Exercise 5

First

[8] Choose :-

(1) "Direction of cosines" = "unit vector"

$\| \vec{d} \| = 1$

$\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = 1$ ⓐ

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(2) Let $A = (4, 3, -5)$, $B = (-2, 1, -8)$

$\vec{d} = \pm \vec{AB} = \pm(-6, -2, -3)$

$\vec{U}_d = \pm \dfrac{(-6, -2, -3)}{\sqrt{6^2 + 2^2 + 3^2}} = \pm \dfrac{(-6, -2; 3)}{7}$

$= \left(\frac{-6}{7}, \frac{-2}{7}, \frac{-3}{7}\right)$ or $\left(\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$ ⓒ

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(3) X - axis $\longrightarrow y = 0, z = 0$

$(1, 0, 0)$ ⓐ

(4) direction angles $= (\theta_x, 60^\circ, 60^\circ)$

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos^2\theta_x + \cos^2 60^\circ + \cos^2 60^\circ = 1$$

$$\cos^2\theta_x = \frac{1}{2}$$

$$\cos\theta_x = \pm\frac{1}{\sqrt{2}}$$

$\theta_x = 45^\circ$ $\qquad$ $\theta_x = 135^\circ \rightarrow$ refused

(c) $\qquad$ $(0 \leq \theta \leq 90)$ in straight lines

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[5] $\vec{d} = (-1, 2, 3)$

$$\|\vec{d}\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$$

$$\vec{d} = \left(\frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$$ (c)

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(6) $\cos 2\theta_x + \cos 2\theta_y + \cos^2\theta_z$

$\cos 2x = \cos^2\theta - \sin^2\theta$
$= 1 - 2\sin^2\theta$
$= 2\cos^2\theta - 1$

$$2\cos^2\theta_x - 1 + 2\cos^2\theta_y - 1 + 2\cos^2\theta_z - 1$$

$$= 2(\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z) - 3$$

$$= 2 \times (1) - 3 = 2 - 3 = -1$$ (b)

(7) $\sin^2\theta_x + \sin^2\theta_y + \sin^2\theta_z$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$= 1 - \cos^2\theta_x + 1 - \cos^2\theta_y + 1 - \cos^2\theta_z$

$= 3 - (\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z)$

$= 3 - 1 = 2$ (d)

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(8) the straigut line perpendicular to xz-plane is $\underline{y\text{-axis}}$ $\rightarrow$ $x = 0,\ z = 0$

$(0, 10)$ (b)

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(9) $\sqrt{\left(\frac{1}{c}\right)^2 + \left(\frac{1}{c}\right)^2 + \left(\frac{1}{c}\right)^2} = 1$

$\frac{1}{c^2} + \frac{1}{c^2} + \frac{1}{c^2} = \frac{3}{c^2} = 1$

$c^2 = 3 \rightarrow c = \pm\sqrt{3}$ (c)

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(10) $A = (-1, 0, 2)$, $\vec{d} = (1, -1, 3)$

$\frac{x+1}{1} = \frac{y}{-1} = \frac{z-2}{3}$ (b)

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(11) $(2, -1, 3)$, $(0, 3, 1)$ $\rightarrow$ two points

معلوم ⊖ عشان اجيب ال $\vec{d}$

$\vec{d} = (2, -4, 2)$

$\vec{r} = A + t\vec{d}$

$= (2, 1, -3) + t(2, -4, 2)$ (a)

(12) two points $\rightarrow (-2,4,2)$

$\rightarrow (7,-2,5)$

هنطرحهم ⊖ بعض
عشان أجيب ال $\vec{d}$

$\vec{d} = \{7-(-2), -2-4, 5-2\}$

$\vec{d} = (9, -6, 3)$ $\boxed{\div 3}$

$\vec{d} = (3, -2, 1)$

$$\frac{x+2}{3} = \frac{y-4}{-2} = \frac{z-2}{1} \quad (a)$$

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(13) $y=5$, $\dfrac{x-4}{3} = \dfrac{2z+1}{6} = t$

- $\dfrac{x-4}{3} = t \rightarrow x-4=3t \rightarrow x = 3t+4$
- $y=5$ جاهزة
- $\dfrac{2z+1}{6} = t \rightarrow 2z+1 = 6t \rightarrow 2z = 6t-1 \rightarrow z = 3t - \frac{1}{2}$

$x = 4+3t,\ y=5,\ z = \frac{-1}{2} + 3t$ (d)

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(14) $\vec{r} = (1,3,9) + t(5,4,2)$

$\vec{r} = A + t\vec{d}$

$A = (1,3,9)$

$\vec{d} = (5,4,2)$

$$\frac{x-1}{5} = \frac{y-3}{4} = \frac{z-9}{2} \quad (b)$$

(15)

$$\frac{x+3}{2} = \frac{2y-1}{5} = \frac{3z+2}{4}$$

$$\frac{x+3}{2} = \frac{y-\frac{1}{2}}{\frac{5}{2}} = \frac{z+\frac{2}{3}}{\frac{4}{3}}$$

A

$\vec{d}$

$A = (-3, \frac{1}{2}, \frac{-2}{3})$

$\vec{d} = (2, \frac{5}{2}, \frac{4}{3})$

$\vec{r} = A + td = (-3, \frac{1}{2}, \frac{-2}{3}) + t(2, \frac{5}{2}, \frac{4}{3})$ (a)

