GEOMETRY

Q1

The point A $(1-k, 2k, 3+k)$ lies on the plane $\mathcal{X}y$, then A = ...

(a) $(-2, 6, 0)$ (b) $(4, 2, 0)$ (c) $(-4, -6, 0)$ (d) $(4, -6, 0)$

Q2

If the points A $(6, 0, 3)$, B $(7, 1, 7)$, C $(9, 3, 15)$ lie on the same straight line , then A divides $\overline{BC}$ in the ratio ....

(a) 3 : 1 internally. (b) 1 : 3 externally. (c) 2 : 3 internally. (d) 1 : 2 externally.

Q3

The image of the point $(-2, 3, 4)$ by reflection in the z-axis is ....

(a) $(-2, 3, 4)$ (b) $(2, 3, 4)$ (c) $(2, -3, -4)$ (d) $(2, -3, 4)$

Q4

The projection of the point $(3, 2, 1)$ on the $\mathcal{X}y$-plane is ....

(a) $(3, 2, 0)$ (b) $(0, 2, 1)$ (c) $(3, 2, 1)$ (d) $(3, 0, 1)$

Q5

(1st Session 2022) If A $(3, 2, 5)$ is a point in space , its projection on $\mathcal{X}y$-plane is point B and its projection on $\mathcal{X}z$-plane is point C , then $\overrightarrow{BC}$ = ...

(a) $-2\hat{i} + 3\hat{j}$ (b) $2\hat{j} - 5\hat{k}$ (c) $-2\hat{j} + 5\hat{k}$ (d) $-2\hat{i} - 3\hat{k}$

Pure

Position vector

* $\overrightarrow{A} = (A_x, A_y, A_z)$

* Length of $\overrightarrow{A} = \|\overrightarrow{A}\| = \sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}$

* $\overrightarrow{AB} = B - A$ * $\overrightarrow{AB} = -\overrightarrow{BA}$ * $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

* $\|k\overrightarrow{A}\| = k\|\overrightarrow{A}\|$ where k is a constant

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Unit vector (direction of Cosines)

* Unit vector of $\overrightarrow{A}$ is $(\overrightarrow{U_A}) = \frac{\overrightarrow{A}}{\|\overrightarrow{A}\|} = \left(\frac{A_x}{\|\overrightarrow{A}\|}, \frac{A_y}{\|\overrightarrow{A}\|}, \frac{A_z}{\|\overrightarrow{A}\|}\right)$

* $\overrightarrow{A} = \|\overrightarrow{A}\|.\overrightarrow{U_A}$

* $\overrightarrow{A} = \|\overrightarrow{A}\|.(\cos\theta_x\ ,\ \cos\theta_y\ ,\ \cos\theta_z)$

* $\overrightarrow{U_A} = (\cos\theta_x\ ,\ \cos\theta_y\ ,\ \cos\theta_z)$

* A_x is The component of $\overrightarrow{A}$ in the direction of x-axis

  $A_x = \cos\theta_x \longrightarrow A_{yz} = \sin\theta_x$

* $\theta_x + \theta_y \geq 90$ لازم

* If the vector makes angle $\theta_x, \theta_y, \theta_z$ with axes

  ➢ $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

* If $\overrightarrow{A}$ makes direction angles $(\theta_x, \theta_y, \theta_z)$

  Then K.$\overrightarrow{A}$ makes angles
  - $(\theta_x, \theta_y, \theta_z)$ if K is positive
  - $(180 - \theta_x, 180 - \theta_y, 180 - \theta_z)$ if K is negative

Pure

Q1

If $\overrightarrow{A}$ is a unit vector where $\overrightarrow{A} = \left(k , \frac{2}{3} , \frac{2}{3}\right)$, then k = ...

(a) ± 3 (b) $\pm \frac{1}{3}$ (c) $\pm\sqrt{3}$ (d) $\pm \frac{1}{\sqrt{3}}$

Q2

(2nd Session 2023) If $\overrightarrow{A} = (-3 , 0 , 4)$, then the direction cosines of the vector $\overrightarrow{A}$ are ....

