥ If $\overleftrightarrow{L_1} \parallel \overleftrightarrow{L_2}$ and $A \in \overleftrightarrow{L_1}$ , $A \in \overleftrightarrow{L_2}$ then $\overleftrightarrow{L_1}$ and $\overleftrightarrow{L_2}$ are coincident

## the length of the $\perp$ from a point to a given st. line

L of the $\perp$ $= \dfrac{\|\overrightarrow{BA} \times \vec{d}\|}{\|\vec{d}\|}$




**Remark**

To find the relation between a st. line $\vec{r} = A + t\vec{d}$ and the sphere $(x-\ell)^2 + (y-k)^2 + (z-n)^2 = r^2$

1. we get $r$ from the equation of sphere
2. we find the length of the $\perp$ from the center to the st. line
   1. if the $\perp$ dist $= r$ then the st. line is tangent
   2. if the $\perp$ dist $< r$ then the st. line is secant
   3. if the $\perp$ dist $> r$ then the st. line is outside

---

## Division of a line segment

Point D divides $\overline{BC}$ in the ratio $3:4$, $B = (5, 4, -1)$, $C = (-3, 1, 2)$ then $D = (\ldots, \ldots, \ldots)$

**Solution**

$B = (5, 4, -1)$, $D(x, y, z)$, $C = (-3, 1, 2)$

B D C

ratio of division $= \dfrac{BD}{DC} = \dfrac{3}{4} = \dfrac{5-x}{x+3} = \dfrac{4-y}{y-1} = \dfrac{-1-z}{z-2}$

$\therefore x = \frac{11}{7}, y = \frac{19}{7}, z = \frac{2}{7} \longrightarrow D = \left(\frac{11}{7}, \frac{19}{7}, \frac{2}{7}\right)$

# Relations between 2 st. lines in space

## 1 Parallel lines

$\overleftrightarrow{L_1} \parallel \overleftrightarrow{L_2}$ iff
1. $\vec{d_1} = K\,\vec{d_2}$, $K \in \mathbb{R}^*$
2. $\vec{d_1} \times \vec{d_2} = \vec{0}$
3. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

---

## 2 perpendicular lines

$\overleftrightarrow{L_1} \perp \overleftrightarrow{L_2}$ iff $\vec{d_1} \cdot \vec{d_2} = 0 \rightarrow a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

---

## 3 Intersecting lines

If $\vec{r_1} = A + t_1 \vec{d_1}$, $\vec{r_2} = B + t_2 \vec{d_2}$

and if $\overrightarrow{AB} \cdot \vec{d_1} \times \vec{d_2} = 0$ then the 2 st. lines ly in the same plane

if $\overrightarrow{AB} \cdot \vec{d_1} \times \vec{d_2} \neq 0$ then the lines are skew not contained in the same plane

1. Try condition of parallelism $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

if not parallel they are intersecting

2. If they are parallel and have common point then they are concident

---

## 4 skew lines iff $\overrightarrow{AB} \cdot \vec{d_1} \times \vec{d_2} \neq 0$

**V.I Remark** To find the intersection point between the 2 st. lines

$$\vec{r_1} = (x_1, y_1, z_1) + t_1(a_1, b_1, c_1)$$
$$\vec{r_2} = (x_2, y_2, z_2) + t_2(a_2, b_2, c_2)$$

First
$$\left.\begin{aligned} x_1 - x_2 &= t_2 a_2 - t_1 a_1 \\ y_1 - y_2 &= t_2 b_2 - t_1 b_1 \\ z_1 - z_2 &= t_2 c_2 - t_1 c_1 \end{aligned}\right\}$$

second: solve any 2 equations to find $t_1, t_2$

third:
- If $t_1, t_2$ satisfies the third equation we get the intersection point
- If $t_1, t_2$ not satisfies the third equation then the 2 lines are skew

**V.I**
1. direction of X-axis is $\hat{i} = (1, 0, 0)$
2. direction of xy plane is $(a, b, 0)$
3. $x = 3, y = 8 \rightarrow$ st.line // z-axis $\perp$ xy-plane
4. $\frac{x-5}{2} = \frac{y-7}{2}, z = 4 \rightarrow$ st.line // xy plane

* equation of st-line // X-axis $\rightarrow y = \text{cont}, z = \text{cont}$
* equation of st-line // xy plane $\rightarrow z = \text{constant}$

eg: the equation of st-line passes through $(2, 3, 5)$
- // X-axis $\rightarrow y = 3, z = 5$
- // xy plane $\rightarrow \frac{x-2}{a} = \frac{y-3}{b}, z = 5$

# Ideas of equation of st. line



$\boxed{1}$ $* \; x = 0 , y = z \rightarrow \vec{d} = (0,1,1)$

because

عزلناها $x = 0 , \frac{y}{1} = \frac{z}{1}$

$y = 0 , x = z \rightarrow \vec{d} = (1,0,1)$

$\boxed{2}$ $\underset{\text{const}}{x = 1} , \underset{\text{const}}{y = 5} \rightarrow \vec{d} = (0, 0, \overset{\text{أي؟}}{c})$ because z is variable

parametric eq. عزلناهم st. line // Z-axis

$x = 2 , z = -1 \rightarrow \vec{d} = (0, b, 0) \rightarrow$ // y-axis

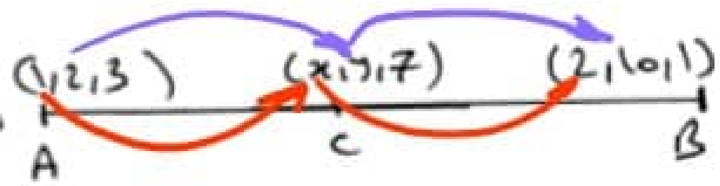
$\boxed{3}$ $3x + 2y = 7 , \underset{\text{عزلناها}}{z = 3} \rightarrow \vec{d} = (2, -3, 0)$

$\boxed{4}$ $2x = 3y = -z$

$, \vec{d} = (-3, -2, 6)$

$\boxed{5}$ $\cos\theta_z = \frac{|A_z|}{\|\vec{A}\|}$

$\boxed{6}$ find c




ratio $= \frac{3-7}{7-1} = \frac{-2}{3} = \frac{1-x}{x-2} = \frac{2-y}{y-10}$

$x = -1$ $\quad y = -14$

③ $3x + 2y = 7, \quad z = 3$

$y - 3.5$

$$\frac{3x}{1/3} = \frac{7 - 2y}{-1/2}, \quad z = 3$$

$$\vec{d} = \left(\frac{1}{3}, \frac{-1}{2}, 0\right)$$

$$\vec{d} = (2, -3, 0)$$

④ $2x = 3y = -z$

$$\frac{x}{1/2} = \frac{y}{1/3} = \frac{z}{-1}$$

$(0, 0, 0)$

$\left(\frac{1}{2}, \frac{1}{3}, -1\right)$

$(3, 2, -6)$

**68** If the two straight lines $L_1 : \overrightarrow{r_1} = (1\ ,2\ ,3) + t_1\ (1\ ,2\ ,3)$ and $L_2 : \overrightarrow{r_2} = (-1\ ,2\ ,0) + t_2\ (m\ ,2\ ,-1)$ are intersecting , then m = ...............

(a) $\frac{-8}{3}$ (b) $\frac{-5}{3}$ (c) zero (d) $-3$

$A = (1, 2, 3)$ , $\overrightarrow{d_1} = (1, 2, 3)$

$B = (-1, 2, 0)$ , $\overrightarrow{d_2} = (m, 2, -1)$

intersecting $\rightarrow$

$\overrightarrow{AB} \cdot \overrightarrow{d_1} \times d_2 = 0$

$\overrightarrow{AB} = (-2, 0, -3)$

$$\begin{vmatrix} -2 & 0 & -3 \\ 1 & 2 & 3 \\ +m & -2 & +(-1) \end{vmatrix} = 0$$

$m \times 6 - 2 \times -3 - 1 \times -4 = 0$

$6m + 6 + 4 = 0$

$6m = -10$

$m = -\frac{10}{6} = \frac{-5}{3}$

59 The two straight lines $x = y = z$ , $2x = 3y = z$ are ............

~~(a) skew.~~

(b) intersecting and perpendicular.

~~(c) parallel.~~

(d) intersecting but not perpendicular.

$\frac{x}{1} = \frac{y}{1} = \frac{z}{1} \rightarrow (0,0,0)$ , $\vec{d_1} = (1,1,1)$

$A$

$\frac{2x}{1} = \frac{3y}{1} = \frac{z}{1}$

$B$

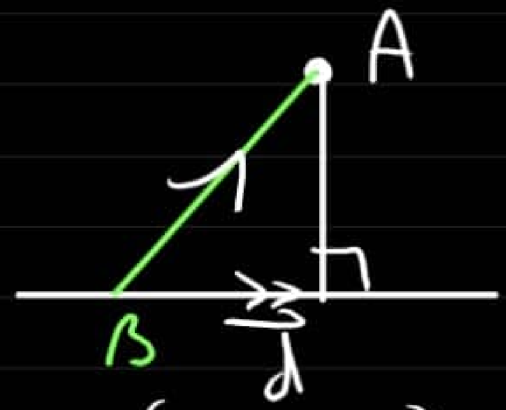
$\frac{x}{\frac{1}{2}} = \frac{y}{\frac{1}{3}} = \frac{z}{1} \rightarrow (0,0,0)$ , $\vec{d_2} = (\frac{1}{2}, \frac{1}{3}, 1)$

$\vec{d_1} \cdot \vec{d_2} = 1 \times \frac{1}{2} + 1 \times \frac{1}{3} + 1 \times 1 \neq 0$ not $\perp$

77 The length of the perpendicular drawn from the point $(-1, 0, 1)$ to the straight line $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z+1}{-1}$ equals ............... length units.