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(16) "passes throug the point" $\rightarrow A = (2, 2, 8)$

"its direction vector" $\rightarrow \vec{d} = (3, 1, 4)$

$$\frac{x-2}{3} = \frac{y-2}{1} = \frac{z-8}{4}$$

هنعوض من بيها فوق

• let the point at x-ais $= (x_1, 0, 0)$

$$\frac{x_1-2}{3} = -2 = -2$$

$x_1 - 2 = -6$ $\boxed{x_1 = -4}$

$(-4, 0, 0)$ (a)

(17) equation of y-axis $\rightarrow x=0, z=0$ (a)

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(18) $\vec{r} = (2, -1, 3) + t(1, 2, -1)$

$$t = \frac{x-2}{1} = \frac{y+1}{2} = \frac{z-3}{-1}$$

$a \rightarrow (1,1,1) \rightarrow \frac{1-2}{1} \neq \frac{1+1}{2}$

$b \rightarrow (0,2,-2) \rightarrow \frac{0-2}{1} \neq \frac{2+1}{2}$

$c \rightarrow (3,1,2) \rightarrow \frac{3-2}{1} = \frac{1+1}{2} = \frac{2-3}{-1}$ // (c)

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(19) let the point $= (2k, k, z_1)$

$$\frac{x}{3} = \frac{y+1}{1} = \frac{z-3}{2} = t$$

$$\frac{x}{3} = \frac{y+1}{1}$$

$\frac{2k}{3} = k+1$, $2k = 3k+3$, $k = -3$

$(-6, -3, z_1)$

$$\frac{x}{3} = \frac{y+1}{1} = \frac{z-3}{2}$$

$\frac{-3+1}{1} = \frac{z_1-3}{2}$, $-4 = z_1 - 3$

$z_1 = -1$

$(-6, -3, -1)$ (a)

(20) $\underbrace{(5,2,4),\ (6,-1,2)}$, $(8,-7,k)$

↳ هجيب منه ال $\vec{d}$

$\vec{d} = (6-5, -1-2, 2-4)$

$\vec{d} = (1, -3, -2)$

$\vec{r} = A + t\,d \rightarrow (5,2,4)$ or $(6,-1,2)$ or $(8,-7,k)$

$\vec{r} = (5,2,4) + t(1,-3,-2)$

$\frac{x-5}{1} = \frac{y-2}{-3} = \frac{z-4}{-2} = t$, at point $(8,-7,k)$

$\frac{-7-2}{-3} = \frac{k-4}{-2}$, $k-4 = -6 \rightarrow \boxed{k=-2}$ (d)

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(21) "makes equal angles with coordinates axes"

$\vec{d} = (1,1,1)$, $A = (2,3,-5)$

$x-2 = y-3 = z+5$ (b)

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(22) $\vec{d} = (k,k,k)$ ال ratio بيكون ثابتة

d → $\frac{x-1}{\sqrt{3}} = \frac{1-y}{\sqrt{3}} = \frac{z-1}{\sqrt{3}}$

$\frac{x-1}{\sqrt{3}} = \frac{y-1}{-\sqrt{3}} = \frac{z-1}{\sqrt{3}}$ ال ratio بيكون مش ثابتة

(23) "moves parallel to x-axis"

$\vec{d} = (1,0,0)$, $A = (2,3,-3)$

$y = 3$, $z = -3$ (b)

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(24) "moves parallel to x-axis"

y, z are constants (c)

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(25) $x = \frac{y+5}{-2} = \frac{z+1}{3}$

$\frac{x}{1} = \frac{y+5}{-2} = \frac{z+1}{3}$, $\vec{d} = (1,-2,3)$ (d)

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(26) vector of y-axis $\rightarrow$ $x = 0$, $z = 0$

$(0,-1,0)$ (b)

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(27) $A = (0,0,0)$ $B = (3,-1,2)$

$B - A = \vec{d} = (3,-1,2)$

$$\vec{u} = \frac{(3,-1,2)}{\sqrt{3^2+(-1)^2+2^2}} = \left(\underset{\cos\theta_x}{\frac{3}{\sqrt{14}}}, \underset{\cos\theta_y}{\frac{-1}{\sqrt{14}}}, \underset{\cos\theta_z}{\frac{2}{\sqrt{14}}}\right)$$

$$\cos\theta_z = \frac{2}{\sqrt{14}}$$

(e)

(28) $$\frac{2x-3}{4} = \frac{y+5}{7} = \frac{3z-8}{6}$$

$$\frac{x-\frac{3}{2}}{2} = \frac{y+5}{7} = \frac{z-\frac{8}{3}}{2}$$

$\vec{d} = (2, 7, 2)$ (c)

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بتاع ال x,y,
يبقا ب 1

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(29) $$\frac{x-2}{3} = \frac{y+3}{2}, \quad \underline{z=4}$$

$\vec{d}\ (3, 2, 0)$ (b)

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(30) $2x+3y=5, \ z=4$ $\qquad (x, y, 0)$

$2x+3y=5$

$2x = 5-3y \quad \div 6$

$$\frac{x}{3} = \frac{\frac{5}{3}-y}{2}$$

$$\frac{x}{3} = \frac{y-\frac{5}{3}}{-2}$$

$(3, -2, 0)$ ✓ (c)

$3y = 5-2x \quad \div 6$

$$\frac{y}{2} = \frac{\frac{5}{2}-x}{3}$$

$$\frac{y}{2} = \frac{x-\frac{5}{2}}{-3}$$

$(-3, 2, 0)$ ✓

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(31) "direction of cosines" $\quad (L, m, n)$

$$L^2+m^2+n^2=1$$

$$L^2+m^2=1-n^2 \quad \text{(d)}$$

(32) "paralle to z-axis"
"passes through (a,b,c)

$(x=0\ ,\ y=0)$ XX   why ?

