Exercise 4

[9] Choose

(1) $\vec{A} = (3, 0, 4)$, $\vec{B} = (1, -2, 3)$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 3 & 0 & 4 \\ 1 & -2 & 3 \end{vmatrix} = i \begin{vmatrix} 0 & 4 \\ -2 & 3 \end{vmatrix} - j \begin{vmatrix} 3 & 4 \\ 1 & 3 \end{vmatrix} + k \begin{vmatrix} 3 & 0 \\ 1 & -2 \end{vmatrix}$$

$$= 8i - (9-4)j + k(-6)$$

$$= 8i - 5j - 6k \quad = (8, -5, -6) \quad (a)$$

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(2) $\vec{A} = (1, -3, 1)$, $\vec{B} = (1, 1, 1)$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 1 & -3 & 1 \\ 1 & 1 & 1 \end{vmatrix} = (-3-1)i - (1-1)j + (1+3)k$$

$$= -4i + 4k$$

$$\vec{A} \times \vec{B} = (-4, 0, 4) \rightarrow \|\vec{A} \times \vec{B}\| = \sqrt{4^2 + 4^2}$$

$$= 4\sqrt{2} \quad (a)$$

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(3) $\|\vec{A}\| = 5$

$\|\vec{B}\| = 4$, $\theta = 30$

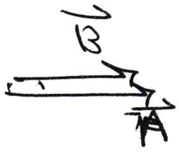
$$\|\vec{A} \times \vec{B}\| = \|\vec{A}\| \, \|\vec{B}\| \sin\theta$$

$$= 5 \times 4 \sin 30$$

$$= 10 \quad (c)$$

(4) $\vec{A} \parallel \vec{B}$ either they are

[have same direction] [have oppisite direction]



$\theta = 0$

$\vec{A} \times \vec{B} = \|\vec{A}\| \, \|\vec{B}\| \sin(0)$

$= \text{Zero}$

(but with direction)

$\theta = 180^\circ$

$\vec{A} \times \vec{B} = \|\vec{A}\| \, \|\vec{B}\| \sin 180$

$= \text{Zero}$

(but with direction)

$\to \vec{0}$ (a)

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(5) $\|\vec{A} \times \vec{B}\| = 36\sqrt{3}$

$\|\vec{A}\| = 8$, $\|\vec{B}\| = 9$

$$\sin\theta = \frac{\|\vec{A} \times \vec{B}\|}{\|\vec{A}\| \, \|\vec{B}\|} = \frac{36\sqrt{3}}{8 \times 9} = \frac{\sqrt{3}}{2}$$

$\theta = \sin^{-1} \frac{\sqrt{3}}{2} = 60^\circ$ or 120 (b)

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(6) $\vec{A} = (1,1,1)$, $\vec{B} = (2,-1,-1)$

* $\vec{A} - \vec{B} = (-1, 2, 2)$

* $\vec{A} \times (\vec{A} - \vec{B}) = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ -1 & 2 & 2 \end{vmatrix}$

$= -3\hat{j} + 3k$

(b)

$\to$ Another Solution

$\vec{A} \times (\vec{A} - \vec{B})$

$= \vec{A} \times \vec{A} - \vec{A} \times \vec{B}$

$= \vec{0} - \vec{A} \times \vec{B} = -\vec{A} \times \vec{B}$

$= - \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & -1 & -1 \end{vmatrix}$

$= 3\hat{j} + 3k$

(7) $(2, k, -3) \parallel (4, 6, -6)$

$$\frac{2}{4} = \frac{k}{6} = \frac{-3}{-6} \rightarrow \underline{\underline{k = 3}} \quad (b)$$

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(8) $\vec{A} = (4, -k, 6)$, $\vec{B} = (2, 2, m)$

$$\frac{4}{2} = \frac{-k}{2} = \frac{6}{m}$$

* $\frac{-k}{2} = 2 \rightarrow k = -4$ * $\frac{6}{m} = 2 \rightarrow m = 3$

* $k + m = -4 + 3 = -1$ (c)

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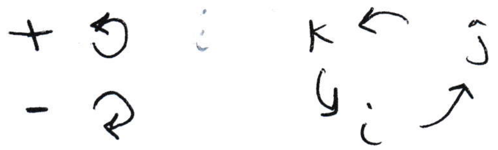
(9) $i \times j = k$

(d)



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(10) * a $\rightarrow \hat{i} \cdot \hat{j} = 0$ "because they are $\underline{\underline{\text{perpendicular}}}$"

* b $\rightarrow \hat{i} \cdot \hat{i} = 1$ ✓

* c $\rightarrow \hat{i} \times \hat{j} = \hat{k}$

* d $\rightarrow i \times (j \times \hat{k}) = i \times -i = \vec{0}$

(b)