(a) $\sqrt{30}$ (b) $\sqrt{6}$ (c) $\frac{\sqrt{30}}{6}$ (d) $2\sqrt{6}$

L of the $\perp$ from $A(-1,0,1) = \dfrac{\|\overrightarrow{BA} \times \vec{d}\|}{\|\vec{d}\|}$




$$\overrightarrow{BA} \times \vec{d} = \begin{vmatrix} i & j & k \\ -2 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix}$$

same opp same

$= (-1, 2, 0)$

$= \dfrac{\sqrt{5}}{\sqrt{6}}$

$= \dfrac{\sqrt{30}}{6}$

$\vec{B} = (1, 1, -1)$

$\overrightarrow{BA} = (-2, -1, 2)$

80 If the length of the perpendicular drawn from the point (2 , – 1 , m) on the straight line

$x - 1 = 3 - y = z$ equals 5 length unit , then number of possible values of m equals ............

$\frac{x-1}{1} = \frac{y-3}{-1} = \frac{z}{1}$

(a) zero (b) 1 (c) 2 (d) infinite number.

L of the ⊥ from $A = (2, -1, m) = \frac{\|\overrightarrow{BA} \times \vec{d}\|}{\|\vec{d}\|}$

$B = (1, 3, 0)$

$\overrightarrow{BA} = (1, -4, m)$

$\vec{d} = (1, -1, 1)$

$\|\vec{d}\| = \sqrt{3}$

$\overrightarrow{BA} \times \vec{d} = \begin{vmatrix} i & j & k \\ 1 & -4 & m \\ 1 & -1 & 1 \end{vmatrix}$

$= (-4 + m, m - 1, 3)$

$\frac{\|\overrightarrow{BA} \times \vec{d}\|}{\|\vec{d}\|} = \frac{\sqrt{(-4+m)^2 + (m-1)^2 + 9}}{\sqrt{3}} = 5 \rightarrow$

$m = -3.4$

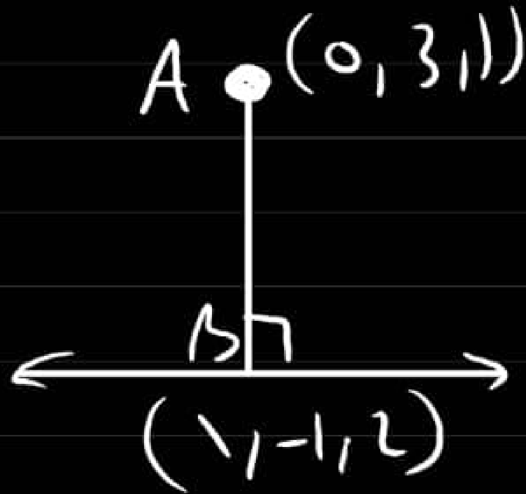
$m = 8.04$

82 If the point (1 , – 1 , 2) is the project of the point (0 , 3 , 1) on the straight line $\frac{x-a}{\ell} = \frac{y-2}{3} = \frac{z-b}{4}$ , then a + b = ............

(a) 8 (b) 13 (c) 14 (d) 15

$\overrightarrow{AB} = (1, -4, 1)$ , $\vec{d} = (\ell, 3, 4)$

$\overrightarrow{AB} \cdot \vec{d} = 0 \rightarrow \ell - 12 + 4 = 0 \rightarrow \ell = 8$




$\frac{x-a}{8} = \frac{y-2}{3} = \frac{z-b}{4}$

$\frac{1-a}{8} = \frac{-1-2}{3} = \frac{2-b}{4} \Rightarrow a = 9$

$b = 6$

the proj of the point $(-1, 2, 5)$ on the st. line $\vec{r} = (3, 4, 5) + t(1, 7, -1)$ is ......

## Solution

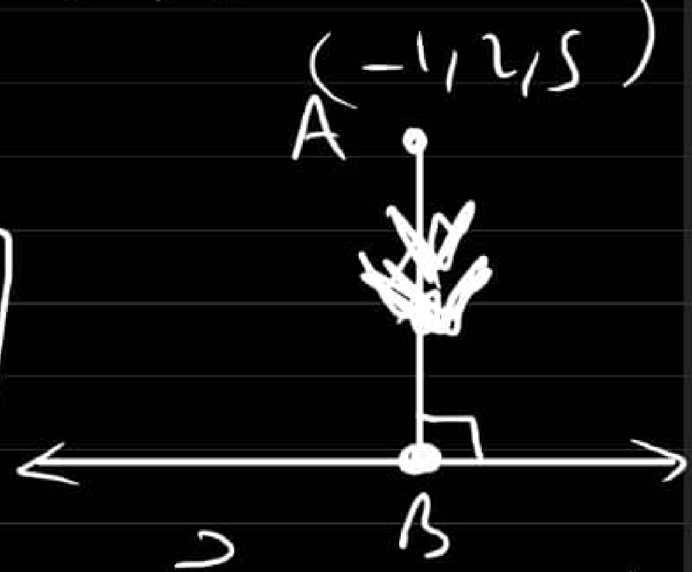
let $B = (3+t, 4+7t, 5-t)$

$\vec{AB} = (\underline{t+4}, 7t+2, -t)$

$\vec{AB} \cdot \vec{d} = t + 4 + 49t + 14 + t = 0$

$t = \frac{-6}{17}$

$B = \left(\frac{45}{17}, \frac{26}{17}, 91\right)$

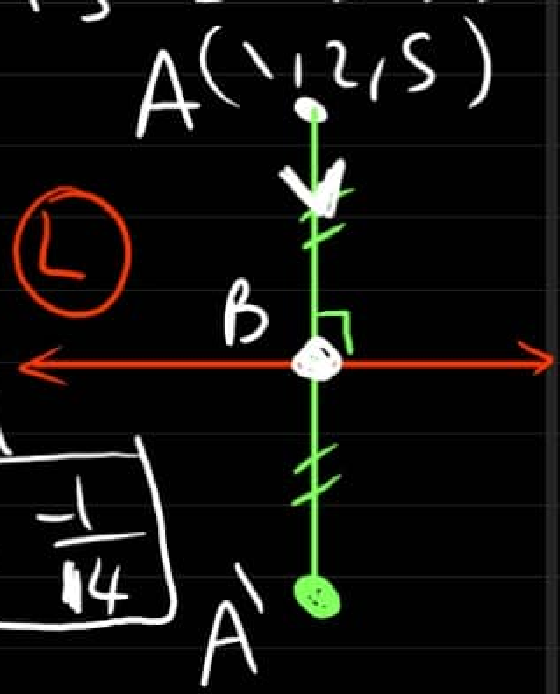



$\vec{d} = (1, 7, -1)$

the image of the point $(1, 2, 5)$ by reflection on the st. line $\frac{x-1}{2} = \frac{4-y}{-3} = \frac{z}{1} = t$ is ....

$\frac{y-4}{3}$

## Solution




$x = 2t+1$

$y = 3t+4 \quad B = (2t+1, 3t+4, t)$

$z = t$

$\overrightarrow{AB} = (2t, 3t+2, t-5), \quad \vec{d} = (2, 3, 1)$

$\overrightarrow{AB} \cdot \vec{d} = 4t + 9t + 6 + t - 5 = 0 \Rightarrow \boxed{t = \frac{-1}{14}}$

$\vec{B} = \left( \frac{6}{7}, \frac{53}{14}, \frac{-1}{14} \right)$

$A' = 2B - A = \left( \frac{5}{7}, \frac{39}{7}, \frac{-36}{7} \right)$

If the angle between the two st. lines

$L_1: \frac{x}{a} = \frac{y}{2} = z$ and $L_2: \frac{x}{2} = \frac{y}{1} = \frac{z}{-1}$ is $60^\circ$

then $a = \ldots\ldots$, $a \in R^+$

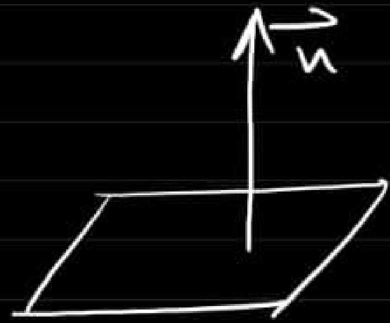
Solution

$$\cos\theta = \frac{|\vec{d_1} \cdot \vec{d_2}|}{\|\vec{d_1}\| \, \|\vec{d_2}\|} = \frac{|(a, 2, 1) \cdot (2, 1, -1)|}{\sqrt{a^2+4+1}\,\sqrt{4+1+1}} = \frac{1}{2}$$

$$\frac{|2a+1|}{\sqrt{6}\sqrt{a^2+5}} = \frac{1}{2} \rightarrow a = 1$$

$= \ominus$ refused.