$(x=a\ ,\ y=b)$ —

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(33) $\vec{oy}=(0,-1,0)$

$\vec{oz}=(0,0,1)$ $A=(2,-1,4)$

the bisector line $\left(\frac{0+0}{2},\ \frac{-1+0}{2},\ \frac{0+1}{2}\right)$

$=\left(0,\ \frac{-1}{2},\ \frac{1}{2}\right)$

$\vec{d}$ // bisector line

$\vec{d}=\left(0,\ \frac{-1}{2},\ \frac{1}{2}\right)\to\times 2$ كرا كر دى ratios

$\vec{d}\ (0,-1,1)$

$A+t\vec{d}=(2,-1,4)+t(0,-1,1)$

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(34) $B=(1,0,2)$, $A=(0,2,1)$

$\vec{d}=\pm\vec{AB}=\pm(1,-2,1)$

$\frac{x-0}{1}=\frac{y-2}{-2}=\frac{z-1}{1}$ or $\frac{x-1}{1}=\frac{y-0}{-2}=\frac{z-2}{1}$

$x=\frac{y-2}{-2}=z-1$ (d) $x-1=\frac{y}{-2}=z-2$

(35) $A=(1,2,4)$, $B=(-2,0,5)$, $C=(1,4,0)$

$$M=\frac{A+B+C}{3}=\left(\frac{1-2+1}{3},\frac{2+0+4}{3},\frac{4+5+0}{3}\right)$$

$$=(0,2,3)$$

$$\vec{d}=\pm\vec{AM}=\pm(-1,0,-1)$$

$$\vec{r}=A+t\vec{d}$$

$$=(1,2,4)+t(-1,0,-1)$$ (c)

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(36) $\vec{r}=(2,1,-3)+t(1,-2,4)$

$$\left.\begin{aligned} x&=2+t\\ y&=1-2t\\ z&=-3+4t\end{aligned}\right\}$$ by changing the value of $\underline{t}$ you get diffrent points on the straight line.

(b)

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(37) $\vec{r}=(1,2,3)+t(-2,1,-1)$

$$\frac{x-1}{-2}=\frac{y-2}{1}=\frac{z-3}{-1}$$

$A=(x_1,y_1,2)$

$$\frac{x_1-1}{-2}=\frac{y_1-2}{1}=\boxed{\frac{2-3}{-1}=1}$$

- $\frac{x_1-1}{-2}=1$, $x_1-1=-2$, $x_1=-1$
- $\frac{y_1-2}{1}=1$, $y_1-2=1$, $y_1=3$

$A=(-1,3,2)$

At $B=(3,y_2,z_2)$

$$\boxed{\frac{3-1}{-2}}=\frac{y_2-2}{1}=\frac{z_2-3}{-1}=(-1)$$

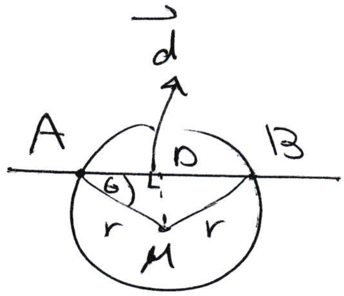
- $\frac{y_2-2}{1}=-1\rightarrow y_2=1$
- $\frac{z_2-3}{-1}=-1\rightarrow z_2=4$

$B=(3,1,4)$

$$\overline{AB}=\sqrt{(3+1)^2+(1-3)^2+(4-2)^2}$$

$$=2\sqrt{6}$$ (c)

(38) $\vec{d} = (2, 0, -2)$, $A = (0, -1, 3)$



eq. of sphere:

$$(x-2)^2 + (y+1)^2 + (z-3)^2 = 4$$

• $M = (2, -1, 3)$ • $r = 2$

→ $AB = 2AD$

$AD = AM \cos\theta$

$AD = 2 \times \dfrac{|\vec{d} \cdot \vec{AM}|}{\|\vec{d}\| \, \|\vec{AM}\|}$

$\vec{AM} = (2, 0, 0)$

$\vec{d} \cdot \vec{AM} = 4 + 0 + 0 = 4$

$\|\vec{d}\| = 2\sqrt{2}$

$\|\vec{AM}\| = 2$

$AD = \cancel{2} \times \dfrac{\cancel{4}\,2}{\cancel{2}\sqrt{2} \times \cancel{2}}$, $AD = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

$AB = 2AD = 2\sqrt{2}$ (b)

Second:-

[A] Choose:-

(1) $\vec{r}_1 = (-2, 5, 7) + k(-6, 6, 8)$

$\vec{d}_1 = (-6, 6, 8)$

$\vec{r}_2 = (1, -2, 3) + k'(4, 12, -6)$

$\vec{d}_2 = (4, 12, -6)$

$$\cos\theta = \frac{|\vec{d}_1 \cdot \vec{d}_2|}{\|\vec{d}_1\| \, \|\vec{d}_2\|} = \frac{|-24 + 72 - 48|}{2\sqrt{34}\,\sqrt{14}} = \frac{0}{2\sqrt{14}\sqrt{34}} = 0$$

