Exercise(3)

[6] choose

(1) $\vec{A} = (2, 3, -1)$, $\vec{B} = (4, -1, 0)$

$\vec{A} \cdot \vec{B} = (2 \times 4) + (3 \times -1) + (-1 \times 0) = 8 - 3 = 5$ (a)

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(2) $\|\vec{A}\| = 5$, $\vec{B} = (-1, 2, 2)$, $\theta = 30$

$\vec{A} \cdot \vec{B} = \|\vec{A}\| \, \|\vec{B}\| \cos\theta$, $\|\vec{B}\| = \sqrt{1^2 + 2^2 + 2^2} = 3$

$\vec{A} \cdot \vec{B} = 5 \times 3 \cos 30' = 7.5\sqrt{3}$ (b)

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(3) $\vec{A} = (1, 1, -2)$, $\vec{B} = (3, 2, -1)$

$\vec{A} + 3\vec{B} = (10, 7, -5)$

$2\vec{A} - \vec{B} = (-1, 0, -3)$

$(\vec{A} + 3\vec{B}) \cdot (2\vec{A} - \vec{B}) = (10 \times -1) + (7 \times 0) + (-5 \times -3)$

$= -10 + 15 = 5$

(4) $A = (2, 1, 3)$, $B = (3, 5, -2)$, $C = (-1, 4, 0)$

$$\overrightarrow{AB} = \vec{B} - \vec{A} = (1, 4, -5)$$

$$\overrightarrow{AC} = \vec{C} - \vec{A} = (-3, 3, -3)$$

$$\overrightarrow{AB} \cdot \overrightarrow{AC} = (1 \times -3) + (4 \times 3) + (-5 \times -3)$$

$$= -3 + 12 + 15 = 24 \quad (b)$$

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(5) $2\vec{A} = (-6, -4, 0)$

$3\vec{B} = (9, 0, -15)$

$$3\vec{A} \cdot 2\vec{B} = 2\vec{A} \cdot 3\vec{B} = 6\vec{A} \cdot \vec{B} = 6(\vec{A} \cdot \vec{B})$$

$$= (-6, -4, 0) \cdot (9, 0, -15) = -6 \times 9$$

$$= -54 \quad (a)$$

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(6) • $\vec{A}$, $\vec{B}$ are two unit vectors

$\downarrow$

$\|A\| = 1$, $\|B\| = 1$

$$\vec{A} \cdot \vec{B} = \|\vec{A}\| \, \|\vec{B}\| \, \cos\theta$$

$$= 1 \times 1 \cos 45 = \frac{1}{\sqrt{2}} \quad (c)$$

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(7) $$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{\|\vec{A}\| \, \|\vec{B}\|} = \frac{-150\sqrt{3}}{15 \times 20} = \frac{-\sqrt{3}}{2}$$

$$\theta = \cos^{-1} \frac{-\sqrt{3}}{2} = 150^\circ \quad (d)$$

(8) $\vec{A} = (2, 0, 2)$, $\vec{B} = (0, 0, 4)$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{\|\vec{A}\| \, \|\vec{B}\|}$$

$\rightarrow \vec{A} \cdot \vec{B} = (2, 0, 2) \cdot (0, 0, 4) = 8$

$\rightarrow \|\vec{A}\| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

$\rightarrow \|\vec{B}\| = \sqrt{4^2} = 4$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{\|\vec{A}\| \, \|\vec{B}\|} = \frac{8}{4 \times 2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

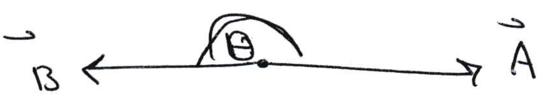
$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$ (b)

---

(9) $\vec{A} = (-2, -6, 1)$, $\vec{B} = (2, 6, -1)$

$\vec{A} = -\vec{B}$ $\rightarrow$ " $\vec{A}$, $\vec{B}$ have the same magnitude but diff. in direction"

$\theta = 180^\circ$ (d)



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(10) $(1, -1, 1) \rightarrow \vec{A} \rightarrow \|\vec{A}\| = \sqrt{3}$

$(1, 2, 1) \rightarrow \vec{B} \rightarrow \|\vec{B}\| = \sqrt{6}$

$\vec{A} \cdot \vec{B} = 1 - 2 + 1 = 0$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{\|\vec{A}\| \|\vec{B}\|} = \frac{0}{\sqrt{3}\sqrt{6}} = \frac{0}{\sqrt{9}} = \frac{0}{3} = 0$$

$\theta = 90^\circ$, $\frac{\pi}{2}$ (d)

(11) $\vec{A} = (1, -3, 0) \rightarrow \|\vec{A}\| = \sqrt{1^2+3^2} = \sqrt{10}$

$\vec{B} = (2, 0, 1) \rightarrow \|\vec{B}\| = \sqrt{2^2+1^2} = \sqrt{5}$

$$\vec{A}\cdot\vec{B} = 2+0+0 = 2$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{\|\vec{A}\|\ \|\vec{B}\|} = \frac{2}{\sqrt{10}\times\sqrt{5}} = \frac{2}{\sqrt{50}} = \frac{2}{5\sqrt{2}}$$

$$\cos\theta = \frac{\sqrt{2}}{5} \quad (a)$$

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(12) $(3, -1) \rightarrow \vec{A} \rightarrow \|\vec{A}\| = \sqrt{10}$