(11) $\vec{A} = (4, -5, 1)$, $\vec{B} = (2, -k, -2)$, $\vec{C} = (-4, 4, m-2)$

$\overrightarrow{AB} = \vec{B} - \vec{A} = (-2, 5-k, -3)$

$\overrightarrow{AB} \parallel \vec{C} \rightarrow \frac{-4}{-2} = \frac{4}{5-k} = \frac{m-2}{-3}$

$* \quad \frac{4}{5-k} = 2 , \quad 5-k = 2 \rightarrow k = 3$

$* \quad \frac{m-2}{-3} = 2 , \quad m-2 = -6 \rightarrow m = -4$

$k - m = 3 + 4 = 7$ (b)

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(12) $(\vec{A} \times \vec{B}) \cdot (\vec{B} \times \vec{A})$ $\quad \vec{A} = (1, 0, 2)$

$= (\vec{A} \times \vec{B}) \cdot -(\vec{A} \times \vec{B})$ $\quad \vec{B} = (2, -1, -2)$

$= - \|\vec{A} \times \vec{B}\|^2$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 1 & 0 & 2 \\ 2 & -1 & -2 \end{vmatrix} = (2, 6, -1)$$

$* \quad \|\vec{A} \times \vec{B}\| = \sqrt{2^2 + 6^2 + 1} = \sqrt{41}$

$- \|\vec{A} \times \vec{B}\|^2 = -(\sqrt{41})^2 = -41$ (d)

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(13) $\|(\vec{A} - \vec{B}) \times (\vec{A} + \vec{B})\|$

$= \|\vec{A} \times \vec{A} + \vec{A} \times \vec{B} - \vec{B} \times \vec{A} - \vec{B} \times \vec{B}\|$

$= \|0 + \vec{A} \times \vec{B} + \vec{A} \times \vec{B} - 0\|$

$= \|2 \vec{A} \times \vec{B}\| = 2 \|\vec{A} \times \vec{B}\|$

$= 2 \|\vec{A}\| \|\vec{B}\| \sin\theta$

$= 2 \times 1 \times 1 \sin 90^\circ$

$= 2$ (d)

$\|\vec{A}\|$ $\|\vec{B}\|$ Are unit vectors

$\vec{A} \perp \vec{B}$

$\theta = 90$

(14) $\|\vec{A} \times \vec{B}\| = 4$

- let the unit vector that is perpendicular on $\vec{A}$ and $\vec{B}$ $= \vec{U}_c$, $\|\vec{U}_c\| = 1$

$\vec{U}_c \propto \vec{A} \times \vec{B}$ or $\vec{U}_c \parallel \vec{A} \times \vec{B}$

$\vec{U}_c = K(\vec{A} \times \vec{B})$ $\longrightarrow$ $\vec{U}_c = \pm \frac{1}{4}(\vec{A} \times \vec{B})$ (b)

$\|\vec{U}_c\| = \|K(\vec{A} \times \vec{B})\|$

$\|\vec{U}_c\| = |K| \, \|\vec{A} \times \vec{B}\|$

$1 = |K| \times 4$, $|K| = \frac{1}{4}$

$K = \pm \frac{1}{4}$

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(15) $(\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j}$

$= \hat{k} \cdot \hat{k} + 0$

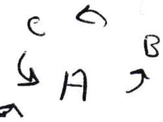
$= \|k\|^2 = 1$ (c)

(16) $\vec{A} \times \vec{B} = \vec{C}$

a) $\rightarrow \underset{\sim\sim\sim}{\vec{B} \times \vec{C} = \vec{A}}$ XX $\rightarrow$ لازم ال angle بين تبقا

90° عشان أقدر أطبق القانون

دا



b) $\rightarrow \vec{A} \times \vec{C} = -\vec{B}$ XX

c) $\rightarrow \vec{A} \times (\vec{B} * \vec{A}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{A}$

$= \vec{C} + \vec{0} = \vec{C}$ ✓✓

d) $\rightarrow \vec{B} \times \vec{A} = -\vec{C}$ XX

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(17) $\vec{A} = (x, y, z)$, $i = (1, 0, 0)$

. $\vec{A} \times \vec{i} = \begin{vmatrix} i & j & k \\ x & y & z \\ 1 & 0 & 0 \end{vmatrix} = -z\hat{j} + yk$

$\| \vec{A} \times \vec{i} \|^2 = z^2 + y^2$

. $\vec{A} \times \vec{j} = \begin{vmatrix} i & j & k \\ x & y & z \\ 0 & 1 & 0 \end{vmatrix} = -z\hat{i} + xk$