$\theta = 90°$ (d)

(2) "Direction of Cosines" $= \vec{d}$

- $\vec{d}_1 = \left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right) \times 3 \to (2,-2,1)$ كدا كدا شغل
- $\vec{d}_2 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \times \sqrt{2} \to (1,1,0)$ ratios

$$\cos\theta = \frac{|\vec{d}_1 \cdot \vec{d}_2|}{\|\vec{d}_1\| \, \|\vec{d}_2\|} = \frac{|2-2+0|}{4\sqrt{2} \times \sqrt{2}} = \frac{0}{8} = 0$$

$\theta = 90^\circ$ (c)

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(3) • $2x = 3y = -z$

لازم ال coff. تاع ال x, y, z يتقابل [1]

$$\frac{2x}{1} = \frac{3y}{1} = \frac{-z}{1}$$

$$\frac{x}{\frac{1}{2}} = \frac{y}{\frac{1}{3}} = \frac{z}{-1} \to \vec{d}_1 = \left(\frac{1}{2}, \frac{1}{3}, -1\right) \times 6$$

$(3, 2, -6)$

• $6x = -y = -4z$

$$\frac{6x}{1} = \frac{-y}{1} = \frac{-4z}{1}$$

$$\frac{x}{\frac{1}{6}} = \frac{y}{-1} = \frac{z}{-\frac{1}{4}} \to \vec{d}_2 = \left(\frac{1}{6}, -1, \frac{-1}{4}\right) \times 12$$

$= (2, -12, -3)$

$$\cos\theta = \frac{|\vec{d}_1 \cdot \vec{d}_2|}{\|\vec{d}_1\| \, \|\vec{d}_2\|} = \frac{|6-24+18|}{7\sqrt{157}} = \frac{0}{7\sqrt{157}} = 0$$

$\theta = 90^\circ$ (c)

(4) $\vec{d}_1 = (1,1,2)$

$\vec{d}_2 = (1,1,4)$

$$\cos\theta = \frac{|\vec{d}_1 \cdot \vec{d}_2|}{\|\vec{d}_1\| \, \|\vec{d}_2\|} = \frac{|1+1+8|}{\sqrt{6}\,\sqrt{18}} = \frac{10}{6\sqrt{3}} = \frac{5\sqrt{3}}{9}$$

$$\theta = \cos^{-1}\left(\frac{5\sqrt{3}}{9}\right) \quad (b)$$

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(5) • $\frac{x-3}{2} = \frac{z+1}{-2}$, $y=1$ $\rightarrow \vec{d}_1 = (2,0,-2)$

• $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z+1}{-2} \rightarrow \vec{d}_2 = (1,2,-2)$

$$\cos\theta = \frac{|\vec{d}_1 \cdot \vec{d}_2|}{\|\vec{d}_1\| \, \|\vec{d}_2\|} = \frac{|2+4|}{2\sqrt{2} \times 3} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\theta = \cos^{-1}\frac{1}{\sqrt{2}} = 45^\circ \quad (c)$$

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(6) • $\frac{x-1}{\sqrt{2}} = \frac{y-\sqrt{2}}{1} = \frac{z+1}{1} \rightarrow \vec{d}_1 = (\sqrt{2}, 1, 1)$

• positive direction of x-axis $\rightarrow \vec{d}_2 = (0,0,1)$

$$\cos\theta = \frac{|\vec{d}_1 \cdot \vec{d}_2|}{\|\vec{d}_1\| \, \|\vec{d}_2\|} = \frac{|1|}{2 \times 1} = \frac{1}{2}$$

$$\theta = 60^\circ \quad (e)$$

(7) $\dfrac{x}{a} = \dfrac{y}{2} = \dfrac{z}{1}$ $\qquad \vec{d_1} = (a, 2, 1)$

$\dfrac{x}{2} = \dfrac{y}{1} = \dfrac{z}{-1}$ $\qquad \vec{d_2} = (2, 1, -1)$

$\cos\theta = \cos 60 = \dfrac{1}{2} = \dfrac{|\vec{d_1} \cdot \vec{d_2}|}{\|\vec{d_1}\| \, \|\vec{d_2}\|}$

$\dfrac{1}{2} = \dfrac{|2a + 2 - 1|}{\sqrt{a^2+5}\,\sqrt{6}}$

$\therefore \sqrt{a^2+5} \times \sqrt{6} = 4a + 2$ $\rightarrow$ shif solve

by squaring

$\therefore (a^2+5) \times 6 = (4a+2)^2$

$\therefore 6a^2 + 30 = 16a^2 + 16a + 4$

$\therefore 10a^2 + 16a - 26 = 0 \rightarrow a = 1$ or $\dfrac{-13}{5}$ (b)

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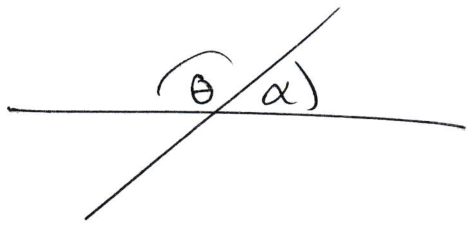
(8) • $x = 0,\ y = z \rightarrow \vec{d_1} = (0, 1, 1)$