$(-4, 6) \rightarrow \vec{B} \rightarrow \|\vec{B}\| = 2\sqrt{13}$

$$\vec{A}\cdot\vec{B} = -12-6 = -18$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{\|\vec{A}\|\ \|\vec{B}\|} = \frac{-18}{\sqrt{10}\times 2\sqrt{13}}$$

$$\theta = \cos^{-1}\left(\frac{-18}{\sqrt{10}\times\sqrt{13}}\right) = 142^\circ\, 7'\, 30.06'' \quad (d)$$

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(13) $\|\vec{A}\| = 4$, $\|\vec{B}\| = 6$, $\theta = 60^\circ$

$$(3\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B}) =$$

$$= 3\vec{A}\cdot\vec{A} - 3\vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{A} - \vec{B}\cdot\vec{B}$$

$$= 3\|A\|^2 - 2\vec{A}\cdot\vec{B} - \|\vec{B}\|^2$$

$$= 3\times 4^2 - 2\times 4\times 6\cos 60 - 6^2$$

$$= 48 - 24 - 36$$

$$= -12 \quad (d)$$

$\vec{A}\cdot\vec{A} = \|\vec{A}\|\,\|\vec{A}\|\cos\theta$

$\theta = 0 \leftarrow \overset{A}{\underset{A}{\rightrightarrows}}$

$\vec{A}\cdot\vec{A} = \|\vec{A}\|\,\|\vec{A}\|\,\overset{1}{\cos 0}$

$\vec{A}\cdot\vec{A} = \|A\|^2$

(14) $\vec{A} \cdot \vec{B} = 0$

$= \|\vec{A}\| \, \|\vec{B}\| \cos\theta = 0$

$\|\vec{A}\| = 0$ refused ; $\|\vec{B}\| = 0$ refused ; $\cos\theta = 0$

$\theta = 90^\circ$

" $\vec{A}, \vec{B}$ are Non-Zero vectors" → perpendicular on eachother

(c)

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(15) $\vec{A} = (1, 3, -2)$, $\vec{B} = (-2, -6, 4)$

$$\frac{1}{-2} = \frac{3}{-6} = \frac{-2}{4} = \frac{1}{-2} \rightarrow \text{parallel}$$

$\vec{A} = -2\vec{B}$ → but have oppisite directions

(b)

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(16) " $\vec{A}, \vec{B}$ are two perpendicular unit vectors"

↳ $\vec{A} \cdot \vec{B} = 0$

↳ $\|\vec{A}\| = 1$, ↳ $\|\vec{B}\| = 1$

$(\vec{A} - 2\vec{B}) \cdot (3\vec{A} + 5\vec{B})$

$= 3\|A\|^2 + 5\vec{A} \cdot \vec{B} - 6\vec{A} \cdot \vec{B} - 10\|B\|^2$

$= 3(1) + 5(0) - 6(0) - 10(1)$

$= 3 - 10 = -7$ (b)

(17) $\vec{A} = (3, 3, m)$, $\vec{B} = (-6, -4, 6)$

" $\vec{A} \perp \vec{B}$ " $\rightarrow \vec{A} \cdot \vec{B} = 0$

$\vec{A} \cdot \vec{B} = -18 - 12 + 6m = 0$

$6m = 30 \rightarrow m = 5$ (b)

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(18) $\vec{A} = (K, -3, 1)$, $\vec{B} = (2, 3, -K)$

$\vec{A} \cdot \vec{B} = 2K - 9 - K = 0$

$K = 9$ (c)

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(19) $\vec{A} = (m, 1, 4)$, $\vec{B} = (2, 6, 3)$

$\vec{A} \cdot \vec{B} = 2m + 6 + 12 = 4$

$2m = -14 \rightarrow m = -7$ (d)

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(20) $3\vec{A} \cdot 2\vec{B} = 12$

$6 \vec{A} \cdot \vec{B} = 12$, $\underline{\vec{A} \cdot \vec{B} = 2}$

$-\vec{B} \cdot 5\vec{A} = -5\,\vec{A} \cdot \vec{B} = -5 \times 2 = -10$ (a)

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(21)

- right $\rightarrow A \cdot B = 0$
- straight $\rightarrow A \cdot B = (+)$ "Same direction"
  $\rightarrow A \cdot B = (-)$ "oppisite direction"
- obtuse $\rightarrow A \cdot B = (-)$
- acute $\rightarrow A \cdot B = (+)$

$A \cdot B = 13 \rightarrow (+)$ acute (a)

(22) $\vec{A} \perp \vec{B} \rightarrow \vec{A} \cdot \vec{B} = 0$

$$(2\vec{A} - 4\vec{C}) \cdot 3\vec{B}$$

$$= 6\,\underbrace{\vec{A} \cdot \vec{B}}_{0} - 12\,\vec{B} \cdot \vec{C} = 108$$

$$\vec{B} \cdot \vec{C} = -9 \quad (c)$$

[19] Choose

(1) $\vec{A} = (5, -4, 0)$, $\vec{B} = (x, y, z)$

$\vec{A} \cdot \vec{B} = 5x - 4y = 0$ → هجرب اختيارات

$\vec{B} = (8, 10, -7)$ (b)

(2) $\vec{A} = (-1, 2, 2) \rightarrow \|\vec{A}\| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$

$\|\vec{B}\| = 1$ "unit vector"

$\vec{A} \cdot \vec{B} = \|A\| \, \|\vec{B}\| \cos\theta$

$= 3 \cos\theta$, $\cos\theta \rightarrow [-1, 1]$

$\vec{A} \cdot \vec{B} = [-3, 3]$

$2 \in [-3, 3]$

(c)

(3) "$\vec{A}, \vec{B}$ are unitvectors" $\rightarrow \|\vec{A}\| = \|\vec{B}\| = 1$