$\| \vec{A} \times \vec{j} \|^2 = z^2 + x^2$

. $\vec{A} \times \vec{K} = \begin{vmatrix} i & j & k \\ x & y & z \\ 0 & 0 & 1 \end{vmatrix} = yi - x\hat{j}$

$\| \vec{A} \times \vec{K} \|^2 = y^2 + x^2$

$\| \vec{A} \times \hat{i} \|^2 + \| \vec{A} \times \hat{j} \|^2 + \| \vec{A} \times \hat{k} \|^2$

$= z^2 + y^2 + z^2 + x^2 + y^2 + x^2$

$= 2(x^2 + y^2 + z^2)$

$= 2\|\vec{A}\|^2$ ⓒ

(18) $\vec{A} \parallel \vec{B}$

* a $\rightarrow$ $\vec{A} = K\vec{B}$ ✓

* b $\rightarrow$ $\dfrac{a_x}{b_x} = \dfrac{a_y}{b_y} = \dfrac{a_z}{b_z}$ ✓

* c $\rightarrow$ $\vec{A} \cdot \vec{B} = \|\vec{A}\| \, \|\vec{B}\| \cos 0 = \|\vec{A}\| \, \|\vec{B}\| \neq 0$ ⓒ

* d $\rightarrow$ $\vec{A} \times \vec{B} = \|\vec{A}\| \, \|\vec{B}\| \sin 0 = \text{Zero}$ ⓓ

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(19) $\vec{A} \cdot \vec{B} = \|\vec{A}\| \, \|\vec{B}\| \cos\theta$

$= 9 \times 16 \cos\theta = -72\sqrt{3}$

$\cos\theta = \dfrac{-72\sqrt{3}}{9 \times 16} = \dfrac{-\sqrt{3}}{2}$

$\theta = 150^\circ$

$\vec{A} \times \vec{B} = \|\vec{A}\| \, \|\vec{B}\| \sin\theta$

$= 9 \times 16 \sin 150 = 72$

$\pm 72 \vec{C}$ ⓒ

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(20) $\vec{A} = (x, y, z)$, $\|A\|^2 = x^2 + y^2 + z^2 = 13$

$\vec{B} = (-2, 3, 5)$ $\quad \|A\| = \sqrt{13}$

$\vec{C} = (1, -1, 4)$

$\vec{D} = (1, 3, -2)$

$\vec{DC} = \vec{C} - \vec{D} = (0, -4, 6)$

$\vec{A} = \|\vec{A}\| \times \vec{U}_A = \|\vec{A}\| \times \vec{U}_{DC} = \sqrt{13} \left(\dfrac{0}{\sqrt{4^2+6^2}}, \dfrac{-4}{\sqrt{4^2+6^2}}, \dfrac{6}{\sqrt{4^2+6^2}} \right)$

$\vec{A} = (0, -2, 3)$

$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 0 & -2 & 3 \\ -2 & 3 & 5 \end{vmatrix} = (-19, -6, -4)$

(21) $\vec{C} = \vec{A} + \vec{B} \quad , \vec{E} = \vec{A} - \vec{B}$

$$\vec{C} \times \vec{E} = 3\vec{e}$$

$$(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})$$

$$= \cancel{\vec{A} \times \vec{A}}^{\,0} - \vec{A} \times \vec{B} + \vec{B} \times \vec{A} - \cancel{\vec{B} \times \vec{B}}^{\,0} = 3\vec{e}$$

$$= -2\,\vec{A} \times \vec{B} = 3\vec{e}$$

$$\vec{A} \times \vec{B} = \frac{3}{-2}\vec{e} \quad (a)$$

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(22) $\vec{A} = \quad \hat{i} \times 2\hat{j} \quad 2\hat{i} \times \hat{k} \quad \hat{j} \times \hat{k}$

$\rightarrow \hat{i} \times 2\hat{j} = 2\hat{k}$

$\rightarrow 2\hat{i} \times \hat{k} = -2\hat{j}$

$\rightarrow \hat{j} \times \hat{k} = \hat{i}$

$$\vec{A} = (1, -2, 2) \rightarrow \|\vec{A}\| = \sqrt{1 + 2^2 + 2^2} = \sqrt{9} = 3$$

$$\vec{U}_A = \frac{\vec{A}}{\|\vec{A}\|} = \frac{(1, -2, 2)}{3} = \left(\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}\right) \quad (d)$$