• $y = 0,\ x = z \rightarrow \vec{d_2} = (1, 0, 1)$

$\cos\alpha = \dfrac{|\vec{d_1} \cdot \vec{d_2}|}{\|\vec{d_1}\| \, \|\vec{d_2}\|} = \dfrac{1}{\sqrt{2} \times \sqrt{2}} = \dfrac{1}{2}$



$\alpha = 60^\circ \rightarrow \theta = 180 - 60 = 120^\circ$

(9) * L_1: $x = 2y+1$, $2y = 1-z$

$x = 2y+1$ ($\div 2$)

$$\frac{x}{2} = \frac{y+\frac{1}{2}}{1}$$

$2y = 1-z$ ($\div 2$)

$$\frac{y}{1} = \frac{1-z}{2}$$

$$\frac{y}{1} = \frac{z-1}{-2}$$

$$\vec{d}_1 = (2, 1, -2)$$

* L_2: $2x+y+z=0$, $z+2=0$

$z = -2$

$2x+y-2=0$

$2x-2=-y$ ($\div 2$)

$$\frac{x-1}{1} = \frac{-y}{2}$$

$$\frac{x-1}{1} = \frac{y}{-2}$$

$$\vec{d}_2 = (1, -2, 0)$$

$$\cos\theta = \frac{|\vec{d}_1 \cdot \vec{d}_2|}{\|\vec{d}_1\| \, \|\vec{d}_2\|} = \frac{|2-2+0|}{3\sqrt{5}} = 0$$

$\theta = 90°$ $\frac{\pi}{2}$ (d)

Third :-

[1] Choose :-

(1). $\vec{A} = (-2, K, -3)$

$\cdot \dfrac{x+2}{4} = \dfrac{y}{8} = \dfrac{z-1}{6} \rightarrow \vec{d} = (4, 8, 6)$

$\dfrac{-2}{4} = \dfrac{K}{8} = \dfrac{-3}{6} \rightarrow \boxed{K = -4}$ (a)

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(2) $\cdot \dfrac{x+3}{2} = \dfrac{y+1}{-6} = \dfrac{z-2}{K} \rightarrow \vec{d_1} = (2, -6, K)$

$\cdot \dfrac{x+2}{4} = \dfrac{y-5}{m} = \dfrac{z-1}{3} \rightarrow \vec{d_2} = (4, m, 3)$

$\dfrac{2}{4} = \dfrac{-6}{m} = \dfrac{K}{3} \rightarrow m = -12$

$\rightarrow K = 1.5$

$K + m = -12 + 1.5 = \dfrac{-21}{2}$ (d)

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(3) $\cdot \dfrac{x+2}{6} = \dfrac{y-1}{m} = \dfrac{z-1}{3} \rightarrow \vec{d_1} = (6, m, 3)$

$\cdot \dfrac{x-9}{-2} = \dfrac{y+8}{1}, \ z = 3 \rightarrow \vec{d_2} = (-2, 1, 0)$

$d_1 \perp d_2 \rightarrow d_1 \cdot d_2 = 0$

$(6, m, 3) \cdot (-2, 1, 0) = -12 + m + 0 = 0$

$m = 12$ (d)

(4) $\cdot L_1: \dfrac{x+2}{-1} = \dfrac{y+3}{3} = \dfrac{z+5}{2}$

$$\vec{d}_1 = (-1, 3, 2)$$

$\cdot L_2: \dfrac{x}{2} = \dfrac{y-5}{k} = \dfrac{z-6}{m}$

$$\vec{d}_2 = (2, k, m)$$

$\vec{d}_1 \perp \vec{d}_2 \rightarrow \vec{d}_1 \cdot \vec{d}_2 = 0$

$$-2 + 3k + 2m = 0$$

$$3k + 2m = 2 \quad (b)$$

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(5) $\cdot L_1: x = 2t_1 - 1 \quad , y = t_1 + 1 \quad , z = t_1 - 1$

$$\vec{d}_1 = (2, 1, 1)$$

$\cdot L_2: x = at_2 - 1 \quad , y = 2t_2 + 1 \quad , z = bt_2 - 2$

$$\vec{d}_2 = (a, 2, b)$$

$\dfrac{2}{a} = \dfrac{1}{2} = \dfrac{1}{b} \quad \rightarrow a = 4$

$\rightarrow b = 2$

$$a + b = 4 + 2 = 6 \quad (c)$$

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(6) $\cdot L_1: \vec{r} = t_1 (-2, m, 7)$

$\vec{d}_1 \perp \vec{d}_2$

$$\vec{d}_1 = (-2, m, 7)$$

$\vec{d}_1 \cdot \vec{d}_2 = 0$

$\cdot L_2: \dfrac{x-1}{n} = \dfrac{1-y}{4} = \dfrac{z-2}{2}$

$\dfrac{x-1}{n} = \dfrac{y-1}{-4} = \dfrac{z-2}{2} \qquad \vec{d}_2 = (n, -4, 2)$

$$-2n - 4m + 14 = 0$$

$2n + 4m = 14 \quad \rightarrow n + 2m = 7 \quad (a)$

(7) . L_1: $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ $\hookrightarrow \vec{d}_1 = (1,2,3)$

. L_2: $\frac{x}{2} = \frac{y+2}{2} = \frac{z-3}{-2}$ $\hookrightarrow \vec{d}_2 = (2,2,-2)$