(a) $\left(\frac{3}{5} , 0 , \frac{-4}{5}\right)$ (b) $\left(\frac{3}{5} , 0 , \frac{4}{5}\right)$ (c) $\left(\frac{-3}{5} , 0 , \frac{4}{5}\right)$ (d) $\left(\frac{-3}{5} , 0 , \frac{-4}{5}\right)$

Q3

If (60° , 135° , 60°) are the direction angles of vector $\overrightarrow{A}$, then the direction angles of vector $-2\overrightarrow{A}$ is ....

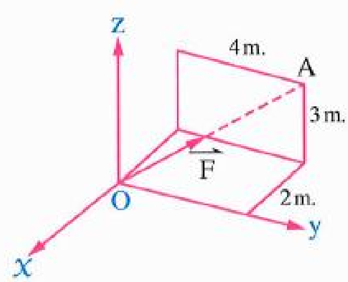
(a) (60° , 135° , 60°) (b) (120° , 270° , 120°)

(c) (30° , 45° , 30°) (d) (120° , 45° , 120°)

Q4

In the opposite figure :

The components of the force $\overrightarrow{F}$ whose magnitude is $12\sqrt{29}$ newtons = ...



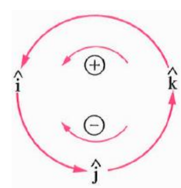
(a) (12 , 12 , 12) (b) (– 24 , 48 , 36)

(c) $\left(-2\sqrt{29} , 4\sqrt{29} , 3\sqrt{29}\right)$ (d) $\left(12\sqrt{29} , 12\sqrt{29} , 12\sqrt{29}\right)$

Pure

(1) Scalar product & vector product comparison:

for $\vec{A} = (A_x, A_y, A_z)$ and $\vec{B} = (B_x, B_y, B_z)$

Scalar Product	Vector product
$\vec{A}\bullet\vec{B}$	$\vec{A}\times\vec{B}$
$\vec{A}\bullet\vec{B} = \lVert\vec{A}\rVert\lVert\vec{B}\rVert\cos\theta$	$\vec{A}\times\vec{B} = \left(\lVert\vec{A}\rVert\lVert\vec{B}\rVert\sin\theta\right)\vec{c}$,where $\vec{c}$ is a unit vector perpendicular to $\vec{A}$ and $\vec{B}$
$\vec{A}\bullet\vec{B} = A_xB_x + , A_yB_y + A_zB_z$	$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$
$\vec{A}\bullet\vec{B} = \vec{B}\bullet\vec{A}$	$\vec{A}\times\vec{B} = -\left(\vec{B}\times\vec{A}\right)$
IF $\vec{A}\bullet\vec{B} = 0$ then $\vec{A}\perp\vec{B}$	IF $\vec{A}\times\vec{B} = \vec{O}$ then $\vec{A} // \vec{B}$
$\vec{A}\bullet\vec{A} = \lVert\vec{A}\rVert^2$	$\vec{A}\times\vec{A} = \vec{O}$
✸$\hat{i}\bullet\hat{i} = \hat{j}\bullet\hat{j} = \hat{k}\bullet\hat{k} = 1$ ✸$\hat{i}\bullet\hat{j} = \hat{j}\bullet\hat{k} = \hat{k}\bullet\hat{i} = 0$ ✸$\hat{j}\bullet\hat{i} = \hat{k}\bullet\hat{j} = \hat{i}\bullet\hat{k} = 0$	✸$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{O}$ ✸$\hat{i}\times\hat{j} = \hat{k}$, $\hat{k}\times\hat{i} = \hat{j}$, $\hat{j}\times\hat{k} = \hat{i}$ ✸$\hat{j}\times\hat{i} = -\hat{k}$, $\hat{i}\times\hat{k} = -\hat{j}$ $\hat{k}\times\hat{j} = -\hat{i}$ 
$\vec{A}\bullet\vec{O} = 0$	$\vec{A}\times\vec{O} = \vec{O}$
$\left(\vec{A}+\vec{B}\right)\bullet\vec{C} = \vec{A}\bullet\vec{C} + \vec{B}\bullet\vec{C}$	$\left(\vec{A}+\vec{B}\right)\times\vec{C} = \vec{A}\times\vec{C} + \vec{B}\times\vec{C}$

Pure

(2) Scalar product:

- $\cos\theta = \dfrac{\vec{A}\bullet\vec{B}}{\|\vec{A}\|\|\vec{B}\|}$, $0 \le \theta \le 180°$

  $\cos\theta = \overrightarrow{U_A}\bullet\overrightarrow{U_B}$

> - If $\theta = 0$ $\vec{A} \,//\, \vec{B}$ and in the same direction
> - If $\theta = 180$ $\vec{A} \,//\, \vec{B}$ and in the opposite direction
> - If $\theta = 90$ $\vec{A} \perp \vec{B}$

- the algebraic component (projection) of $\vec{A}$ in the direction of $\vec{B}$ is

$$A_B = \|\vec{A}\|\cos\theta = \frac{\vec{A}\bullet\vec{B}}{\|\vec{B}\|} = \vec{A}\bullet\overrightarrow{U_B}$$

- the vector component (projection) of $\vec{A}$ in the direction of $\vec{B}$ is

$$\overrightarrow{A_B} = \left(\frac{\vec{A}\bullet\vec{B}}{\|\vec{B}\|^2}\right)\vec{B}$$

- work done $= w = \vec{F}\bullet\vec{S} = \|\vec{F}\|\|\vec{S}\|\cos\theta$

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(3) Vector product:

- $\sin\theta = \dfrac{\|\vec{A}\times\vec{B}\|}{\|\vec{A}\|\|\vec{B}\|}$, $0 \le \theta \le 180°$

- The unit vector $\perp$ to the plane that contains $\vec{A}$, $\vec{B}$ is $\vec{C} = \pm\dfrac{\vec{A}\times\vec{B}}{\|\vec{A}\times\vec{B}\|}$

- To prove that $\vec{A}\perp\vec{B}$, then $\vec{A}\bullet\vec{B} = 0$

- To prove that $\vec{A} \,//\, \vec{B}$

  - $\vec{A}\times\vec{B} = \vec{O}$ **OR** - $\dfrac{A_x}{B_x} = \dfrac{A_y}{B_y} = \dfrac{A_z}{B_z}$ **OR** - $\vec{A} = k\vec{B}$

Pure

Q1

If $\overrightarrow{A} = (-1, 4, 2)$, $\overrightarrow{B} = (2, 2, 1)$, then the component of $\overrightarrow{A}$ in direction of $\overrightarrow{B}$ = ...

(a) $\frac{4}{3}$ (b) $\frac{-8}{3}$ (c) $\frac{8}{3}$ (d) 5

Q2

If $\|\overrightarrow{A}\| = 11$, $\|\overrightarrow{B}\| = 23$, $\|\overrightarrow{A} - \overrightarrow{B}\| = 30$, then $\|\overrightarrow{A} + \overrightarrow{B}\|$ = ...

(a) 5 (b) 10 (c) 20 (d) 30

Q3

If (2 , 5 , 3) , (4 , 6 , – 1) , (8 , 8 , k) are three collinear points then k = ...

(a) 8 (b) 7 (c) – 9 (d) – 7

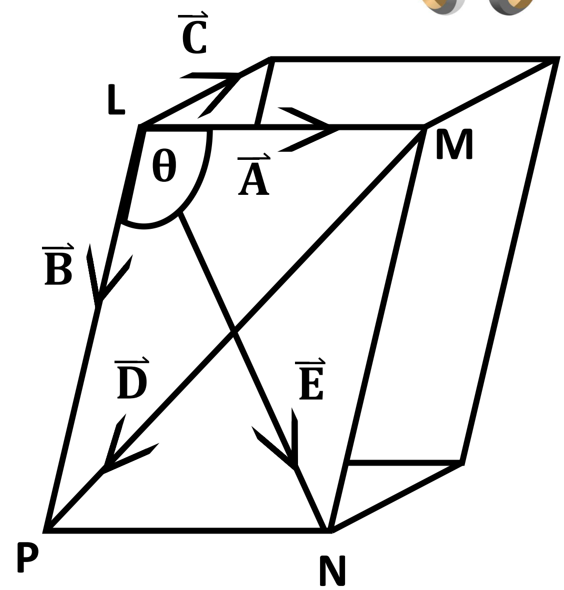
Q4

ABCDEF is a uniform hexagon with side length 2 cm. , then $\|\overrightarrow{AB} \times \overrightarrow{AF}\|$ = ...