# The equation of the plane in space




① vector form: $\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}$ , $\vec{n} \perp$ the plane

$(a, b, c) \cdot \vec{r} = (a, b, c) \cdot (x_1, y_1, z_1)$

② standard form: $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

③ general form: $ax + by + cz = d$ $\quad 2x + 3y - 3z = 8$

$d = ax_1 + by_1 + cz_1$

eg ① equation of the plane passes through the point $A(1, -2, 5)$ and $\vec{n} = (2, 1, 3) \perp$ to it is....

solution

$\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}$

$(2, 1, 3) \cdot \vec{r} = (2, 1, 3) \cdot (1, -2, 5)$

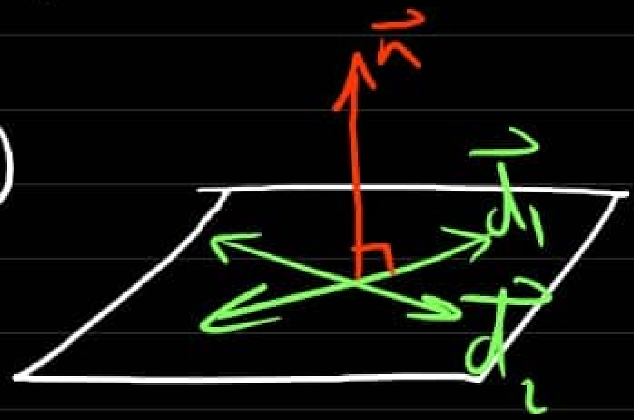
$(2, 1, 3) \cdot \vec{r} = 15 \rightarrow 2x + y + 3z = 15$

eg (2) equation of plane contains the 2 st. lines

$L_1 : \frac{x+1=0}{-1} = \frac{y-2=0}{-1} = \frac{z-1=0}{3}$, $L_2 : \frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-1}{-1}$

is . . . . . .

Solution




$\vec{n} = \vec{d_1} \times \vec{d_2} = (-1,-1,3) \times (1,-2,-1)$

$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 3 \\ 1 & -2 & -1 \end{vmatrix} = (7,2,3)$

$\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}$

$(7,2,3) \cdot \vec{r} = (7,2,3) \cdot (-1,2,1)$

$7x + 2y + 3z = 0$

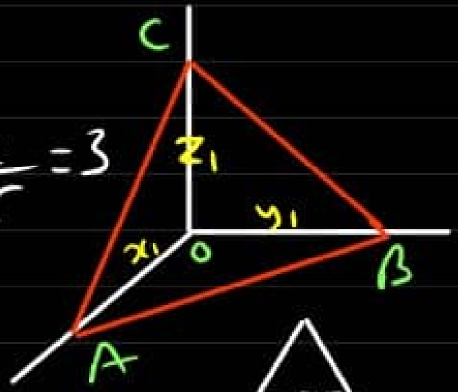
## V.I Remarks

① If the plane cuts the coordinate axis at the points $(x_1, 0, 0)$, $(0, y_1, 0)$, $(0, 0, z_1)$ then the

* equation of the plane is $\frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = 1$

* the centroid of $\triangle ABC$ is $(p, q, r)$

  equation of the plane is $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$




② * Volume of the Pyramid $OABC = \frac{1}{6}|x_1 y_1 z_1|$

$V = \frac{1}{3}$ base area $\times h$


* Area of $\triangle ABC = \frac{1}{2} \| \overrightarrow{AB} \times \overrightarrow{AC} \|$

$$= \frac{1}{2}\sqrt{(x_1 y_1)^2 + (y_1 z_1)^2 + (z_1 x_1)^2}$$

* height from O to plane ABC $= \dfrac{|x_1 y_1 z_1|}{\sqrt{(x_1 y_1)^2 + (y_1 z_1)^2 + (z_1 x_1)^2}}$

$$h = \frac{3 \times V}{\text{base area}}$$

(3) If the equation of the plane is $ax+by+cz=d$

(1) $d=0$ $\longrightarrow$ the plane passes through the origin

(2) $a=0$ $\longrightarrow$ the plane parallel to $x$-axis and $\perp$ to $yz$-plane

(3) $d=0$, $a=0$ $\longrightarrow$ the plane contains $x$-axis and $\perp$ to $yz$-plane

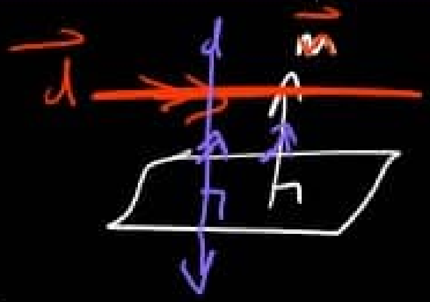
(4) $x=5$ is a plane parallel to $yz$ plane, $\perp$ to $x$-axis

$x=0$ is the equation of $yz$ plane, $\perp$ to $x$-axis

# Relations between st.line and plane

① st.line // plane

$\vec{d} \cdot \vec{n} = 0$




② st.line ⊥ plane

$\vec{d} \,//\, \vec{n} \longrightarrow$
- $\vec{d} = K\vec{n}$
- $\vec{d} \times \vec{n} = \vec{0}$
- $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

## To find the relation between the st.line and the plane

① $\vec{r} = (x_1, y_1, z_1) + t(a, b, c)$

$\hookrightarrow \vec{r} = (x_1 + at, y_1 + bt, z_1 + ct)$

② substitute in the plane equation

1) if $t$ has only one value the the st.line intersects the plane

2) If $t$ has more than one value then the st.line is contained in the plane

3) If $t$ has no real value then $\vec{L}$ // the plane

## To find the intersection between the st.line and the plane

we solve the 2 equations

① $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = t \longrightarrow \begin{aligned} x &= at + x_1 \\ y &= bt + y_1 \\ z &= ct + z_1 \end{aligned}$

② Substitute in the plane equation to get the value of $t$

③ Sub by the value of $t$ in the st.line equation

we get the intersection point.

## ③ Two planes

let the two planes

$a_1x + b_1y + c_1z = d_1$ , $a_2x + b_2y + c_2z = d_2$

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then they are intersecting at a line

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{d_1}{d_2}$ they are concident

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \neq \frac{d_1}{d_2}$ they are parallel

1 the equation of the plane passing through the point $(1,-2,5)$ and its normal vector $(2,1,3)$ is....

ⓐ $2x+y+3z=1$

ⓑ $2x+y+3z=15$

ⓒ $x-2y+5z=15$

ⓓ $x+y+z=4$

$\vec{n}\cdot\vec{r}=\vec{n}\cdot\vec{A}$

$(2,1,3)\cdot(x,y,z)=15$

$2x+y+3z=15$

2 Equation of the plane containing the 2 st. lines

$L_1: \frac{x+1}{-1} = \frac{y-2}{-1} = \frac{z-1}{3}$ , $L_2: \frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-1}{-1}$ is...

$x+1=0 \quad y-2=0 \quad z-1=0$

point

$(-1, 2, 1)$

$\vec{n}$

$\vec{d_1} \times \vec{d}$

$(-1,-1,3) \times (1,-2,-1)$

$(7, 2, 3)$

(a) $5x - 4y + 3z = 7$

(b) $2x + y + z = 1$

(c) $7x - 5y - z = 4$

(d) $7x + 2y + 3z = 0$

3 Equation of the plane containing the 2 st. lines

$L_1: x = 4 - 2t_1, \; y = 3 + t_1, \; z = 1 + 3t_1$ → $A = (4, 3, 1)$

$L_2: x = 5 + 2t_2, \; y = 1 - t_2, \; z = 1 - 3t_2$ is ... $B = (5, 1, 1)$

(a) $x + y + z + 2 = 0$

(b) $2x + y + z = 36$

(c) $6x + 3y + 3z = 36$

(d) $x - 2y + 3z = 1$

Point

$(4, 3, 1)$

$\vec{n}$

$\vec{d_1} \times \vec{d_2}$

$\vec{AB} \times (2, -1, -3)$

$(1, -2, 0) \times (2, -1, -3)$

$\vec{n} = (6, 3, 3)$

$\vec{n} = (2, 1, 1)$

$\vec{n} \cdot \vec{A}$

$(6, 3, 3) \cdot (4, 3, 1)$

$= 36$

$\boxed{4}$ the equation of the plane passing through the points

$A(2,3,5)$, $B=(-1,3,1)$, $c=(4,3,-2)$ is . . . .

(a) $x+y+z=0$

(b) $x=1$

(c) $y=3$

(d) $z=-3$

$$-\begin{vmatrix} x-2 & y-3 & z-5 \\ -3 & 0 & -4 \\ 5 & 0 & -3 \end{vmatrix}$$

$$-(y-3)(9+20)=0$$

$$y=3$$

5 Equation of the plane containing the st. line

$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ and the point $(2,-3,4)$ is ....