. a) $\vec{d}_1 \cdot \vec{d}_2 = 2+4-6 = 0$

they are perpendicular (a)

---

(8) $\vec{r}_1 = \vec{A}_1 + t_1\vec{d}_1$

$r_2 = \vec{A}_2 + t_2\vec{d}_2$

$\vec{d}_1 = L\,\vec{d}_2$ $\quad L \neq 0$ (a)

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(9) $\vec{d}_1 = (a,b,c)$, $\vec{d}_2 = (a',b',c')$

$\vec{d}_1 \perp \vec{d}_2 \longrightarrow \vec{d}_1 \cdot \vec{d}_2 = 0$

$aa' + bb' + cc' = 0$ (b)

(10) • L_1: $\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{7}$

$\to \vec{d}_1 = (2,4,7)$

$\to \boxed{A = (1,2,3)}$

• L_2: $\frac{x-1}{4} = \frac{y-2}{5} = \frac{z-3}{7}$

$\to \vec{d}_2 = (4,5,7)$

$\to \boxed{A = (1,2,3)}$

∵ they have the same point

∴ they are intersecting (b)

---

(11) • $x = y = z \to \frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ $\to \boxed{A = (0,0,0)}$

$\to \vec{d} = (1,1,1)$

• $2x = 3y = z$ $(\div 6)$ $\to \frac{x}{3} = \frac{y}{2} = \frac{z}{6}$ $\to \boxed{A = (0,0,0)}$

$\to \vec{d} = (3,2,6)$

∵ they have the same point

∴ they are intersecting $\to$ a, c xxx

$\vec{d}_1 \cdot \vec{d}_2 = 3 + 2 + 6 = 11 \neq 0$ "Not perpendicular"

Intersecting but Not perpendicular (d)

(12) $\vec{r}_1 = (1,2,3) + t_1(1,2,3)$

$\vec{r}_2 = (-1,2,0) + t_2(m,2,-1)$

$$\left.\begin{aligned} x_1 &= 1 + t_1 \\ y_1 &= 2 + 2t_1 \\ z_1 &= 3 + 3t_1 \end{aligned}\right\} \quad \begin{aligned} x_2 &= -1 + mt_2 \\ y_2 &= 2 + 2t_2 \\ z_2 &= 0 - t_2 \end{aligned}$$

- At $y_1 = y_2 \rightarrow 2 + 2t_1 = 2 + 2t_2$

$\boxed{t_1 - t_2 = 0} \rightarrow$ 1

- At $z_1 = z_2 \rightarrow 3 + 3t_1 = 0 - t_2$

$\boxed{3t_1 + t_2 = -3} \rightarrow$ 3

From 1, 2 $\rightarrow t_1 = \frac{-3}{4}$ $\quad t_2 = \frac{-3}{4}$

- At $x_1 = x_2$

$1 + t_1 = -1 + mt_2$

$1 - \frac{3}{4} = -1 - \frac{3}{4}m$

$-\frac{3}{4}m = \frac{5}{4} \rightarrow m = \frac{-5}{3}$ (b)

Fourth :-

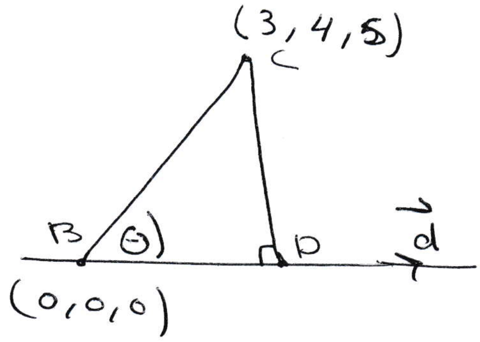
[1] Choose :-

(1) • direction of y-axis is $\vec{d} = (0,1,0)$

• $(0,0,0) \in$ y-axis

$$CD = \frac{\|\vec{BC} \times \vec{d}\|}{\|\vec{d}\|}$$

$$CD = \frac{\sqrt{5^2+3^2}}{1} = \sqrt{34} \quad (c)$$



• $\vec{BC} = (3,4,5)$

• $\vec{BC} \times \vec{d}$

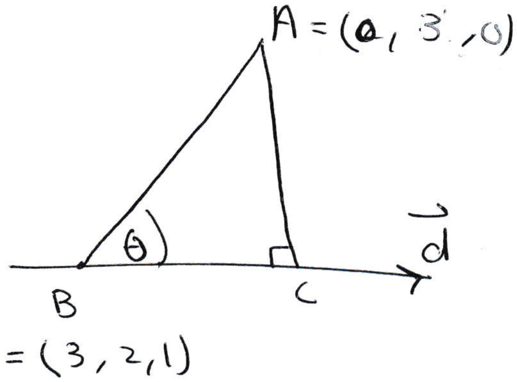
$$= \begin{vmatrix} 3 & 4 & 5 \\ 0 & 1 & 0 \end{vmatrix}$$

$$= (-5, 0, 3)$$

---

(2) $\vec{r} = (3,2,1) + t(2,-1,4)$

• $AC = \dfrac{\|\vec{AB} \times \vec{d}\|}{\|\vec{d}\|}$



• $\vec{d} = (2,-1,4)$, $\|\vec{d}\| = \sqrt{21}$

• $\vec{AB} = (3,-1,1)$

$$\vec{AB} \times \vec{d} = \begin{vmatrix} 3 & -1 & 1 \\ 2 & -1 & 4 \end{vmatrix} = (-3, -10, -1)$$