$\vec{A} \cdot \vec{B} = \|\vec{A}\| \, \|\vec{B}\| \cos\theta = \frac{-1}{2}$

$\cos\theta = \frac{-1}{2}$, $\theta = \cos^{-1}\left(\frac{-1}{2}\right)$

$\theta = 120^\circ$ (c)

(4) " $\vec{A}, \vec{B}$ are unit vectors " $\rightarrow \|\vec{A}\| = \|\vec{B}\| = 1$

$\vec{A} \cdot \vec{B} = \|\vec{A}\| \, \|\vec{B}\| \cos\theta$

$= \cos\theta \rightarrow [-1, 1]$ ©

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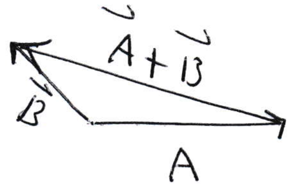
(5) $\|\vec{A}\| = \|\vec{B}\| = \|\vec{A} + \vec{B}\| = 1$

the form equilateral Δ
and its side length $= 1$



$\theta = 120$ $\quad \frac{2\pi}{3}$ ©

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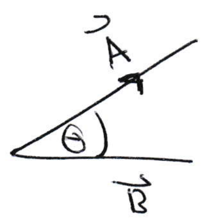
(6) "Component of $\vec{A}$ in the direction of $\vec{B}$ " $\rightarrow A_B$

$A_B = \|A\| \cos\theta = \frac{\vec{A} \cdot \vec{B}}{\|\vec{B}\|}$

©



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(7) $B_A = \|B\| \cos\theta$

$= 6 \cos 30 = 3\sqrt{3}$ ©

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(8) $\vec{A} = (-1, 4, 2)$, $B = (2, 2, 1)$

$\vec{A}_B = \|A\| \cos\theta = \frac{\vec{A} \cdot \vec{B}}{\|\vec{B}\|}$

$\rightarrow \vec{A} \cdot \vec{B} = -2 + 8 + 2 = 8$

$\rightarrow \|\vec{B}\| = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9} = 3$

$\vec{A}_B = \frac{\vec{A} \cdot \vec{B}}{\|\vec{B}\|} = \frac{8}{3}$ ©

(9) $\vec{A} = (4, -3, 5)$

z-axis $\rightarrow (0, 0, 1) = \vec{B}$

$\vec{A} \cdot \vec{B} = 5$, $\|\vec{B}\| = 1$

$$\vec{A}_B = \frac{\vec{A} \cdot \vec{B}}{\|\vec{B}\|} = \frac{5}{1} = 5 \quad (d)$$

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(10) "The Same Idea, He wants A_B"

$\vec{A} = (2, 3, -1)$, $\vec{B} = (3, 4, 0)$

$$\vec{A}_B = \frac{\vec{A} \cdot \vec{B}}{\|\vec{B}\|} = \frac{6 + 12}{\sqrt{3^2 + 4^2}} = \frac{18}{5} \quad (b)$$

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(11) $\vec{F} = (2, -3, 5)$, $A = (1, 4, 0)$, $\vec{B} = (-1, 2, 3)$

$\vec{AB} = \vec{B} - \vec{A} = (-2, -2, 3)$

$$\cdot\ \vec{F}_{AB} = \frac{\vec{F} \cdot \vec{AB}}{\|\vec{AB}\|} = \frac{-4 + 6 + 15}{\sqrt{2^2 + 2^2 + 3^2}} = \frac{17}{\sqrt{17}} = \sqrt{17} \quad (d)$$

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(12) $\vec{E} \cdot \vec{F} = 0$

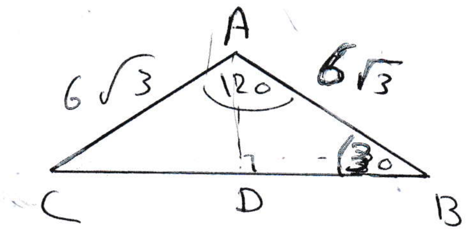
$(\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 0$

$= \|A\|^2 - \|B\|^2 = 0$

$\|A\|^2 = \|B\|^2$

$\|A\| = \|B\| \quad (d)$

(13) $\vec{BA}\cdot\vec{BC} = \|\vec{AB}\|\,\|\vec{BC}\|\cos\theta$



$BC = 2\,BD$

$= 2 \times AB\cos 30$

$= 2\times 6\sqrt{3}\cos 30 = 18$

$\vec{BA}\cdot\vec{BC} = 6\sqrt{3}\times 18\cos 30 =$

$= 162$ (d)

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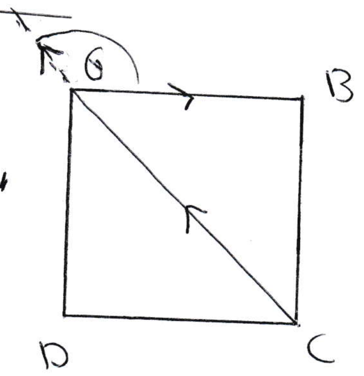
(14) $\vec{AB}\cdot\vec{CA} = \|\vec{AB}\|\,\|\vec{AC}\|\cos\theta$



- $\|\vec{AB}\| = 10$
- $\|\vec{AC}\| = 10\sqrt{2}$

"لازم يكونوا طالعين من نفس النقطة"

- $\theta = 135$

$\vec{AB}\cdot\vec{CA} = 10\times 10\sqrt{2}\cos 135 = -100$ (c)

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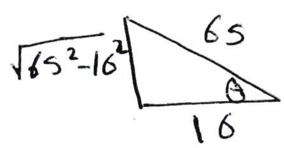
(15) $\vec{A} = (3,-4)$, $\vec{B} = (12,5)$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{\|\vec{A}\|\,\|\vec{B}\|}$$