A

(a) $2x+y+z=0$

(b) $2x+z=0$

(c) $-2x+z=0$

(d) $2x-y+z=0$

Point

$A(2,-3,4)$

$B(1,-1,2)$

$\vec{n}$

$\vec{d_1} \times \vec{AB}$

$(2,3,4)\times(-1,2,-2)$

$\vec{n}=(-14,0,7)$

$\vec{n}=(-2,0,1)$

$\vec{n}\cdot\vec{r}=\vec{n}\cdot\vec{A}$

$(-2,0,1)\cdot(x,y,z)=0$

---

6 the general equation of the plane whose equation

$\vec{r}=(2,3,5)+t_1(-1,3,4)+t_2(6,1,-2)$ is. ......

(a) $10x-22y+19z=49$

(b) $22x-10y+19z=49$

(c) $19x-22y+10z=9$

(d) $19x-10y+22z=49$

Point $(2,3,5$

$\vec{n}=\vec{d_1}\times\vec{d_2}$

$\vec{n}=(-1,3,4)\times(6,1,-2)$

$\vec{n}=(10,-22,19)$

7 the general equation of the plane whose equation

$x = 2 + 2t_1 - 4t_2$ , $y = 4 - t_2$ , $z = 2 + 3t_1$ is....

a) $3x - 12y - 2z + 46 = 0$

b) $2x + 4y + 2z = 8$

c) $4x + y - 3z = 16$

d) $4x - 3y + 7z + 45 = 0$

Point $(2, 4, 2)$

$\vec{n} = \vec{d_1} \times \vec{d_2}$

$= (2, 0, 3) \times (-4, -1, 0)$

$= (3, -12, -2)$

8 Equation of the plane which touches the sphere $x^2+y^2+z^2=9$ at the point $(2,\overset{A}{,}-1,2)$ is...

(a) $2x-y+2z=3$

(b) $x+y+z=9$

(c) $2x-y+2z=9$ ✓

(d) $x+y+z=3$

point

$(2,-1,2)$

$\vec{n}$

$(2,-1,2)$


$M=(0,0,0)$, $A=(2,-1,2)$

$\vec{MA}=(2,-1,2)=\vec{n}$

$\vec{n}\cdot\vec{A}=(2,-1,2).(2,-1,2)$

9) Vector $\vec{A}$ makes angle $45°$ with x-axis, 60 with y-axis, acute angle with z-axis, $\|\vec{A}\| = 8$ units, If $\vec{A}$ perpendicular to the plane passing through the point $(\sqrt{2}, -1, 1)$ then the vector form of the equation of the plane is...

(a) $\vec{r} \cdot (4, 4\sqrt{2}, 4) = 8$

(b) $\vec{r} \cdot (1, \sqrt{2}, 1) = 2$

(c) $\vec{r} \cdot (\sqrt{2}, 1, 1) = 8$

(d) $\vec{r} \cdot (\sqrt{2}, 1, 1) = 2$

$\theta_x = 45, \ \theta_y = 60, \ \theta_z = ?$

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\theta_z = 60 \rightarrow \cos\theta_z = \frac{1}{2}$

$\vec{A} = \|\vec{A}\| \times \vec{u}_A = 8(\cos 45, \cos 60, \cos 60)$

$\vec{A} = (4\sqrt{2}, 4, 4) = \vec{n}$

# Remarks

① If plane A parallel to plane B then $\vec{n_1} = K\vec{n_2}$

② If plane A perpendicular to each of plane B, plane C then plane A // $\vec{n_1}, \vec{n_2}$     $\vec{n_1} \quad \propto \vec{n_2}$

$\therefore$ $\vec{n}$ of plane A $= \vec{n_1} \times \vec{n_2}$

③ If plane A perpendicular to the st. line $\overleftrightarrow{CD}$ then $\vec{n} \parallel \vec{CD} \longrightarrow \vec{n} = K\,\vec{CD}$

④ If plane A // $\overleftrightarrow{L_1}, \overleftrightarrow{L_2}$ then

$\vec{n} \perp$ to the plane containing $\overleftrightarrow{L_1}, \overleftrightarrow{L_2}$

$\vec{n} = K(\vec{d_1} \times \vec{d_2})$

1. Equation of the plane passes through the point $c(-1,2,1)$ and perpendicular to the st. line passes through the 2 points $A(-3,1,2)$, $B=(2,3,4)$ is ....

point $c(-1,2,1)$

$\vec{n} = \vec{AB} = (5,2,2)$

(a) $5x+2y+2z=1$

(b) $x+2y+2z=10$

(c) $-x+2y+z=7$

(d) $2x+3y+4z=5$

$\boxed{2}$ Equation of the plane containing the st. line $L_1: \vec{r} = (1,2,4) + t_1(4,1,1)$ and perpendicular to the st. line $L_2: \vec{r} = (4,15,8) + t_2(2,3,-1)$ is....

ⓐ $2x + 3y - z = 4$

ⓑ $4x + y + z = 39$

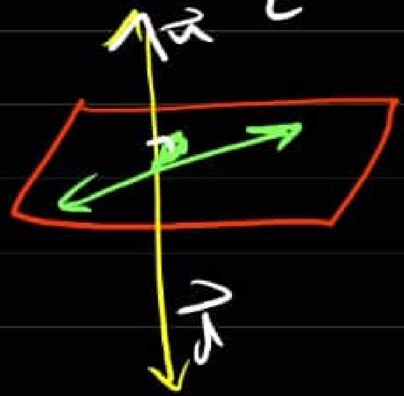
ⓒ $x + 2y + 4z = 15$

ⓓ $2x + 3y - z + 4 = 0$

$A(1,2,4)$

$B(4,15,8)$

$\vec{n} = \vec{d_2} = (2,3,-1)$



3 Equation of the plane passes through the point A(1, -1, 2) and parallel to the plane $x-2y+2z=1$ is...

ⓐ $x-2y+2z=3$

ⓑ $x-2y+2z=5$

ⓒ $x-2y+2z=7$ ✓

ⓓ $x-2y+2z=0$

Point $(1,-1,2)$

$\vec{n}=(1,-2,2)$

4 Equation of the plane contains the st. line

$L_1: \vec{r} = (0,3,-5) + t_1(6,-2,-1)$ parallel to the

st. line $L_2: \vec{r} = (1,7,-4) + t_2(1,-3,3)$ is ....

(a) $x - 2y - z = 5$

(b) $x - 3y + 3z = 7$

(c) $x + y + z = 5$

(d) $9x + 19y + 16z + 23 = 0$

Point

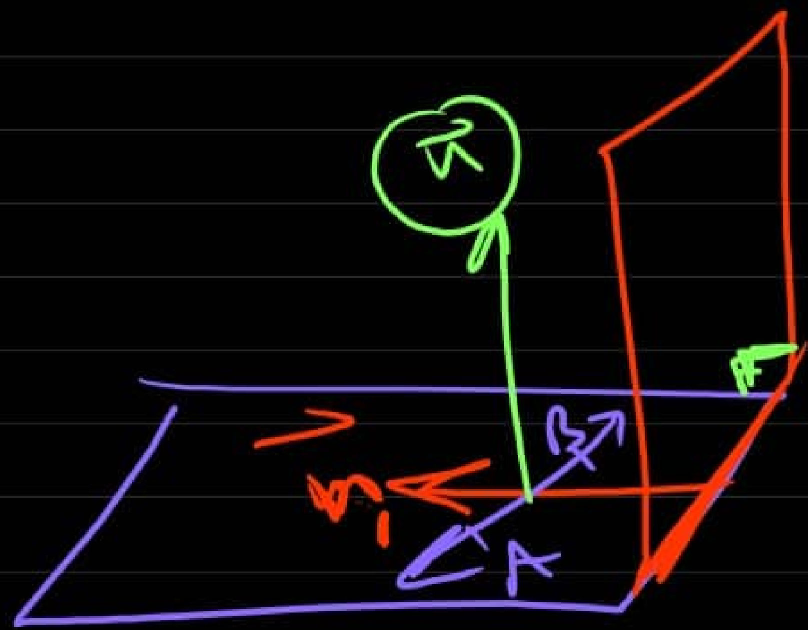
$(0,3,-5)$


$\vec{n} = \vec{d}_1 \times \vec{d}_2$

$= (-9,-19,-16)$

$= (9,19,16)$

N.B Remark



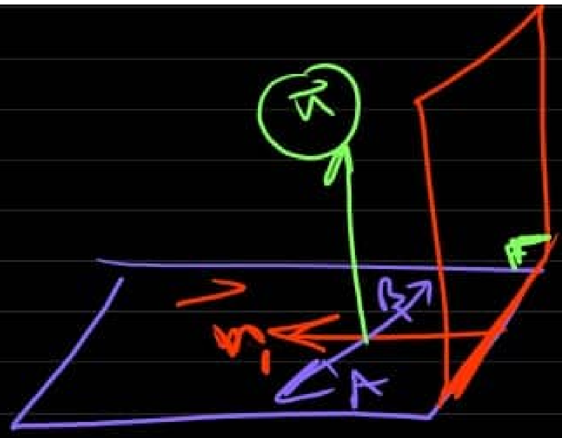

given

2 perpendicular planes

$\therefore$ $\vec{n_1}$ is included in the other plane

$\therefore$ $\vec{n} = \vec{n_1} \times \vec{AB}$

## V.I Remark




given

2 perpendicular planes

$\therefore$ $\vec{n}_1$ is included in the other plane

$\therefore$ $\vec{n} = \vec{n}_1 \times \vec{AB}$

$\boxed{5}$ Equation of the plane passes through the 2 points $A(1,2,-3)$, $B=(2,3,-4)$ and perpendicular to the plane $\overset{\vec{n}=(1,1,1)}{x+y+z+1=0}$ is ......