$$\|\vec{AB} \times \vec{d}\| = \sqrt{3^2+10^2+1^2} = \sqrt{110}$$

$$AC = \frac{\sqrt{110}}{\sqrt{21}} = \sqrt{\frac{110}{21}} \quad (d)$$

(3) $\cdot\ 2x-4 = \dfrac{2y-8}{3} = \dfrac{2z-14}{5}$ $\quad \div 2$

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-7}{5}$$

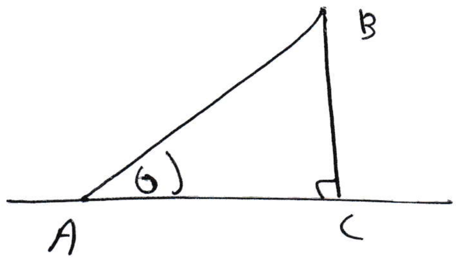
$\rightarrow A = (2,4,7)$

$\hookrightarrow \vec{d} = (1,3,5)$

$\cdot$ let $B = (2,4,7)$

$BC = AB \sin\theta$

$= AB \times \dfrac{\|\vec{AB} \times \vec{d}\|}{\|\vec{AB}\|\ \|\vec{d}\|}$



$BC = \dfrac{\|\vec{AB}\times\vec{d}\|}{\|\vec{d}\|}$

$, \|\vec{d}\| = \sqrt{35}$

$, \vec{AB} = (0,0,0)$

$\cdot\ \vec{AB}\times\vec{d} = \begin{vmatrix} 0 & 0 & 0 \\ 1 & 3 & 5 \end{vmatrix} = (0,0,0)$

$\|\vec{AB}\times\vec{d}\| = 0$

$BC = \dfrac{0}{\sqrt{35}} = \text{Zero}$ (a)

* point B lies on the straight line بالنظر

$A = B = (2,4,7)$

---

(4)

$\sqrt{(\text{الالل متقالش})^2} = \sqrt{b^2+c^2}$

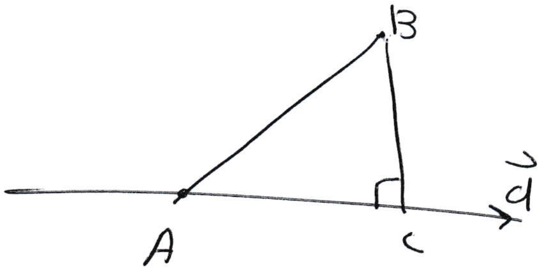
(d)

(5) $\cdot \dfrac{x-1}{2} = \dfrac{y-1}{1} = \dfrac{z+1}{-1}$ $\rightarrow A = (1, 1, -1)$

$\rightarrow \vec{d} = (2, 1, -1)$

$\cdot$ let $B = (-1, 0, 1)$

$\cdot\ BC = \dfrac{\|\vec{AB} \times \vec{d}\|}{\|\vec{d}\|}$



$\cdot\ \|\vec{d}\| = \sqrt{6}$

$\cdot\ \vec{AB} = (-2, -1, 2)$

$\cdot\ \vec{AB} \times \vec{d} = \begin{vmatrix} -2 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix} = (-1, 2, 0)$

$\cdot\ \|\vec{AB} \times \vec{d}\| = \sqrt{1 + 2^2 + 0^2} = \sqrt{5}$

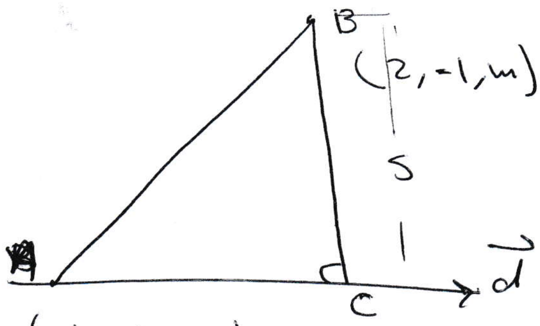
$BC = \dfrac{\sqrt{5}}{\sqrt{6}} = \dfrac{\sqrt{30}}{6}$ c

---

(6) $x - 1 = 3 - y = z$

$\dfrac{x-1}{1} = \dfrac{3-y}{1} = \dfrac{z}{1}$

$= \dfrac{x-1}{1} = \dfrac{y-3}{-1} = \dfrac{z}{1}$ $\rightarrow A = (1, 3, 0)$



$\rightarrow \vec{d} = (1, -1, 1)$, $\|\vec{d}\| = \sqrt{3}$

$BC = \dfrac{\|AB \times \vec{d}\|}{\|\vec{d}\|} = 5 = \dfrac{\|\vec{AB} \times \vec{d}\|}{\sqrt{3}}$ c 2

$\vec{AB} = (1, -4, m)$

$\vec{AB} \times \vec{d} = \begin{vmatrix} 1 & -4 & m \\ 1 & -1 & 1 \end{vmatrix}$