[30] Choose

(1) $(1, 1, 1) \rightarrow \vec{A} \rightarrow \|\vec{A}\| = \sqrt{3}$

$(2, -1, 2) \rightarrow \vec{B} \rightarrow \|\vec{B}\| = 3$

$$\|\vec{A} \times \vec{B}\| = \|A\| \, \|B\| \sin\theta$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 2 \end{vmatrix} = (3, 0, -3) \rightarrow \|\vec{A} \times \vec{B}\| = 3\sqrt{2}$$

$$\sin\theta = \frac{3\sqrt{2}}{3\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}} \quad (b)$$

(2) $\vec{A} \times \vec{B} = \vec{0} \longrightarrow$ " $\vec{A} \parallel \vec{B}$ "

$\vec{A} \cdot \vec{C} = 0 \longrightarrow$ " $\vec{A} \perp \vec{B}$ "

$\vec{B} \cdot \vec{C}$ " $B \perp C$ "

$\vec{B} \cdot \vec{C} = 0$ (a)

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(3) $\hat{i} + \hat{j} \rightarrow (1, 1, 0) \rightarrow \vec{A}$

$\hat{j} + \hat{k} \longrightarrow (0, 1, 1) \rightarrow \vec{B}$

$\vec{A} \times \vec{B} = \vec{C}$ "where $\vec{C}$ is perpendicular on both $\vec{A}$, $\vec{B}$ "

$$\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = (1, -1, 1) \rightarrow \|\vec{C}\| = \sqrt{3}$$

$$\vec{U}_C = \frac{(1, -1, 1)}{\sqrt{3}} = \frac{i - \hat{j} + \hat{k}}{\sqrt{3}} \quad \text{(b)}$$

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(4) $\vec{A} = (3, -6, -1)$

$\vec{B} = (1, 4, -3)$

$\vec{C} = (3, -4, -12)$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & -1 \\ 1 & 4 & -3 \end{vmatrix}$$

$$= (22, 8, 18)$$

$$\vec{A} \times \vec{B}_C = \frac{(\vec{A} \times \vec{B}) \cdot \vec{C}}{\|\vec{C}\|}$$

$$= \frac{66 - 32 - 216}{\sqrt{3^2 + 4^2 + 12^2}}$$

$$= \frac{-182}{13}$$

$$= -14 \quad \text{(b)}$$

$$C \cdot (\vec{A} \times \vec{B}) = \begin{vmatrix} 3 & -4 & -12 \\ 3 & -6 & -1 \\ 3 & -4 & -12 \end{vmatrix}$$

ممكن تحلها بطريقة
ال determinant

(5) $\vec{A}\cdot\vec{B} = \|\vec{A}\times\vec{B}\|$

$\|\vec{A}\|\,\|\vec{B}\|\cos\theta = \|\vec{A}\|\,\|\vec{B}\|\sin\theta$

$\tan\theta = 1 \rightarrow \theta = 45^\circ$ (b)

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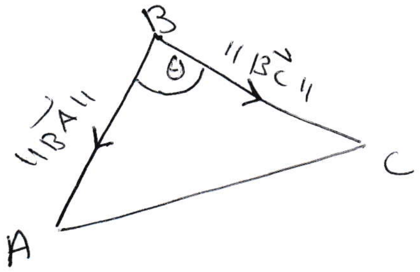
(6) $\|\vec{BA}\times\vec{BC}\|$

$= \|\vec{BA}\| \times \|\vec{BC}\|\sin\theta$



Area $= \frac{1}{2}ab\sin\theta$

$= 24 = \frac{1}{2}\|\vec{BA}\| \times \|\vec{BC}\|\sin\theta)$

$\|\vec{BA}\| \times \|\vec{BC}\|\sin\theta = 48$ (d)

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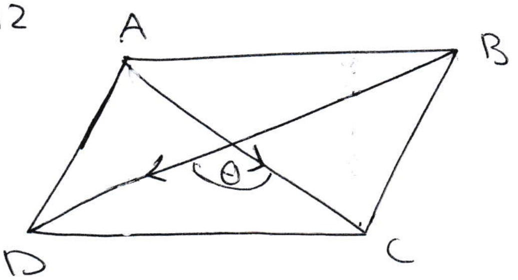
(7) Area of the $\square = 12$

$= \frac{1}{2}d_1 d_2 \sin\theta$



$= \frac{1}{2}\|\vec{AC}\|\,\|\vec{BD}\|\sin\theta$

$= 12$

$\|\vec{AC}\|\,\|\vec{BD}\| = 24$ (c)

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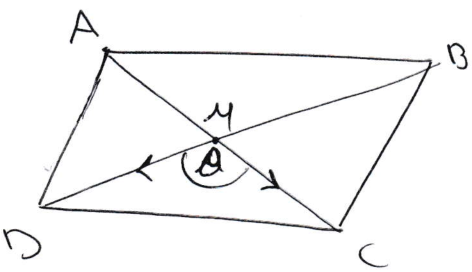
(8) $\|\vec{MD}\times\vec{MC}\| = \|\vec{MD}\|\,\|\vec{MC}\|\sin\theta$