(a) $x-y+1=0$

(b) $x+2y-2z+1=0$

(c) $2x+3y-4z+1=0$

(d) $x+y+z=0$

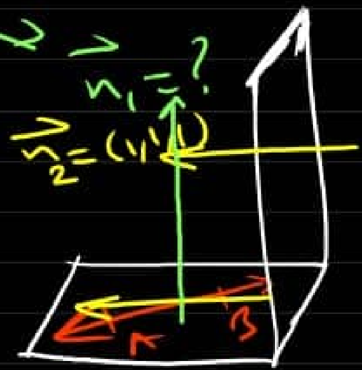
point

$A(1,2,-3)$

$B(2,3,-4)$

$\vec{AB}=(1,1,-1)$

Required plane // $(1,1,1)$




$\vec{n}_1 = \vec{n}_2 \times \vec{AB}$
$= (2,-2,0)$

6 Equation of the plane contains the st. line $x = \frac{y-3}{2} = \frac{z-5}{3}$ and is perpendicular to the plane $2x+7y-3z=1$ is ........

(a) $-9x+3y+z=14$

(b) $2x+7y-3z=6$

(c) $x+2y+3z=6$

(d) $3y+5z=8$

$\vec{n}_2$ included in the first plane

$\vec{n}_2 = (2,7,-3)$

$\vec{d}_1 = (1,2,3)$

$\vec{n} = \vec{n}_2 \times \vec{d}_1 = (-27,9,3)$

$= (-9,3,1)$

7 Equation of the plane passing through the 2 points $A(1,-2,5)$, $B=(2,2,3)$ and parallel to the direction $\vec{d}=(3,-1,0)$ is . . .

(a) $2x+6y+13z=55$

(b) $x-2y+5z=4$

(c) $2x+2y+3z=7$

(d) $3x-y=$ zero

$\therefore (3,-1,0)$ is included in the plane.

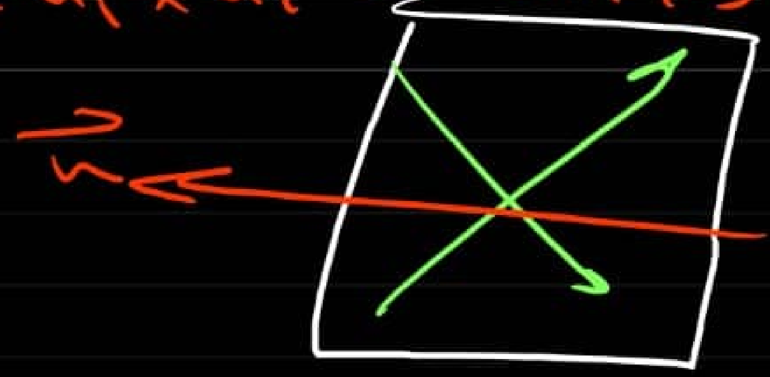
$$\vec{n}=\vec{d}\times\vec{AB}$$
$$=(3,-1,0)\times(1,4,-2)$$
$$=(2,6,13)$$

$\boxed{8}$ the equation of the plane passing through the point $(1,2,3)$ and parallel to the 2 directions $\vec{u}_1 = (2,1,-1)$, $\vec{u}_2 = (3,6,-2)$ is ....

(a) $4x+y+9z=33$

$\vec{n} = \vec{u}_1 \times \vec{u}_2 = (4,1,9)$

(b) $4x+y+9z=0$




(c) $x+2y+3z=33$

(d) $2x+y-z=33$

V. I Remark plane $\perp$ to 2 planes

$\therefore \vec{n}_1, \vec{n}_2$ included in the req. plane

$\therefore \vec{n} = \vec{n}_1 \times \vec{n}_2$

V.I Remark plane $\perp$ to 2 planes

$\therefore \vec{n}_1, \vec{n}_2$ included in the req. plane

$\therefore \vec{n} = \vec{n}_1 \times \vec{n}_2$

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[9] the equation of the plane passing through the Point $(1,4,3)$ and perpendicular to the 2 planes $2x+4y+7z=4$, $x-y+2z=3$

(a) $5x+y-2z=3$

(b) $x+4y+3z=10$

(c) $2x+4y+7z=15$

(d) $x-y+2z=13$

$\vec{n}_1, \vec{n}_2$ included in the required plane

$\vec{n} = \vec{n}_1 \times \vec{n}_2$

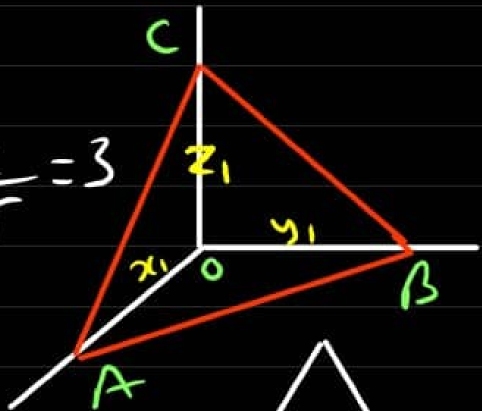
$(15,3,-6) = (5,1,-2)$

## V.I Remarks

① If the plane cuts the coordinate axis at the points $(x_1, 0, 0), (0, y_1, 0), (0, 0, z_1)$ then the

* equation of the plane is $\frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = 1$
* the centroid of $\Delta ABC$ is $(p, q, r)$

  equation of the plane is $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$




② * Volume of the Pyramid $OABC = \frac{1}{6}|x_1 y_1 z_1|$

$V = \frac{1}{3}$ base area $\times h$


* Area of $\Delta ABC = \frac{1}{2}\|\overrightarrow{AB} \times \overrightarrow{AC}\|$

$$= \frac{1}{2}\sqrt{(x_1 y_1)^2 + (y_1 z_1)^2 + (z_1 x_1)^2}$$

* height from O to plane $ABC = \frac{|x_1 y_1 z_1|}{\sqrt{(x_1 y_1)^2 + (y_1 z_1)^2 + (z_1 x_1)^2}}$

$h = \frac{3 \times V}{\text{base area}}$

* height from O to plane ABC $= \dfrac{|x_1 y_1 z_1|}{\sqrt{(x_1 y_1)^2 + (y_1 z_1)^2 + (z_1 x_1)^2}}$

↳ $h = \dfrac{3 \times V}{\text{base area}}$

③ If the equation of the plane is $ax + by + cz = d$

① $d = 0$ ⟶ the plane passes through the origin

② $a = 0$ ⟶ the plane parallel to x_axis and ⊥ to yz-plane

③ $d = 0, a = 0$ ⟶ the plane contains x-axis and ⊥ to yz-plane

④ $x = 5$ is a plane parallel to yz plane, ⊥ to x_axis

$x = 0$ is the equation of yz plane, ⊥ to x-axis

① the equation of the plane passing through the points $A(0,0,-4)$, $B=(0,5,0)$, $C=(-2,0,0)$ is

ⓐ $2x-5y+4z=0$

ⓑ $10x+4y-5z=20$

ⓒ $10x-4y+5z+20=0$

ⓓ $a(x+2)+b(y-5)+c(z+4)=0$

$$\frac{x}{-2}+\frac{y}{5}+\frac{z}{-4}=1$$

$x-20$

① the equation of the plane passing through the points $A(0,0,-4)$, $B=(0,5,0)$, $C=(-2,0,0)$ is

ⓐ $2x-5y+4z=0$

ⓑ $10x+4y-5z=20$

ⓒ $10x-4y+5z+20=0$

ⓓ $a(x+2)+b(y-5)+c(z+4)=0$

$$\frac{x}{-2}+\frac{y}{5}+\frac{z}{-4}=1$$

$x-20$

② equation of the plane that intersects equal parts of the coordinate axis is.