$\rightarrow \vec{A}\cdot\vec{B} = 36 - 20 = 16$

$\rightarrow \|A\| = \sqrt{3^2+4^2} = 5$ $\rightarrow \|\vec{B}\| = \sqrt{12^2+5^2} = 13$

$$\cos\theta = \frac{16}{5\times 13} = \frac{16}{65} \rightarrow$$



$$\tan\theta = \frac{\sqrt{65^2-16^2}}{16} = \frac{63}{16}$$ (c)

(16) $\vec{A}_B$ $\underrightarrow{\text{vector}}$ Component $\vec{A}_B\left(\frac{\vec{B}}{\|\vec{B}\|}\right)$

$$\vec{A}_B = \left(\frac{A \cdot B}{\|B\|^2}\right)\vec{B}$$

$$\vec{A} = (1,-2,1) \quad , \vec{B} = (-2,1,-2)$$

- $\vec{A}\cdot\vec{B} = -2-2+2 = -2$
- $\|\vec{B}\|^2 = \left(\sqrt{2^2+1^2+2^2}\right)^2 = 9$

$$\vec{A}_B = \frac{-2}{9}(-2,1,2)$$

$$= \frac{-2}{9}(-2,1,2) = \left(\frac{4}{9}, \frac{-2}{9}, \frac{-4}{9}\right) \quad (a)$$

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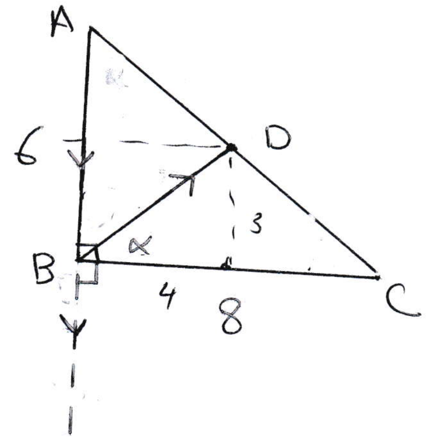
(17)

$$\vec{BD}_{AB} = \|\vec{BD}\| \cos\theta$$



$$BD = \sqrt{4^2+3^2} = 5$$

$$\alpha = \tan^{-1}\frac{3}{4}$$

$$\theta = 90+\alpha$$

$$\vec{BD}_{AB} = \|\vec{BD}\| \cos\theta$$

$$= 5\cos\left(90+\tan^{-1}\left(\tfrac{3}{4}\right)\right)$$

$$= -3 \quad (b)$$

(18) $\vec{w} = \vec{F}\cdot\vec{S} = \|\vec{F}\|\|\vec{S}\|\cos\theta$

Soon in dynamics

$\vec{F} = (3,0,7)$, $\vec{A} = (1,1,2)$, $B = (7,3,5)$

$\vec{S} = \vec{AB} = \vec{B} - \vec{A} = (6,2,3)$

$\vec{F}\cdot\vec{S} = 18 + 21 = 39$ (b)

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(19) $\vec{w} = \vec{F}\cdot\vec{S} = \|\vec{F}\|\,\|\vec{S}\|\cos\theta$

$\vec{F} = (5,-4,1)$, $\vec{S} = (2,1,-3)$

$\vec{w} = \vec{F}\cdot\vec{S} = 10 - 4 - 3 = 3$ (b)

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(20) $\vec{w} = \vec{F}\cdot\vec{S} = \|\vec{F}\|\,\|\vec{S}\|\cos\theta$

- $\|\vec{F}\| = 15$ $\quad A = (2,3,-1)$, $B = (3,5,1)$
- $\theta = 180°$ "moved inthe oppisite direction"
- $\|\vec{S}\| = \|\vec{AB}\| = \|\vec{B} - \vec{A}\|$

$\vec{B} - \vec{A} = (1,2,2) \rightarrow \|\vec{B} - \vec{A}\| = 3$

$\vec{w} = 15 \times 3 \cos 180 = -45$

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(21) $\vec{A} + \vec{B} + \vec{C} = \vec{0} \rightarrow \vec{C} = -\vec{A} - \vec{B}$

$\vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

$= \vec{A}\cdot\vec{B} + \vec{A}\cdot(-\vec{A} - \vec{B}) = \cancel{\vec{A}\cdot\vec{B}} - \|A\|^2 - \cancel{\vec{A}\cdot\vec{B}}$

$= -\|A\|^2 = -(2)^2 = -4$ (c)

(22) $\vec{A}_B = \dfrac{\vec{A}\cdot\vec{B}}{\|\vec{B}\|} = \dfrac{0}{\|\vec{B}\|} = \text{Zero}$ (d)

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(23) $\cdot\ \vec{A}_B = \dfrac{\vec{A}\cdot\vec{B}}{\|\vec{B}\|} = 3 = \dfrac{\vec{A}\cdot\vec{B}}{4} = 3$

$\vec{A}\cdot\vec{B} = 12$

$\cdot\ \vec{B}_A = \dfrac{\vec{A}\cdot\vec{B}}{\|\vec{A}\|} = \dfrac{12}{6} = 2$ (b)

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(24) $\vec{A} = 2\vec{B} \rightarrow \|\vec{A}\| = 2\|\vec{B}\|$