$\|\vec{MC}\times\vec{MB}\|$



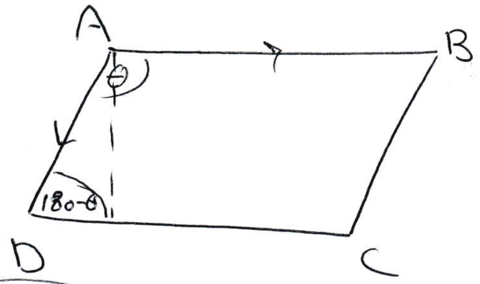
$= \|\vec{MC}\|\,\|\vec{MB}\|\sin(180-\theta)$

$= \|\vec{MC}\|\,\|\vec{MD}\|\sin\theta$

$\|\vec{MB}\| = \|\vec{MD}\|$

(c)

(9) A. of ▱



$= \text{base} \times \text{height}$

$= \|\overrightarrow{AB}\| \times \|\overrightarrow{AD}\| \sin(180-\theta)$

$= \|\overrightarrow{AB}\| \times \|\overrightarrow{AD}\| \sin\theta$

$= \cancel{\|\overrightarrow{AB}\|} \times \cancel{\|\overrightarrow{AD}\|} \times \dfrac{\|\overrightarrow{AB} \times \overrightarrow{AD}\|}{\cancel{\|\overrightarrow{AB}\|}\,\cancel{\|\overrightarrow{AD}\|}}$

$m(\angle A) = 180 - m(\angle D)$

$m(\angle D) = 180 - m(\angle A)$

$\sin\theta = \sin(180-\theta)$

$= \overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ -1 & 2 & -3 \end{vmatrix} = (-4, 7, 6)$

$\|\overrightarrow{AB} \times \overrightarrow{AD}\| = \sqrt{4^2+7^2+6^2} = 101$ (d)

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(10) $\vec{M} = (2, -1)$, $\vec{N} = (4, -5)$

A. of ▱ $= \frac{1}{2} d_1 d_2 \sin\theta$

$= \frac{1}{2} \|\vec{M}\| \|\vec{N}\| \sin\theta$

$= \frac{1}{2} \cancel{\|\vec{M}\|}\,\cancel{\|\vec{N}\|} \times \dfrac{\|\vec{M} \times \vec{N}\|}{\cancel{\|\vec{M}\|}\,\cancel{\|\vec{N}\|}}$

$= \frac{1}{2} \|\vec{M} \times \vec{N}\|$

$\|\vec{M} \times \vec{N}\| = (2, -1) \times (4, -5) = |-10 + 4|$

$= |-6| = 6$

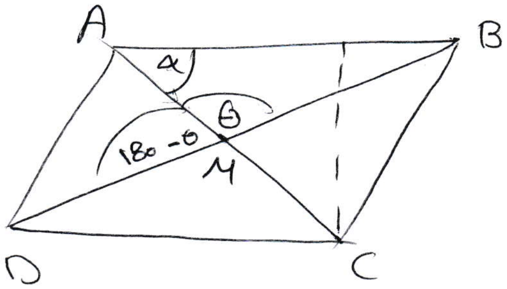
أو نقول إن $\vec{M} = (2, -1, 0)$

$\vec{N} = (4, -5, 0)$

و نشتغل عادي

$\frac{1}{2} \|\vec{M} \times \vec{N}\| = \frac{1}{2} \times 6 = \underline{\underline{3}}$ (d)

(11)



$* \; a \rightarrow \| \vec{AM} \times \vec{BM} \|$

$= \| \vec{AM} \| \, \| \vec{BM} \| \sin\theta$ ✗✗

$* \; b \rightarrow \| \vec{AB} \times \vec{AM} \|$

$(\sin\theta = \sin(180 - \theta))$

$= \| \vec{AB} \| \times \| \vec{AM} \| \sin\alpha$ ✗✗

$* \; c \rightarrow \| \vec{AB} \times \vec{AC} \|$

$= \underbrace{\| \vec{AB} \|}_{\text{base}} \underbrace{\| \vec{AC} \| \sin\alpha}_{\text{height}}$ ✓

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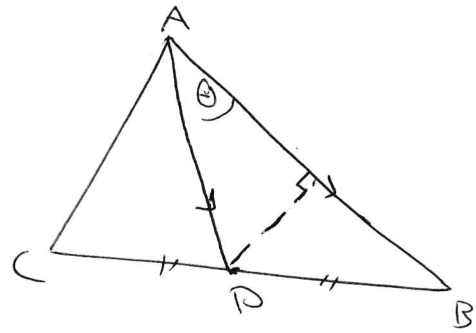
(12) $\| \vec{AD} \times \vec{AB} \|$