ⓐ $x+y+z=a$

ⓑ $x+y-2z=5$

ⓒ $2x+y-z=2$

ⓓ $-x+2y+z=3$

$$\frac{x}{a}+\frac{y}{a}+\frac{z}{a}=1$$

$\times a$

$x+y+z=a$

③ If the plane $\frac{x}{4}+\frac{y}{2}+\frac{z}{2}=1$ intersects the coordinate Axis at the points A, B, C then the area of $\Delta ABC = \ldots\ldots$

ⓐ 4

ⓑ 6

ⓒ 10

ⓓ 12

$A(4,0,0)$ , $B(0,2,0)$ , $C=(0,0,2)$

$$\text{Area} = \frac{1}{2}\sqrt{(4\times 2)^2+(2\times 2)^2+(4\times 2)^2}$$

$$=6$$

---

④ If the plane $20x+15y+12z=60$ intersects the coordinate axis at the points A, B, C then the volume of the pyramid ABCO $= \ldots\ldots$

ⓐ 10

ⓑ 30

ⓒ 60

ⓓ 90

$$\frac{20x}{60}+\frac{15y}{60}+\frac{12z}{60}=\frac{60}{60} \rightarrow \div 60$$

$$\frac{x}{3}+\frac{y}{4}+\frac{z}{5}=1$$

$$V=\frac{1}{6}\left|3\times 4\times 5\right|=10$$

# Remarks

the general form of the equation is

$ax + by + cz + d = 0$ then:

1. If $d = 0$ then the plane passes through the origin

   eg. $2x + 3y + 2z = 0$

2. If $a = 0$ then the plane // x-axis and $\perp$ the plane $yz$

   eg: $5y - 2z + 7 = 0$

3. If $d = 0$, $a = 0$ then the plane contains X-axis, $\perp$ plane $yz$

   $5y - 2z = 0$

Eg:

① $x = 5$ is a plane parallel to $yz$ plane
   $y = 2$ " " " " $xz$ plane
   $z = 3$ " " " " $xy$ plane

② $x = 0$ is the equation of $yz$ plane
   $y = 0$ is the equation of $xz$ plane
   $z = 0$ is the equation of $xy$ plane

③ $2x + 3y + 4z + 7 = 0 \rightarrow$ plane does not pass throug the origin
   $2x + 3y + 4z = 0 \rightarrow$ plane passing through the origin

④ $2x + 3y + 7 = 0 \rightarrow$ plane not pass $(0,0)$, // Z-axis $\perp$ $xy$ plane
   $2x + 3y = 0 \rightarrow$ plane passes $(0,0)$ Contains Z axis, $\perp$ $xy$ plane

# Position of a point with respect to a plane

find the position of each of the following points with respect to the plane $2x - 3y + 4Z - 5 = 0$

$A(-3, 3, 5)$, $B(6, 5, 1)$, $C(6, -5, 1)$

## Solution

by subistituting $A(-3, 3, 5) \to 2(-3) - 3(3) + 4(5) - 5 = 0 \therefore A \in$ the plane

$B(6, 5, 1) \longrightarrow 2(6) - 3(5) + 4(1) - 5 = -4 \therefore A \notin$ the plane

$C(6, -5, 1) \longrightarrow 2(6) - 3(-5) + 4(1) - 5 = 26 \therefore A \notin$ the plane

$\therefore B, C \notin$ the plane and in different sides of the plane.

1) Equation of the plane passing by the point $(1, 2, 3)$ parallel to the coordinate axis X, y is.

Cont المعامل الناقص

$z = 3$

(a) $x + y = 3$

(b) $x = 1$

(c) $y = 2$

(d) $z = 3$

---

2) the 2 st. lines $\overleftrightarrow{XX'}$, $\overleftrightarrow{ZZ'}$ form the plane ....

(a) $x = 0$

(b) $y = 0$

(c) $z = 0$

(d) $x + y + z = 5$

$\boxed{XZ\text{-plane}}$

Zero = المعامل الناقص

(3) Equation of the plane parallel to the xz-plane and passes through the point $(0, 5, 0)$ is ----

$y = 5$

(a) $x = 5$

(b) $y = 5$

(c) $z = 5$

(d) $x + y + z = 5$

(4) All the following points belong to the plane

$\boxed{\vec{n} \cdot (3, -5, 4) = 5}$ except ----

$3x - 5y + 4z \cancel{=5}$

$= 0$

(a) $(1, 2, 1)$

(b) $(0, -1, 2)$

(c) $(2, -3, 1)$

(d) $(1, 2, 4)$

# Relations between st.line and plane

① st.line // plane

$$\vec{d} \cdot \vec{n} = 0$$

② st.line ⊥ plane

$$\vec{d} \,//\, \vec{n} \rightarrow \begin{cases} \vec{d} = K\vec{n} \\ \vec{d} \times \vec{n} = \vec{0} \\ \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2} \end{cases}$$

## To find the relation between the st.line and the plane

① $\vec{r} = (x_1, y_1, z_1) + t(a, b, c)$

↳ $\vec{r} = (x_1 + at, y_1 + bt, z_1 + ct)$

② Substitute in the plane equation

1) if $t$ has only one value the the st.line intersects the plane

2) If $t$ has more than one value then the st.line is contained in the plane

3) If $t$ has no real value then $L$ // the plane

## To find the intersection between the st.line and the plane

we solve the 2 equations

① $\dfrac{x - x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{z - z_1}{c} = t \longrightarrow \begin{array}{l} x = at + x_1 \\ y = bt + y_1 \\ z = ct + z_1 \end{array}$

② Substitute in the plane equation to get the value of $t$

③ Sub by the value of $t$ in the st.line equation

we get the intersection point.

① If st. line $x = 3y = az$ parallel to the plane $x + 3y + 2z + 4 = 0$ then $a = \ldots$

ⓐ $-1$

ⓑ $1$

ⓒ $6$

ⓓ $7$

$\vec{d} = (1, \frac{1}{3}, \frac{1}{a})$

$\vec{n} = (1, 3, 2)$

$\vec{d} \cdot \vec{n} = 0$

$\longrightarrow a = -1$

$\div 3$

$\frac{x}{1} = \frac{3y}{1} = \frac{az}{1}$

$\frac{x}{1} = \frac{y}{\frac{1}{3}} = \frac{z}{\frac{1}{a}}$

---

② If the st. line $\frac{x-2}{k} = \frac{y+1}{3} = \frac{1-z}{2}$ parallel to the plane $4x - 4y + (k+1)z = 5$ then $k = \ldots$

ⓐ $4$

ⓑ $5$

ⓒ $6$

ⓓ $7$

$\frac{z-1}{-2}$

$\cdot = 0$

$\vec{d} = (k, 3, -2)$

$\vec{n} = (4, -4, k+1)$

$\vec{d} \cdot \vec{n} = 0 \longrightarrow k = 7$

③ the st. line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$ is parallel

to the plane whose equation: $dst = 0$

ⓐ $2x + y - 2z = 0$

ⓑ $x + y + z = 0$

ⓒ $3x + 4y + 5z = 7$

ⓓ $2x + 3y + 4z = 0$

---

④ 1) the st. line $\overleftrightarrow{L}$: $\frac{x-3}{5} = \frac{y+1}{5} = \frac{z+4}{7}$ and the

plane $P$: $3x + 4y - 5z = 25$ then - - -

ⓐ $L \parallel P$ $\vec{d_1} = (5, 5, 7)$, $n = (3, 4, -5)$

ⓑ $L \perp P$ $Pnt\ (3, -1, -4)$ $\vec{d_1} \cdot \vec{n} = 0$

ⓒ $L \subset P$ $L \subset P$ Parallel included.

ⓓ $L$ intersects $P$ at a point

ⓓ L intersects P at a point

2) the st. line L : $\vec{r} = (3,1,3) + t(1,-2,2)$ and the plane P: $\vec{r} \cdot (2,0,-1) = 3$ then

ⓐ L // P

ⓑ L ⊥ P

ⓒ L ⊂ P

ⓓ L intersects P at a point.

$2x - z = 3$

$3 = 3$

$\vec{d} = (1,-2,2)$ , $\vec{n} = (2,0,-1)$

$\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}$

$\vec{d} \cdot \vec{n} = 0 \rightarrow L // P$

$\rightarrow L \subset P$ ✓

3) the st. line L : $\vec{r} = \hat{k} + t(2\hat{i} + 3\hat{j} + 4\hat{k})$ and the plane P: $x + \frac{3}{2}y + 2z = 5$ then

ⓐ L // P

ⓑ L ⊥ P

ⓒ L ⊂ P

ⓓ L intersects P at a point.

$\vec{r} = (0,0,1) + t(2,3,4)$

$\vec{n} = (1,\frac{3}{2},2)$

$\vec{n} = (2,3,4)$

$\vec{d} = (2,3,4)$

(5) If the st. line $\frac{x-3}{2}=\frac{y-4}{3}=\frac{z-5}{4}$ lies in the plane $4x+4y-Kz-m=0$ then $K+m=$ ....