$\|\vec{C}\| = 2\|\vec{B}\|$, $\|\vec{A}\| = \|\vec{C}\|$

$\cdot\ A\cdot(\vec{B} - \vec{C}) = \text{zero}$

$\vec{A}\cdot\vec{B} - \vec{A}\cdot\vec{C} = \text{Zero}$

$\frac{1}{2}\vec{A}\cdot\vec{A} = \vec{A}\cdot\vec{C}$

$\frac{1}{2}\|A\|^2 = \|A\|\,\|C\| \cos\theta$

$\frac{1}{2}\cancel{\|A\|^2} = \cancel{\|A\|}\ \cancel{\|A\|} = \cos\theta$

$\cos\theta = \frac{1}{2} \rightarrow \theta = 60 \quad \frac{\pi}{3}$ (b)

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(25) $\|2\vec{A} - 3\vec{B}\| \xrightarrow{\text{by squaring}} \|2\vec{A} - 3\vec{B}\|^2$

$\|2\vec{A} - 3\vec{B}\|^2 = 2^2\vec{A}\cdot\vec{A} - 2\times2\times3\,\vec{A}\cdot\vec{B} + 3^2\vec{B}\cdot\vec{B}$

$= 4\|A\|^2 - 12\|A\|\,\|B\|\cos\theta + 9\|B\|^2$

$= 4\times2^2 - 12\times2\times3\times\frac{2}{3} + 9\times3^2$

$\|2\vec{A} - 3\vec{B}\|^2 = 49$

$\|2\vec{A} - 3\vec{B}\| = \sqrt{49} = 7$ (b)

(26) $\|\vec{A}-\vec{B}\| = \sqrt{19}$ by squaring

$\|\vec{A}-\vec{B}\|^2 = 19$

$\|A\|^2 - 2\vec{A}\cdot\vec{B} + \|B\|^2 = 19$

$\|\vec{A}\|^2 - 2\|\vec{A}\|\,\|\vec{B}\|\cos\theta + \|\vec{B}\|^2 = 19$

$4 - 2\times 2\times 3\cos\theta + 9 = 19$

$-12\cos\theta = 6$

$\cos\theta = \frac{-6}{12} = \frac{-1}{2}$, $\theta = 120^\circ$ (d)

---

(27) $\vec{A}\perp\vec{B} \rightarrow \vec{A}\cdot\vec{B} = 0$

$\|\vec{A}+\vec{B}\|^2 = 12^2$

$\|\vec{A}\|^2 + 2\cancel{\vec{A}\cdot\vec{B}}^{\,0} + \|B\|^2 = 12^2$

$\|\vec{A}\|^2 + \|B\|^2 = 12^2 \longrightarrow (1)$

• $\|\vec{A}-\vec{B}\|^2 = \|\vec{A}\|^2 - 2\vec{A}\cdot\vec{B} + \|\vec{B}\|^2$

$= \|\vec{A}\|^2 + \|\vec{B}\|^2$

from (1)

$\|\vec{A}-\vec{B}\|^2 = 12^2$

$\|\vec{A}-\vec{B}\| = 12$ (b)

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(28) $\vec{B}_A = \frac{\vec{A}\cdot\vec{B}}{\|\vec{A}\|} = 3 = \frac{\vec{A}\cdot\vec{B}}{5}$

$\vec{A}\cdot\vec{B} = 15$ (a)

(29) $\vec{A}+\vec{B}=\vec{C}$

$\vec{A}=\vec{C}-\vec{B} \rightarrow \|\vec{A}\|=\|\vec{C}-\vec{B}\|$

by squaring

$\|\vec{A}\|^2=\|\vec{C}-\vec{B}\|^2$

$\|\vec{A}\|^2=\|\vec{C}\|^2-2\|\vec{B}\|\|\vec{C}\|\cos\theta+\|\vec{B}\|^2$

$\rightarrow \|\vec{A}\|=\|\vec{B}\| \qquad \|\vec{C}\|=\sqrt{3}\|\vec{A}\|$

$\|A\|^2=3\|\vec{A}\|^2-2\sqrt{3}\|A\|^2\cos\theta+\|A\|^2$

$\cancel{\|A\|^2}=\cancel{\|A\|^2}(3-2\sqrt{3}\cos\theta+1)$

$2\sqrt{3}\cos\theta=3$

$\cos\theta=\frac{3}{2\sqrt{3}}=\frac{\sqrt{3}}{2} \rightarrow \theta=30, \frac{\pi}{6}$ a

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(30) $\|\vec{A}+\vec{B}\|=\|\vec{A}\|$ by squaring

$\|\vec{A}+\vec{B}\|^2=\|\vec{A}\|^2$

$=\cancel{\|\vec{A}\|^2}+2\|\vec{A}\|\|\vec{B}\|\cos\theta+\|\vec{B}\|^2=\cancel{\|A\|^2}$

$2\|\vec{A}\|\cancel{\|\vec{B}\|}\cos\theta+\|\vec{B}\|^{\cancel{2}}=0 \qquad \times\frac{1}{\|\vec{B}\|}$

$2\|\vec{A}\|\cos\theta=-\|\vec{B}\|, \quad \cos\theta=\frac{-1}{2}\cdot\frac{\|\vec{B}\|}{\|\vec{A}\|}$

$*(2\vec{A}+\vec{B})\cdot\vec{B}=2\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{B}$

$=2\|\vec{A}\|\|\vec{B}\|\boxed{\cos\theta}+\|\vec{B}\|^2=$

$=\cancel{2}\cancel{\|\vec{A}\|}\|\vec{B}\|\times\frac{-1}{\cancel{2}}\frac{\|\vec{B}\|}{\cancel{\|\vec{A}\|}}+\|\vec{B}\|^2=-\|\vec{B}\|^2+\|\vec{B}\|^2$