$= \| \vec{AD} \| \, \| \vec{AB} \| \sin\theta$

$= 2$ A. of Δ ABD

A. of ABC $= 2$ A. of Δ ABD

$\| \vec{AD} \times \vec{AB} \| = \underline{1} \times$ A. of ΔABC (c)

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(13) $\vec{A} = (1, -1, 2)$, $\vec{B} = (3, -2, 0)$, $\vec{C} = (0, 2, 4)$

$$\vec{A} \cdot \vec{B} \times \vec{C} = \begin{vmatrix} 1 & -1 & 2 \\ 3 & -2 & 0 \\ 0 & 2 & 4 \end{vmatrix}$$

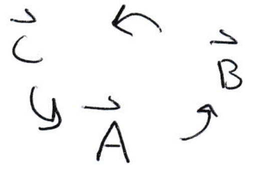
$$= 1 \begin{vmatrix} -2 & 0 \\ 2 & 4 \end{vmatrix} + 1 \begin{vmatrix} 3 & 0 \\ 0 & 4 \end{vmatrix} + 2 \begin{vmatrix} 3 & -2 \\ 0 & 2 \end{vmatrix}$$

$$= 1(-8) + 1(12) + 2(6) = -8 + 12 + 12 = 16$$ (d)

(14) $\vec{A}\cdot(\vec{B}\times\vec{C})$

$= \vec{B}\cdot(\vec{C}\times\vec{A})$

$= \vec{C}\cdot(\vec{A}\times\vec{B})$ (d)



$$\boxed{\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{B}\times\vec{C})\cdot\vec{A}}$$

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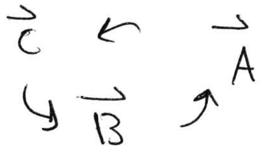
(15) $\vec{B}\cdot(\vec{A}\times\vec{C})$

$= \vec{A}\cdot(\vec{C}\times\vec{B})$

$= \vec{C}\cdot(\vec{B}\times\vec{A}) = (\vec{B}\times\vec{A})\cdot\vec{C}$



(a)

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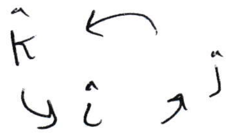
(16) $\hat{i}\cdot(\hat{j}\times\hat{k}) + \hat{j}\cdot(\hat{k}\times\hat{i}) + K\cdot(\hat{i}\times\hat{j})$

$= \hat{i}\cdot i + \hat{j}\cdot\hat{j} + \hat{k}\cdot k$

$= 1 + 1 + 1 = 3$ (b)



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(17) $\vec{A} = (3,-4,0)$, $\vec{B} = (0,-4,3)$, $\vec{C} = (0,0,5)$

Volume of parallelepiped $= |\vec{A}\cdot(\vec{B}\times\vec{C})|$

$$= \begin{vmatrix} 3 & -4 & 0 \\ 0 & -4 & 3 \\ 0 & 0 & 5 \end{vmatrix} = |-60| = 60$$ (c)

(18) $\vec{A} = \|\vec{A}\| \vec{U}_A$

$\vec{A} = 2(\cos 135, \cos 60, \cos 120)$

$\vec{A} = (-\sqrt{2}, 1, -1)$

$\vec{B} = (1, \sqrt{2}, 0)$

$\vec{C} = (\sqrt{2}, 3, 5)$

Area of parallel piped $= |A \cdot (\vec{B} \times \vec{C})|$

$$= \begin{vmatrix} -\sqrt{2} & 1 & -1 \\ 1 & \sqrt{2} & 0 \\ \sqrt{2} & 3 & 5 \end{vmatrix}$$

$$= -\sqrt{2} \begin{vmatrix} \sqrt{2} & 0 \\ 3 & 5 \end{vmatrix} - 1 \begin{vmatrix} 1 & 0 \\ \sqrt{2} & 5 \end{vmatrix} - 1 \begin{vmatrix} 1 & \sqrt{2} \\ \sqrt{2} & 3 \end{vmatrix}$$

$= |-10 - 5 - 1| = |-16| = 16$ (a)

(19)

$$\frac{\|\vec{A} \times \vec{B}\|}{\vec{A} \cdot \vec{B}} = \frac{\cancel{\|\vec{A}\| \|\vec{B}\|} \sin\theta}{\cancel{\|\vec{A}\| \|\vec{B}\|} \cos\theta} = \tan\theta$$

(c)