$5+3=8$

(a) $-12$

(b) $-8$

(c) $8$

(d) $12$

put $(\underset{x}{3},\underset{y}{4},\underset{z}{5}) \in$ plane $\rightarrow$

$12+16-5K-m=0 \rightarrow m+5K=28$

$\vec{d}\cdot\vec{n}=0 \longrightarrow (2,3,4)\cdot(4,4,-K)=0$

$K=5$

$m=3$

⑥ the st. line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ lies in the plane $ax+by+cz+d=0$ if

point نعوض بال plane في ال

ⓐ $al+bm+cn=0$

ⓑ $ax_1+by_1+cz_1+d=0$

ⓒ $(l,m,n)\times(a,b,c)=\vec{0}$

ⓓ $al+bm+cn=0$ , $ax_1+by_1+cz_1+d=0$

Parallel بين st. line , plane → dot = 0

$\vec{n}\cdot\vec{d}=0 \rightarrow (a,b,c)\cdot(l,m,n)=0$

V.I

st. line $\subset$ plane
① point ال نعوض
② $\vec{d}\cdot\vec{n}=0$

---

## More Examples

① the st. line $\frac{x+1}{1}=\frac{y+3}{3}=\frac{z-2}{-2}$ is $\perp$ to the plane

ⓐ $x-3y-2z+7=0$

ⓑ $2x+6y-4z+3=0$

ⓒ $x-3y-2z=0$

ⓓ $2x+6y+4z=5$

parallel ال نشر

$\vec{d}=(1,3,-2)$

+, +, −

+ves

② the st. line $\frac{x-3}{2} = \frac{y-1}{6} = \frac{z-1}{K+3}$ $\perp$ to the plane $x + 3y + 5z = 3$ then $K = \ldots\ldots$

ⓐ 4

ⓑ 5

ⓒ 6

ⓓ 7

$$\frac{2}{1} = \frac{6}{3} = \frac{K+3}{5}$$

$$K = 7$$

---

③ the projection of the point $A(1, 2, -1)$ on the plane $2x + 3y - z + 5 = 0$ is ......

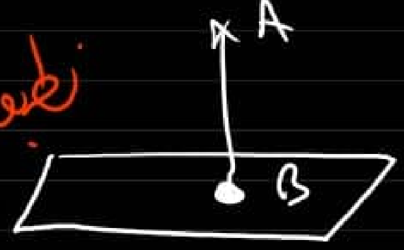
$n = (2, 3, -1)$

ⓐ $(-1, -1, 0)$ ✓ perpendicular

ⓑ $(1, 1, 0)$ parallel // ناظم

ⓒ $(1, 0, 1)$ $\vec{AB} = (2, 3, -1)$

ⓓ $(-1, 0, -1)$ $= \vec{n}$




جرّب الخيارات

④ If $B(2,1,6)$, the plane $x+y-2z=3$

$A \in$ Plane such that $\overline{BA} \perp$ the plane then $A = \ldots$

$\vec{n} = (1,1,-2)$

ⓐ $(4,5,3)$

ⓑ $(4,3,2)$ ✓ $\overrightarrow{AB} = (2,2,-4)$

$= (1,1,-2)$

ⓒ $(-2,1,-2)$

ⓓ $\left(\frac{5}{2}, \frac{5}{2}, \frac{3}{2}\right)$

---

⑤ the image of the point $(-2,3,-4)$ by reflection on the XZ-plane is . . . . . .

ⓐ $(-2,3,-4)$

ⓑ $(-2,-3,-4)$

ⓒ $(2,-3,4)$

ⓓ $(2,3,4$

نغير إشارة y

$(-2,-3,-4)$

إلا لو ماقالش

نغير إشارة

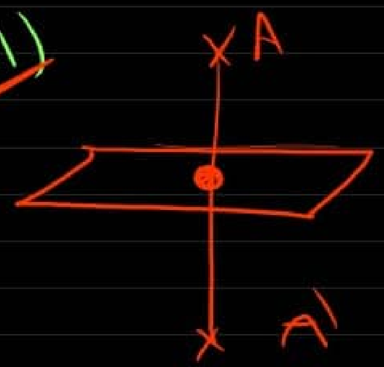
6) the image of the point $(1,2,3)$ by reflection on the plane $x+2y+4z-59=0$ is ...

(a) $(5,10,19)$ mid-pt $(3,6,11)$

(b) $(-1,-2,-3)$

(c) $(19,10,5)$

(d) $(-3,-2,-1)$

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7) the equation of the st. line passing through the point $A(1,1,1)$ and perpendicular to the plane $2x+3y+z+5=0$ is ...

(a) $\frac{x-1}{-1}=\frac{y-1}{1}=\frac{z-1}{1}$

(b) $\frac{x-1}{2}=\frac{y-1}{3}=\frac{z-1}{1}$

(c) $\frac{x-1}{3}=\frac{y-1}{2}=\frac{z-1}{1}$

(d) $x-1=\frac{y-1}{3}=\frac{z-1}{2}$

point $(1,1,1)$, direct $\vec{d}=\vec{n}$ $(2,3,1)$

① the intersection point between the st. line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = t$ with the plane $x-y+z=5$ is

ⓐ $(-1, 2, 2)$

ⓑ $(2, -1, 2)$

ⓒ $(-2, -1, 2)$

ⓓ $(2, -1, -2)$

$x = 2+3t$
$y = -1+4t$
$z = 2+2t$

$t = \text{zero}$

$x = 2$
$y = -1$
$z = 2$

---

② the intersection point between the st. line $\vec{r} = (1, 4, 2) + t(3, 2, 2)$ with the plane $\vec{n} \cdot (3, 2, 2) + 2 = \text{zero}$ is .....

ⓐ $(2, -2, 0)$

ⓑ $(-2, 2, 0)$

ⓒ $(0, 2, -2)$

ⓓ $(0, -2, 2)$

$x = 1+3t$
$y =$
$z =$

$3x + 2y + 2z + 2 = 0$

$t =$

---

(3) the intersection point between the st. line passes through the 2 points A (3, -4, -5), B (2, -3, 1) and the plane passes through the points C (2, 2, 1), D (3, 0, 1), E (4, -1, 0) is ---

(a) (1, -2, 7)

(b) (-2, 1, 7)

(c) (7, -2, 1)

(d) (7, 1, -2)

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ & & \\ & & \end{vmatrix} = 0$$

(4) the point (-1, -5, -10) is far from the intersection point of the st. line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ with the plane $x-y+z=5$ by ---- unit length

(a) 5

(b) 12

(c) 13

(d) 17

intersection point (2, -1, 2)

L dist

$= \sqrt{\phantom{x}}$

$= 13$ ✓

# Angle between a st.line and plane

Angle between
- 2 vectors → $\cos\theta = \frac{\vec{A}\cdot\vec{B}}{\|\vec{A}\|\ \|\vec{B}\|}$
- 2 lines → $\cos\theta = \frac{|\vec{d_1}\cdot\vec{d_2}|}{\|\vec{d_1}\|\ \|\vec{d_2}\|}$
- 2 planes → $\cos\theta = \frac{|\vec{n_1}\cdot\vec{n_2}|}{\|\vec{n_1}\|\ \|\vec{n_2}\|}$

the angle between line and plane

$$\sin\theta = \frac{|\vec{n}\cdot\vec{d}|}{\|\vec{n}\|\ \|\vec{d}\|}$$

1. the measure of the angle between the st.line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z}{3}$ and the plane $3x+2y+z=8$ is...

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$

(d) $\frac{2\pi}{3}$

(2) If the measure of the smaller angle between the st. line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z+3}{-2}$ and the plane $mx+y+4=0$ is $45^\circ$ then $m \in$ ....

(a) $\{1\}$

(b) $\{7\}$

(c) $\{1,7\}$

(d) $\{2,6\}$

---

(3) If the measure of the angle between the st. line $\vec{r}=(-1,3,0)+t(-1,\sqrt{2},1)$ and the plane $x+\sqrt{2}y+mz=3$ is $30$ then $m=$ ....

(a) 1

(b) 2

(c) 3

(d) 4

4 If the tangent of the angle between the St. line $\frac{x-2}{K} = \frac{y-2}{-1}$, $z = 5$ and the plane $2x + y - 2z + 3 = 0$ is $\frac{1}{2}$ then $K \in \ldots\ldots$

(a) $\{2\}$

(b) $\left\{\frac{-2}{11}\right\}$

(c) $\left\{2, -\frac{2}{11}\right\}$

(d) $\left\{-2, \frac{2}{11}\right\}$

## ③ Two planes

let the two planes

$a_1x + b_1y + c_1z = d_1$ , $a_2x + b_2y + c_2z = d_2$

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then they are intersecting at a line

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{d_1}{d_2}$ they are coincident

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \neq \frac{d_1}{d_2}$ they are parallel

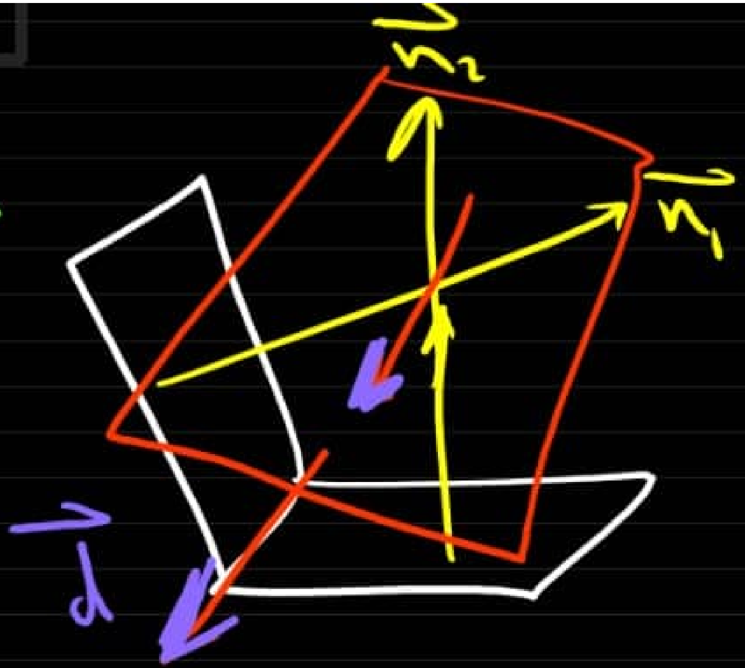
## To find the equation of the intersection line between 2 planes

point, direction, $\vec{r} = A + t\vec{d}$

① $\vec{d} = \vec{n_1} \times \vec{n_2}$

② to find point → put $x = 0$ in the 2 equations of the planes then solve the 2 equations to get $y, z$ the intersection point $A = (0, y, z)$