$=$ Zero

b perpendicular

(31) $\vec{B'} = (3, 5, 4) \rightarrow \|\vec{OB'}\| = \sqrt{3^2+4^2+5^2} = 5\sqrt{2}$

$\vec{B} = (3, 5, 0) \rightarrow \|\vec{OB}\| = \sqrt{34}$

* $\vec{OB} \cdot \vec{OB'} = \|\vec{OB}\| \, \|\vec{OB'}\| \cos\theta$

$9 + 25 = 5\sqrt{2} \, \sqrt{34} \cos\theta$

$\cos\theta = \dfrac{\sqrt{17}}{5}$, $\theta \simeq 34°$ (b)

---

(32) السؤال غلط $\boxed{AB' \rightarrow D'B'}$

$\vec{D'B'} \cdot \vec{BD} = \|\vec{D'B'}\| \, \|\vec{BD}\| \cos\theta$

$= 1 \times 1 \cos 180$

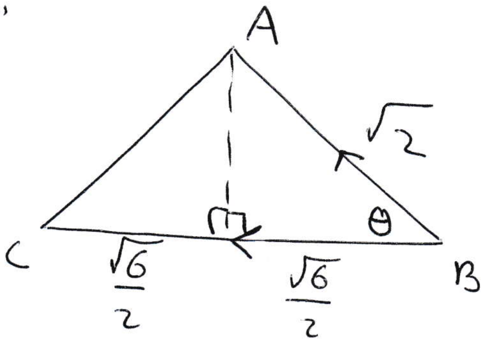
$= -1$ (a)

---

(33) $\vec{BA} \cdot \vec{BC} = \|\vec{BA}\| \, \|\vec{BC}\| \cos\theta$

?

$\|\vec{BA}\| = \sqrt{1^2+1^2} = \sqrt{2}$



$\cos\theta = \dfrac{\frac{\sqrt{6}}{2}}{\sqrt{2}}$

$\vec{BA} \cdot \vec{BC} = \cancel{\sqrt{2}} \times \sqrt{6} \times \dfrac{\frac{\sqrt{6}}{2}}{\cancel{\sqrt{2}}}$

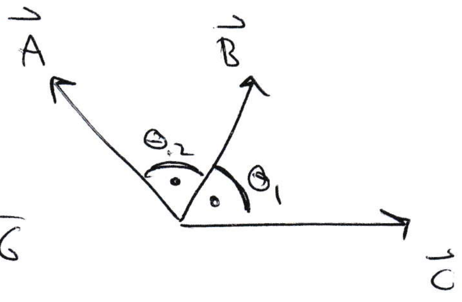
$= \sqrt{6} \times \dfrac{\sqrt{6}}{2} = \dfrac{6}{2} = 3$ (c)

(34) $\theta_1 = \theta_2$

$\vec{A} = (1, 2, -1) \rightarrow \|\vec{A}\| = \sqrt{6}$

$\vec{B} = (6, K, 0)$

$\vec{C} = (4, 0, 2\sqrt{2}) \rightarrow \|\vec{C}\| = 2\sqrt{6}$



$$\cos\theta_1 = \frac{\vec{B}\cdot\vec{C}}{\cancel{\|\vec{B}\|}\|\vec{C}\|} = \cos\theta_2 = \frac{\vec{A}\cdot\vec{B}}{\cancel{\|\vec{B}\|}\|\vec{A}\|}$$

$$\frac{\vec{B}\cdot\vec{C}}{\|\vec{C}\|} = \frac{\vec{A}\cdot\vec{B}}{\|\vec{A}\|}$$

$$\frac{24}{2\cancel{\sqrt{6}}} = \frac{6+2k}{\cancel{\sqrt{6}}} \quad , 24 = 12 + 4K$$

$4K = 12 \rightarrow K = 3$ (b)

---

(35) $\vec{OA} = (-2, 4, 3)$ $\quad \vec{OB} = (-2, 4, 0)$

$$\vec{F} = \|\vec{F}\| \times \vec{U}_{OA} = 12\cancel{\sqrt{29}} \times \frac{(-2, 4, 3)}{\cancel{\sqrt{(2)^2 + 4^2 + 3^2}}}$$

$\vec{F} = (-24, 48, 36)$

$$\vec{F}_B = \frac{\vec{F}\cdot\vec{B}}{\|\vec{B}\|} = \frac{48 + 192}{2\sqrt{5}} = 24\sqrt{5} \text{ (b)}$$

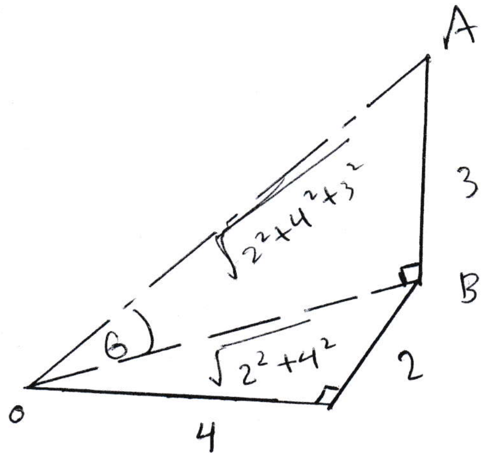
→ Another solution

$\vec{F}_B = \|\vec{F}\| \cos\theta$

$$= 12\cancel{\sqrt{29}} \times \frac{\sqrt{2^2+4^2}}{\sqrt{2^2+4^2+3^2}}$$

$= 12 \times 2\sqrt{5}$

$= 24\sqrt{5}$ (b)