(a) $2\sqrt{3}$ (b) $\sqrt{3}$ (c) 2 (d) 1

Pure

(5) Vectors related to geometric shapes:



$\vec{A}, \vec{B}, \vec{C}, \vec{D}$ and $\vec{E}$ are vectors in Space, then

1. Area of the Parallelogram LMNP =

- $\|\vec{A} \times \vec{B}\| = \|\vec{A}\|\|\vec{B}\| \sin\theta$
- $\frac{1}{2}\|\vec{E} \times \vec{D}\|$ (valid for any quadrilateral)

2.Area of the Triangle LMN=

- $\frac{1}{2}\|\vec{A} \times \vec{B}\| = \frac{1}{2}\|\vec{A}\|\|\vec{B}\| \sin\theta$
- $\frac{1}{4}\|\vec{E} \times \vec{D}\|$

3.Volume of the parallelepiped= $\left|\vec{A} \bullet \vec{B} \times \vec{C}\right| = \begin{vmatrix} A_x & A_y & A_y \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$

1.Mode 6, 1, 1
2.fill the matrix
3.AC
4.shift 4, 7
5.Shift 4, 3
6. volume ^_^

4.height $= \frac{\text{volume}}{\text{Base Area}} = \frac{\|\vec{A} \bullet \vec{B} \times \vec{C}\|}{\|\vec{A} \times \vec{C}\|}$

5.Volume of quadrilateral pyramid= $\frac{1}{3}\left|\vec{A} \bullet \vec{B} \times \vec{C}\right|$

6.Volume of triangular pyramid= $\frac{1}{6}\left|\vec{A} \bullet \vec{B} \times \vec{C}\right|$

Pure

Q1

Area of Δ ABC where : A (5 , 1 , – 2) , B (4 , – 4 , 3) , C (2 , 4 , 0) ≈ .... square unit.

(a) 16.7 (b) 18.2 (c) 13.9 (d) 14.27

Q2

Area of ▱ ABCD where A (2 , 1 , 3) , B (1 , 4 , 5) , C (2 , 5 , 3) = ... square unit.

(a) $3\sqrt{2}$ (b) $4\sqrt{5}$ (c) $2\sqrt{5}$ (d) $2\sqrt{7}$

Q3

(Trial 2021) If $\vec{A}$, $\vec{B}$, $\vec{C}$ are three connected edges in a parallelepiped where $\|\vec{A}\| = 2$ The direction cosines of the vector $\vec{A}$ are (135° , 60° , 120°) and $\vec{B} = (1 , \sqrt{2} , 0)$, $\vec{C} = (\sqrt{2} , 3 , 5)$, then the volume of the parallelepiped = ... cubic units

(a) 16 (b) $6\sqrt{2}$ (c) 11 (d) $16\sqrt{2}$

(1) Equation of straight line

For $A = (x_1, y_1, z_1)$ and $\vec{d} = (a, b, c)$

1. $\vec{r} = A + t\vec{d}$, where t is a constant number

$(x, y, z) = (x_1, y_1, z_1) + t(a, b, c)$ "Vector form"

2. $x = x_1 + at$

$y = y_1 + bt$ "Parametric form"

$z = z_1 + ct$

3. $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = t$ "Cartesian form"

Pure

Q1

The equation of the straight line passing through the point A $(-1, 0, 2)$ and has direction vector $\vec{d} = (1, -1, 3)$ is ....

ⓐ $\frac{x+1}{3} = \frac{y}{1} = \frac{z-2}{-1}$

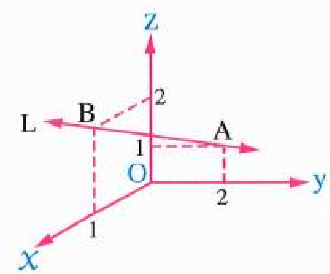
ⓑ $\frac{x+1}{1} = \frac{y}{-1} = \frac{z-2}{3}$

ⓒ $\frac{x-1}{3} = \frac{y}{-1} = \frac{z}{1}$

ⓓ $\frac{x+2}{1} = \frac{y-1}{-1} = \frac{z}{2}$

Q2

In the opposite figure :



The equation of the straight line is ....