③ $\vec{r} = A + t\vec{d}$

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## Equation of plane passes through the intersection of 2 planes

$$a_1 x + b_1 y + c_1 z + d_1 = 0$$
$$a_2 x + b_2 y + c_2 z + d_2 = 0$$

is $a_1 x + b_1 y + c_1 z + d_1 + K(a_2 x + b_2 y + c_2 z + d_2) = 0$

To find K → ① given point → Subistitute

② // given line → $\vec{n} \cdot \vec{d} = 0$

$$n = (a_1 + K a_2, \; b_1 + K b_2, \; c_1 + K c_2)$$

③ ⊥ the plane $ax + by + cz + d = 0$

$$\vec{n} \cdot \vec{n_2} = 0$$

1) the 2 planes $x-3y+mZ=5$, $3x+Ky+6Z=10$ If the 2 planes are parallel then $K \times m = \ldots\ldots$

a) $-18$

b) $-7$

c) $7$

d) $18$

---

2) the 2 planes $-2x+3y-5Z=3$, $-4x+6y-10Z=10$ are

a) parallel

b) perpendicular

c) coincident

d) intersecting

---

3) If the 2 planes $3x-y+2Z+3=0$, $Kx-4y+Z=5$ are perpendicular then $K = \ldots\ldots$

a) $-3$

b) $-2$

c) $2$

d) $3$

---

4. the 2 planes $2x-y+2z=8$, $3x+4y-z=7$ are

(a) parallel

(b) perpendicular

(c) intersecting

(d) coincident

5. the 2 planes $ax+3y-bz=5$, $2x+9y+6z=c$ are coincident then $3a+2b+c=$ ---

(a) 17

(b) 15

(c) 13

(d) 11

6) the intersection line between the 2 planes $\vec{n}_1 \cdot (3,-1,1)=1$, $\vec{n}_2 \cdot (1,1,-2)=-2$ is parallel to the vector ..........

(a) $(2,7,1)$

(b) $(-2,7,13)$

(c) $(1,7,4)$

(d) $(7,13,1)$

$\vec{d} = \vec{n}_1 \times \vec{n}_2$

---

7) Equation of the plane passes through the intersection line of the 2 planes $6x+4y+3z+5=0$, $2x+y+z=2$ and passes through the point $(2,-3,2)$

(a) $16x+7y+8z=27$

(b) $8x+7y+4z=27$

(c) $16x+7y+8z+27=0$

(d) $8x+7y+4z+27=0$

$6x+4y+3z+5+k(2x+y+z-2)=0 \rightarrow$

نعوض

$\boxed{k=-11}$

8. The equation of the plane passes through the intersection line of the 2 planes $x+y+2z+1=0$, $2x+y-z+1=0$ and parallel to the st. line joining the 2 points $A(2,5,-3)$, $B(3,-2,2)$ is ....

(a) $9x+7y+8z+7=0$

(b) $x+7y+8z+25=0$

(c) $7x+9y+8z-15=0$

(d) $9x+8y-7z=37$

$\vec{d}=\overrightarrow{AB}=(1,-7,5)$

dot $=0$

$(1,-7,5)\cdot(1+2K,\ 1+K,\ 2-K)=0$

$K=\frac{2}{5}$

$x+y+2z+1+K(2x+y-z+1)=0$

$\vec{n}=(1+2K,\ 1+K,\ 2-K)$

(9) the equation of the plane passes through the intersection line of the 2 planes $x+2y+3z=4$, $2x+y-z+8=0$ ~~and~~ perpendicular to the plane $3x+2y+z+6=0$ is - - -

(a) $-13x+4y+31z=108$

(b) $13x+14y+31z=75$

(c) $13x-4y+31z+108=0$

(d) $-13x+4y+31z=0$

Angle between

→ 2 vectors → $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{\|\vec{A}\|\,\|\vec{B}\|}$

→ 2 lines → $\cos\theta = \dfrac{|\vec{d}_1\cdot\vec{d}_2|}{\|\vec{d}_1\|\,\|\vec{d}_2\|}$

→ 2 planes → $\cos\theta = \dfrac{|\vec{n}_1\cdot\vec{n}_2|}{\|\vec{n}_1\|\,\|\vec{n}_2\|}$

the angle between line and plane

$$\sin\theta = \frac{|\vec{n}\cdot\vec{d}|}{\|\vec{n}\|\,\|\vec{d}\|}$$

the measure of the angle between the 2 planes

1. $x+\sqrt{2}y+z=5$, $x-\sqrt{2}y+z=1$ is ....

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{2}$

---

2. $\vec{r}\cdot(1,1,2)=7$, $2x-y+z=6$ is ...

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{2}$

---

5. If the angle is 45 between the 2 planes $3x-6y+mz=4$, $x+z=7$ then $m=$...

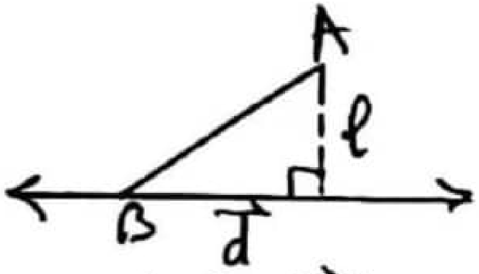
(a) 2

(b) 4

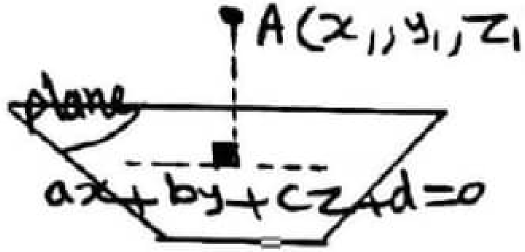
(c) 6

(d) 8

# Length of the perpendicular from a given point to a plane




Lg ⊥ from a given point




$$\text{lg}\perp = \frac{|\overrightarrow{BA} \times \vec{d}\,|}{\|\vec{d}\,\|}$$

$$l = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}$$

Note: the perpendicular distance between 2 parallel planes $P_1: ax+by+cz+d_1=0$

$P_2: ax+by+cz+d_2=0$

$$\text{lg}\perp = \frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}}$$

in Condition of $a=a$, $b=b$, $c=c$

# Dividing line segment

$\boxed{1}$ xy plane divides the line joining the 2 points $(2,4,5)$, $(-4,3,-2)$ in the ratio = ....

Solution

xy plane $\rightarrow z=0$ $(x, y, 0)$ point of division

ratio of division $= \dfrac{5-0}{0--2} = (+)\dfrac{5}{2}$ internally

$\boxed{2}$ the plane $ax+by+cz+d=0$ divides the line joining $(x_1, y_1, z_1)$, $(x_2, y_2, z_2)$ in the ratio $\left(-\dfrac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}\right) = \begin{array}{l}(+) \rightarrow \text{internally} \\ (-) \rightarrow \text{externally}\end{array}$

## To find the projection of a point on a plane

The projection of the point $A(1,2,3)$ on the plane $x+2y+4z=59$ is ......

ⓐ $(3,6,11)$

ⓑ $(6,3,11)$

ⓒ $(11,6,3)$

ⓓ $(3,11,6)$

then find the image of point A by reflection in the plane $x+2y+4z=59$

Solution

let the projection is $B(x_1, y_1, z_1)$

$\vec{n} \parallel \overleftrightarrow{AB} \rightarrow$ equation of $\overleftrightarrow{AB}$

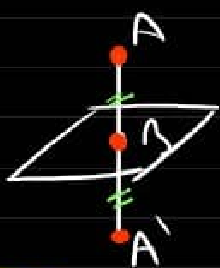
$\vec{r} = A + t\vec{d} \rightarrow \vec{r} = (1,2,3) + t(1,2,4)$

$B = (x_1, y_1, z_1) = (1+t, 2+2t, 3+4t)$

$\because B \in$ plane $x+2y+4z=49$

$\therefore 1+t+2(2+2t)+4(3+4t) = 59 \rightarrow t=2$

$\therefore B = (3,6,11)$




$\grave{A} = 2B - A$

$= 2(3,6,11) - (1,2,3)$

$= (5,10,19)$

1. the length of the ⊥ from point (2, 3, 1) to the plane $2x - 2y + z = 5$ is ......

a) 1

b) 2

c) 3

d) 4