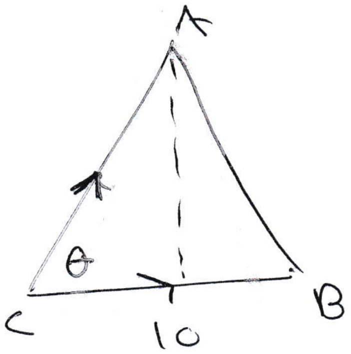


(36) $\vec{CA} \cdot \vec{CB} = \|\vec{CA}\| \, \|\vec{CB}\| \cos\theta$

$= \cancel{A}C \times BC \times \dfrac{\frac{1}{2} BC}{\cancel{A}C}$

$= \frac{1}{2} BC^2$

$= \frac{1}{2} \times 10^2 = 50$

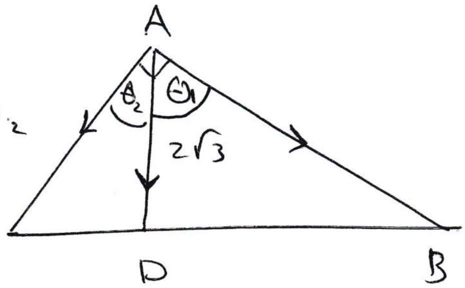


(37) $\vec{AD} \cdot (\vec{AB} + \vec{AC})$

$= \vec{AD} \cdot \vec{AB} + \vec{AD} \cdot \vec{AC}$

$= AD \times AB \cos\theta_1 + AD \times AC \cos\theta_2$

$= AD \times A\cancel{B} \times \dfrac{AD}{A\cancel{B}} + AD \times A\cancel{C} \times \dfrac{AD}{A\cancel{C}}$



$= AD^2 + AD^2 = 2AD^2 = 2(2\sqrt{3})^2$

$= 2 \times 12 = 24$ (b)

(38) $\vec{AD} \cdot \vec{AC} = \|\vec{AD}\| \, \|\vec{AC}\| \cos(90 - \theta_1 - \theta_2)$

$= AD \times AC \cos(90 - \theta_1 - \theta_2)$

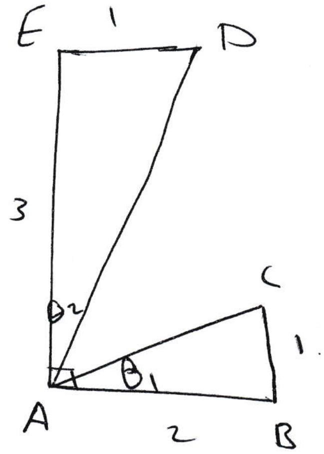
- $\theta_1 = \tan^{-1}\left(\frac{1}{2}\right)$
- $\theta_2 = \tan^{-1}\left(\frac{1}{3}\right)$

$\theta_1 + \theta_2 = 45$

- $AC = \sqrt{2^2 + 1}$

  $AC = \sqrt{5}$
- $AD = \sqrt{3^2 + 1}$

  $AD = \sqrt{10}$



$AD \times AC \cos(90 - 45)$

$= AD \times AC \cos 45$

$= \sqrt{10} \times \sqrt{5} \cos 45 = \sqrt{50} \times \frac{1}{\sqrt{2}}$

$= 5\cancel{\sqrt{2}} \times \dfrac{1}{\cancel{\sqrt{2}}} = 5$ (a)

(39) $\|\overrightarrow{AB}+\overrightarrow{BC}\| = \|\overrightarrow{AB}-\overrightarrow{BC}\|$ by squaring

$\|\overrightarrow{AB}+\overrightarrow{BC}\|^2 = \|\overrightarrow{AB}-\overrightarrow{BC}\|^2$

$\cancel{AB^2} + 2\,AB \times BC \cos\theta + \cancel{BC^2} = \cancel{AB^2} - 2 \times AB \times BC \cos\theta + \cancel{BC^2}$

$4\,AB \times BC \cos\theta = \text{zero}$

$\cos\theta = 0 \rightarrow \theta = 90^\circ \;\; \frac{\pi}{2}$ (a)

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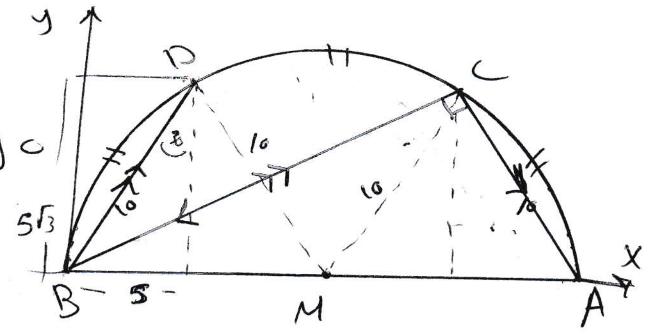
(40) * First



$\overrightarrow{CA} \cdot \overrightarrow{CB} = CA \times CB \cos 90$

$= \text{Zero}$ (a)

*Second

$B = (0,0)$

$D = (5, 5\sqrt{3})$

$C = (15, 5\sqrt{3})$

$\overrightarrow{BD} = (5, 5\sqrt{3})$

$\overrightarrow{BC} = (15, 5\sqrt{3})$

$\overrightarrow{BD} \cdot \overrightarrow{BC}$

$= 15 \times 5 + 5\sqrt{3} \times 15\sqrt{3}$

$= 150$ (c)

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(41) $\overrightarrow{AD} \cdot (\underbrace{\overrightarrow{AB} + \overrightarrow{AC}})$