ⓐ $x = \frac{y-2}{2} = z-1$

ⓑ $x = \frac{y+2}{2} = \frac{z-1}{2}$

ⓒ $\frac{x}{2} = \frac{y-2}{-2} = z-1$

ⓓ $x = \frac{y-2}{-2} = z-1$

Q3

The direction cosines of the straight line $\frac{\sqrt{2}\,x - 3\sqrt{2}}{1} = \frac{2\sqrt{2} - \sqrt{2}\,y}{2}$, $z + 1 = 0$

could be ....

ⓐ $\left(\frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}, \text{zero}\right)$

ⓑ $\left(\frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, \text{zero}\right)$

ⓒ $\left(\frac{2}{\sqrt{5}}, \text{zero}, \frac{1}{\sqrt{5}}\right)$

ⓓ $\left(\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}, \text{zero}\right)$

(3) Relation between two St.lines in space

1.The angle between two St.lines

✸ $\cos\theta = \frac{\overrightarrow{d_1} \bullet \overrightarrow{d_2}}{\|\overrightarrow{d_1}\|\|\overrightarrow{d_2}\|}$

2. Parallel lines $(\overleftrightarrow{L_1} \,//\, \overleftrightarrow{L_2})$ if ✸ $\overrightarrow{d_1} = k\overrightarrow{d_2}$

✸ $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ✸ $\overrightarrow{d_1} \times \overrightarrow{d_2} = \overrightarrow{O}$

3. Perpendicular lines

$(\overleftrightarrow{L_1} \perp \overleftrightarrow{L_2})$ if $\overrightarrow{d_1} \bullet \overrightarrow{d_2} = 0$ ⟶ $a_1a_2 + b_1b_2 + c_1c_2 = 0$

4. If $\overleftrightarrow{L_1} \,//\, \overleftrightarrow{L_2}$ and $A \in \overleftrightarrow{L_1}$ and $A \in \overleftrightarrow{L_2}$ then the two points are coincident

5. Intersecting lines $\overleftrightarrow{L_1}: \overrightarrow{r_1} = A + t_1\overrightarrow{d_1}$ $\overleftrightarrow{L_2}: \overrightarrow{r_2} = B + t_2\overrightarrow{d_2}$

✸ $\overrightarrow{AB} \bullet \overrightarrow{d_1} \times \overrightarrow{d_2} = 0$ intersecting lines (the two lines lie on the same plane)

✸ $\overrightarrow{AB} \bullet \overrightarrow{d_1} \times \overrightarrow{d_2} \neq 0$ Skew lines (the two lines doesn't lie on the same plane)

✸To find the intersection point between two St.Lines

$\overleftrightarrow{L_1}: \overrightarrow{r_1} = (x_1, y_1, z_1) + t_1(a_1, b_1, c_1)$ $\overleftrightarrow{L_2}: \overrightarrow{r_2} = (x_2, y_2, z_2) + t_2(a_2, b_2, c_2)$

I. Let $x_1 = x_2$, $y_1 = y_2$

II. $t_1a_1 - t_2a_2 = 0$
$t_1b_1 - t_2b_2 = 0$ Solving the two equation "Mode, 5, 1"

III. Get t_1 , t_2

IV. Get $z_1 = t_1c_1$ & $z_2 = t_2c_2$

V. If $z_1 = z_2$ (then the two straight lines are intersecting)
If $z_1 \neq z_2$ (then the two straight lines are skew)

Q1

The vector $\overrightarrow{A} = (-1, 1, 1)$ is perpendicular to the straight line that has the equation ....

(a) $\frac{x+1}{-2} = \frac{y-2}{3} = \frac{z-1}{-1}$

(b) $\frac{x+2}{1} = \frac{y-1}{2} = \frac{1-z}{-3}$

(c) $\frac{x+1}{-2} = \frac{y+2}{-1} = \frac{z+2}{3}$

(d) $\frac{x-1}{2} = \frac{y-2}{4} = \frac{2-z}{2}$

Q2

The measure of the angle between the two straight lines whose direction cosines are $\left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right)$ and $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ equals ....