(20) $\|\vec{A} \times \vec{B}\|^2 + (\vec{A} \cdot \vec{B})^2 = 144$

$= (\|\vec{A}\| \|\vec{B}\| \sin\theta)^2 + (\|\vec{A}\| \|\vec{B}\| \cos\theta)^2 = 144$

$= \|A\|^2 \|\vec{B}\|^2 \sin^2\theta + \|A\|^2 \|\vec{B}\|^2 \cos^2\theta = 144$

$\|\vec{A}\|^2 \|\vec{B}\|^2 (\sin^2\theta + \cos^2\theta) = 144$

$4^2 \|\vec{B}\|^2 = 144$, $\|\vec{B}\|^2 = 9$, $\|\vec{B}\| = 3$ (c)

(21) $\|\vec{A}\times\vec{B}\|^2+(\vec{A}\cdot\vec{B})^2$

$= \|\vec{A}\|^2\|\vec{B}\|^2\sin^2\theta + \|\vec{A}\|^2\|\vec{B}\|^2\cos^2\theta$

$= \|\vec{A}\|^2\|\vec{B}\|^2(\sin^2\theta+\cos^2\theta)$

$= \|\vec{A}\|^2\|\vec{B}\|^2 = a^2b^2$ (a)

---

(22) $2\|\vec{A}\times\vec{B}\|\,(\vec{A}\cdot\vec{B})$

$\vec{A},\vec{B}$ are unit vectors. $\rightarrow \|\vec{A}\|=\|\vec{B}\|=1$

$= 2\,\|\vec{A}\|\|\vec{B}\|\sin\theta \quad \|\vec{A}\|\|\vec{B}\|\cos\theta$

$= 2\sin\theta\cos\theta = \sin 2\theta$ (d)

---

(23) $\vec{A}\times\vec{B} = \vec{A}\times\vec{C}$

a) $\rightarrow$ $\vec{B}=\vec{C}$ مش شرط xx

b) $\rightarrow$ $\vec{A}=\vec{B} \rightarrow \vec{A}\times\vec{B}=\vec{0}$ xx

c) $\rightarrow$ $\vec{A},\vec{B},\vec{C}$ are mutually $\perp$

$\vec{A}\times\vec{B}=\vec{C}$

$\vec{A}\times\vec{C}=-\vec{B}$ xx

$\vec{C} \leftarrow \vec{B}$, A

d) $\rightarrow$ $\vec{A},\vec{B},\vec{C}$ are on the same plane ✓

(d)

(24) $\vec{A} \perp \vec{B}$, $\vec{A} \perp \vec{C}$, $\|\vec{A}\| = 4\sqrt{2}$

$\downarrow$ $\downarrow$

$\vec{A} \cdot \vec{B} = 0$ $\vec{A} \cdot \vec{C} = 0$

$\vec{B} = (2, 3, 2)$, $\vec{C} = (1, 2, 1)$

a) $\rightarrow (2, 3, 1) \rightarrow \|\vec{A}\| = \sqrt{2^2 + 3^2} + 1 \neq 4\sqrt{2}$ xx

b) $\rightarrow \vec{A} \cdot \vec{B} = -8 + 8 = 0 \rightarrow A \perp B$

$\rightarrow \vec{A} \cdot \vec{C} = -4 + 4 = 0 \rightarrow A \perp C$ ✓✓ (b)

---

(25) $\vec{A} = (2, 1, -2)$

$\vec{A} + \vec{B} = \vec{A} \times \vec{B}$ $\rightarrow$ يا إما نحلها تجربة اختيارات

$\vec{0} = \vec{0}$

يا إما نلاحظ
إن الاختيارات أرقامها
زى $\vec{A}$ فهنفكر
ازاى نخلى الطرفين
بـ zero

$B = (-2, -1, 2)$

$\swarrow$ $\downarrow$

$\vec{A} + \vec{B} = \vec{0}$ $\vec{A} \parallel \vec{B}$

$\vec{A} \times \vec{B} = \vec{0}$

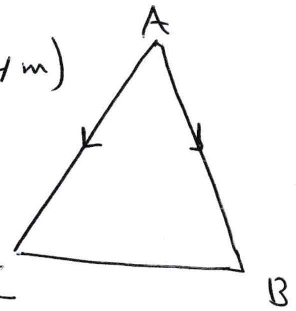
---

(26) A. of $\triangle ABC = \frac{1}{2} \|\vec{AC} \times \vec{AB}\|$ $\rightarrow (\frac{1}{2} ab \sin\theta)$, $A = (4, 0, 0)$