$= (-4 + m, m - 1, 3)$

$\|\vec{AB} \times \vec{d}\|^2 = (-4 + m)^2 + (m-1)^2 + 3^2$

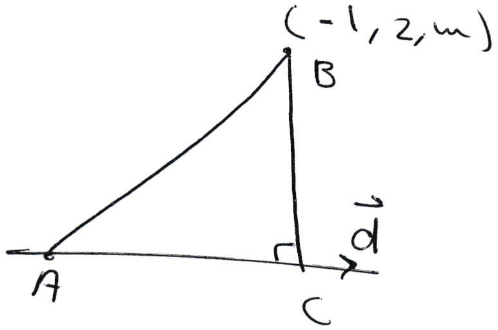
$= (5\sqrt{3})^2$

$16 - 8m + m^2 + m^2 - 2m + 1 + 9 = 75$

$2m^2 - 10m - 49 = 0$

$m = -3.045$, $m = 8$

(7) $\vec{r} = (-1, 3, 0) + t(0, -3, 0)$



$\vec{d} = (0, -3, 0)$, $\|\vec{d}\| = 3$

$A = (-1, 3, 0)$

$BC = \dfrac{\|\vec{AB} \times \vec{d}\|}{\|\vec{d}\|} = 8 = \dfrac{\|\vec{AB} \times \vec{d}\|}{3}$, $\underline{\|\vec{AB} \times \vec{d}\| = 24}$

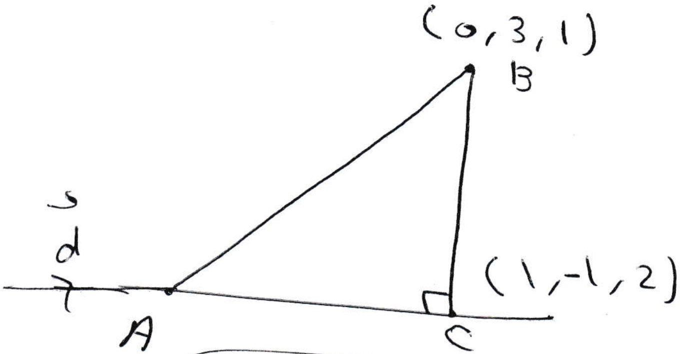
$\vec{AB} = (0, -1, m)$

$\vec{AB} \times \vec{d} = \begin{vmatrix} 0 & -1 & m \\ 0 & -3 & 0 \end{vmatrix} = (3m, 0, 0)$

$\|\vec{AB} \times \vec{d}\|^2 = 9m^2 = 24^2$

$m^2 = 64 \rightarrow m = 8$ $\quad$ $m = -8$ refused

(8) $\dfrac{x-a}{l} = \dfrac{y-2}{3} = \dfrac{z-b}{4}$



$A = (a, 2, b)$

$\vec{d} = (l, 3, 4)$

$\because \vec{BC} \perp \vec{d}$

$\therefore \vec{BC} \cdot \vec{d} = 0$

$\vec{BC} = (1, -4, 1)$

$1(l) - 4 \times 3 + 4(1) = l - 12 + 4 = 0 \rightarrow \underline{l = 8}$

$\vec{d} = (8, 3, 4)$

$\dfrac{x-a}{8} = \dfrac{y-2}{3} = \dfrac{z-b}{4} = t \quad \rightarrow$ نعوض بال C $\quad$ (d)

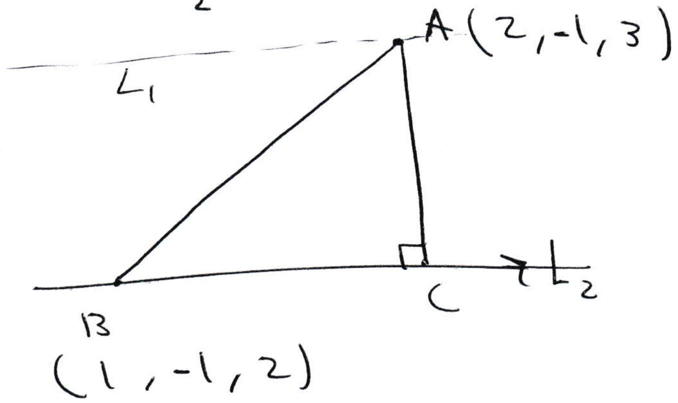
$\dfrac{1-a}{8} = \dfrac{-3}{3} = \dfrac{2-b}{4} = -1$ $\quad \left\{ \begin{array}{l} \dfrac{1-a}{8} = -1 \rightarrow a = 9 \\ \dfrac{2-b}{4} = -1 \rightarrow b = 6 \end{array} \right.$ $\quad \left\{ \begin{array}{l} a + b \\ = 9 + 6 \\ = 15 \end{array} \right.$

(9) $L_1: \vec{r} = (2,-1,3) + t_1(-4,-4,2)$

$L_2: \vec{r} = (1,-1,2) + t_2(2,2,-1)$

$\vec{d_1} = (-4,-4,2) \quad , \vec{d_2} = (2,2,-1)$

$\because \frac{-4}{2} = \frac{-4}{2} = \frac{2}{-1} \quad \therefore L_1 \parallel L_2$



• $AC = \frac{\|\vec{BA} \times \vec{d_2}\|}{\|\vec{d_2}\|}$

• $\|\vec{d_2}\| = 3$

• $\vec{BA} = (1,0,1)$

$\vec{BA} \times \vec{d_2} = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & -1 \end{vmatrix} = (-2,3,2)$

$\|\vec{BA} \times \vec{d_2}\| = \sqrt{17}$

$AC = \frac{\|\vec{BA} \times \vec{d_2}\|}{\|\vec{d_2}\|} = \frac{\sqrt{17}}{3}$ c

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