(a) 60°　(b) 30°　(c) 90°　(d) 120°

Q3

If $\ell_1 : \frac{x-3}{2} = \frac{-y-1}{6} = \frac{z}{k}$ parallel to $\ell_2 : \frac{x+2}{6} = \frac{y-z}{m} = \frac{z-1}{3}$, then k + m = ...

(a) – 17　(b) – 10　(c) 10　(d) 17

Q4

The two straight lines : $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$, $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{6}$ are ....

(a) intersecting.　(b) coincident.　(c) parallel.　(d) skew.

(4) Equation of Plane

For $\mathrm{A} = (x_1, y_1, z_1)$ and $\vec{\mathrm{n}} = (\mathrm{a, b, c})$

1. $\vec{\mathrm{n}} \bullet \vec{\mathrm{r}} = \vec{\mathrm{n}} \bullet \vec{\mathrm{A}}$

$(\mathrm{a, b, c}) \bullet (x, y, z) = (\mathrm{a, b, c}) \bullet (x_1, y_1, z_1)$ "Vector form"

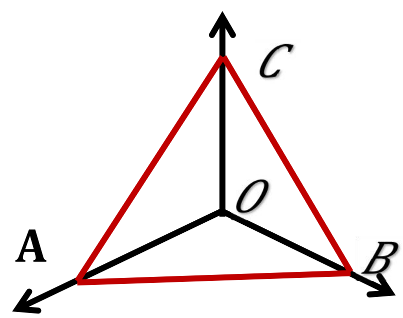
2. $ax + by + cz = \vec{\mathrm{n}} \bullet \vec{\mathrm{A}}$ "General form"

3. $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ "Standard form"

If the plane cuts the coordinate axis at the points $(x_1, 0, 0), (0, y_1, 0), (0, 0, z_1)$ then the equation of plane equals:

✸ $\frac{x}{x_1} + \frac{y}{x_1} + \frac{z}{x_1} = 1$

✸ $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$ (where (p, q, r) is the Centroid of the triangle ABC)



If the equation of plane is $ax + by + cz = d$

✸ $d = 0$ ⟶ the plane passes through the origin point

✸ $a = 0$ ⟶ the plane is parallel to x-axis

✸ $a = 0$ ⟶ the plane is perpendicular to yz-plane

✸ $d = 0, a = 0$ ⟶ the plane contains x-axis and ⊥ yz-plane

Pure

Q1

The plane : $8x + 2y + 4z = 8$ intersects the coordinate axes x , y , z at the points A , B and C respectively , find the area of Δ ABC , then find volume of the pyramid OABC

Q2

If the length of the perpendicular drawn from the origin point to the plane P is 7 length units and the direction ratios of the straight line that carries it are -3 , 2 , 6 which of the following equations is the equation of the plane P ?

(a) $-3x + 2y + 6z - 7 = 0$

(b) $-3x + 2y + 6z - 49 = 0$

(c) $3x - 2y + 6z + 7 = 0$

(d) $-3x + 2y - 6z - 49 = 0$

Pure

Q3

If the plane $2x - y + 2z = 4$ moves parallel to itself 3 units in $\overrightarrow{ox}$ direction , then its equation will be ....

(a) $2x - y + 2z = 1$

(b) $2x - y + 2z = 7$

(c) $2x - y + 2z = 10$

(d) $2x - y + 2z = -7$

Q4

The equation of the line of intersection of the two planes P_1 : $2x - y + z - 1 = 0$, $P_2 : x - 3y - z + 2 = 0$ is ....

(a) $\frac{x+1}{-1} = \frac{y}{2} = \frac{z}{3}$

(b) $\frac{x-1}{1} = -\frac{y}{3} = \frac{z-5}{1}$

(c) $\frac{x-2}{1} = \frac{y-3}{-2} = \frac{z}{-1}$

(d) $\frac{x-1}{4} = \frac{y-1}{3} = \frac{z}{-5}$

Q5

The point A is the image of the point N (1 , 2 , 3) by reflection on the plane x y and B is the image of A by rotation about the origin with an angle of measure 180° and C is the image of B by translation 5 units in positive direction of y-axis , then C = ...

(a) (1 , 7 , – 3) (b) (– 1 , 7 , – 3) (c) (– 1 , – 2 , 8) (d) (– 1 , 3 , 3)