$\vec{AC} = (-4, 0, 2)$ $B = (0, m, 0)$

$\vec{AB} = (-4, m, 0)$ $C = (0, 0, 2)$

$$\vec{AC} \times \vec{AB} = \begin{vmatrix} i & j & \hat{k} \\ -4 & 0 & 2 \\ -4 & m & 0 \end{vmatrix} = (-2m, -8, -4m)$$



$\|\vec{AC} \times \vec{AB}\| = (2m)^2 + 8^2 + (4m)^2 = (2 \times 6)^2$

$= 4m^2 + 64 + 16m^2 = 144$ $\rightarrow \boxed{m = \pm 2}$ (a)

(27) $\vec{A}, \vec{B}, \vec{C}$ are unit vectors

let $\vec{A} = (1,0,0)$, $\vec{B} = (0,1,0)$, $\vec{C} = (0,0,1)$

همكن تبقا بال ⊖ عادي

أهم حاجة $\|\vec{A}\| = \|\vec{B}\| = \|\vec{C}\| = 1$

$$\vec{A} \cdot \vec{B} \times \vec{C} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = |1| = 1 \quad (a)$$

---

(28) $\vec{A} \cdot \{(\vec{B} + \vec{C}) \times (\vec{A} + \vec{B} + \vec{C})\}$

$$= \vec{A} \cdot [\vec{B} \times \vec{A} + \cancel{\vec{B} \times \vec{B}}^{\,0} + \cancel{\vec{B} \times \vec{C}} + \vec{C} \times \vec{A} + \cancel{\vec{C} \times \vec{B}} + \cancel{\vec{C} \times \vec{C}}^{\,0}]$$

$$= \vec{A} \cdot [\vec{B} \times \vec{A} + \vec{C} \times \vec{A}]$$

$$= \vec{A} \cdot [(\vec{B} + \vec{C}) \times \vec{A}] = \text{Zero}$$

(a)

لما تكتبهم في صيغ الـ deter هتلاقي في صفين شبه بعض فبالتالي = Zero

---

(29) $\underbrace{\vec{A} \cdot \vec{B} \times \vec{C}}_{\text{volume of parallepiped}} = \underbrace{\|\vec{A}\| \, \|\vec{B}\| \, \|\vec{C}\|}_{\text{volume of cuboid}}$

Angle between them is $90°$

$A \perp B$, $A \perp C$, $B \perp C$

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{C} = \vec{C} \cdot \vec{A} = \text{Zero}$$

(30) $\vec{A}\cdot\vec{B} = 0 \rightarrow A \perp B$

$\vec{A}\cdot\vec{C} = 0 \rightarrow A \perp C$ $\} \rightarrow \vec{B}\times\vec{C} \parallel \vec{A}$

$\boxed{\|\vec{A}\| = 1}$

$\vec{A} = k(\vec{B}\times\vec{C}) \rightarrow \vec{A} = \pm\sqrt{2}(\vec{B}\times\vec{C})$ (c)

unit vector

$\boxed{\|\vec{B}\| = 1}$

$\boxed{\|\vec{C}\| = 1}$

$\|\vec{A}\| = \|k(\vec{B}\times\vec{C})\|$

$\|\vec{A}\| = |k|\,\|\vec{B}\times\vec{C}\|$

$\|\vec{A}\| = |k|\,\|\vec{B}\|\,\|\vec{C}\|\cos\theta$

$1 = |k|\cos 45, \quad |k| = \sqrt{2}, \quad \boxed{k = \pm\sqrt{2}}$

---

(31) "Coplanar" → "on the same plane"

a) → $\vec{A}\cdot(\vec{B}\times\vec{C}) = 0 \xrightarrow{XX}$ لازم يكونوا مش points

b) → $\vec{AB}\cdot(\vec{AC}\times\vec{AD}) = \text{Zero}$ ✓✓

---

(32) volume of pyramid $= \frac{1}{6}$ volume of prallelpiped

$= \frac{1}{6}|A\cdot\vec{B}\times\vec{C}|$ (d)

(33) $\vec{A} = (1, 1, 1)$, $\vec{B} = (5, 2, L)$, $\vec{C} = (L, 4, 1)$

Volume of parallelpiped $= |A \cdot \vec{B} \times \vec{C}|$

$$\vec{A} \cdot \vec{B} \times \vec{C} = \begin{vmatrix} 1 & 1 & 1 \\ 5 & 2 & L \\ L & 4 & 1 \end{vmatrix} = (2 - 4L) - (5 - L^2) + (20 - 2L)$$

$$V = 2 - 4L - 5 + L^2 + 20 - 2L$$

$$V = L^2 - 6L + 17$$

$$V' = 2L - 6 \quad , \; At \; V' = 0$$

$$2L - 6 = 0 \rightarrow L = 3$$

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الـ Calculus كدا