$\downarrow$

$\overrightarrow{AD} \cdot \underbrace{2\overrightarrow{AD}}$

$= 2\|AD\|^2$

$= 2 \times 4^2$

$= 2 \times 16$

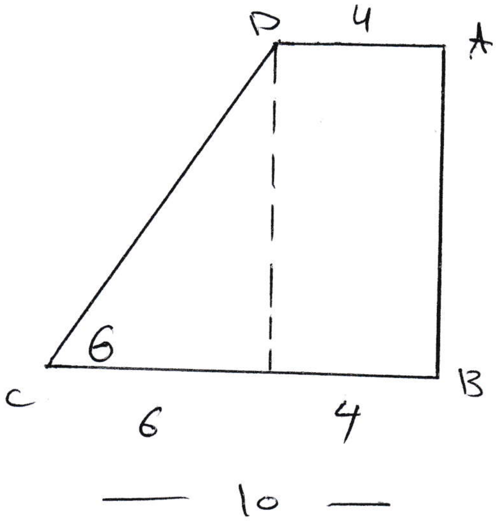
$= 32$ (d)

(42) $\vec{CD} \cdot \vec{CB}$

$= CD \times CB \cos\theta$

$= \cancel{CD} \times 10 \times \frac{6}{\cancel{CD}}$

$= 60$ (e)



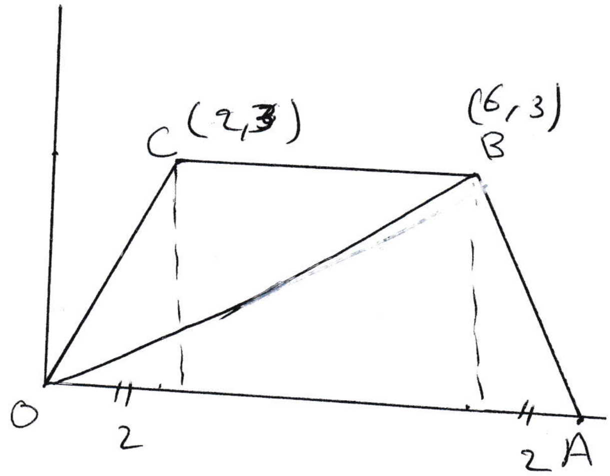
(43) $\vec{OB} = \vec{B}$

$\vec{OC} = \vec{C}$

$\vec{OB} \cdot \vec{OC} = \vec{B} \cdot \vec{C}$

$= (6,3) \cdot (2,3)$

$= 12 + 9 = 21$ (e)



(44) $AB = BC = CA = 4$

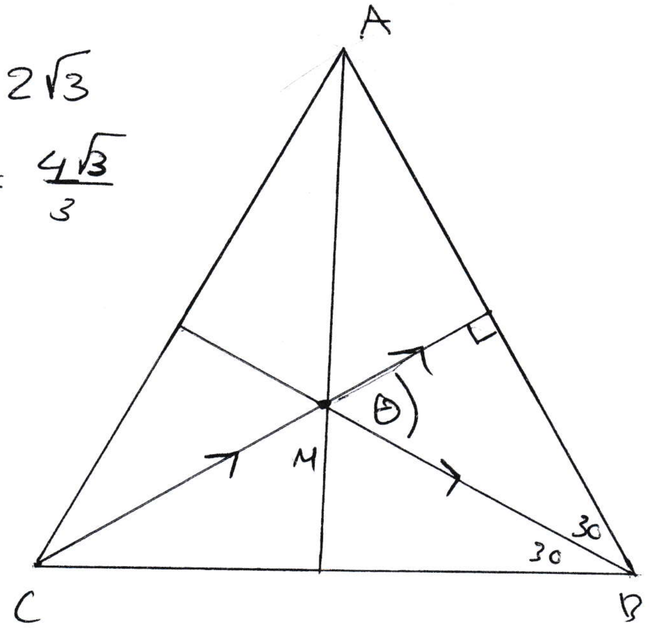
$AD = BF = EC = \sqrt{4^2 - 2^2} = 2\sqrt{3}$

$MA = MC = MA = \frac{2}{3} \times 2\sqrt{3} = \frac{4\sqrt{3}}{3}$

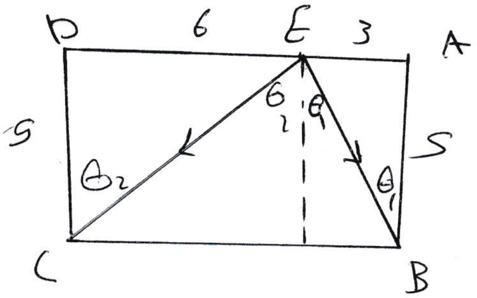
$\vec{MB} \cdot \vec{CM} = MB \times CM \cos 60$

$= \frac{4\sqrt{3}}{3} \times \frac{4\sqrt{3}}{3} \cos 60$

$= \frac{8}{3}$ (e)



(45)



$\vec{EB} \cdot \vec{EC}$

$= EB \times BC \cos(\theta_1 + \theta_2)$

- $EB = \sqrt{5^2 + 3^2}$
- $BC = \sqrt{5^2 + 6^2}$
- $\theta_1 = \tan^{-1}\left(\frac{3}{5}\right)$
- $\theta_2 = \tan^{-1}\left(\frac{6}{5}\right)$

$\vec{EB} \cdot \vec{EC} = EB \times BC \cos(\theta_1 + \theta_2)$

$= \sqrt{5^2+3^2} \times \sqrt{5^2+6^2} \cos\left(\tan^{-1}\frac{3}{5} + \tan^{-1}\frac{6}{5}\right)$

$= 7$ (a) رقم جميل

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(46) circ. $= 12\pi = 2\pi r$

$r = 6$

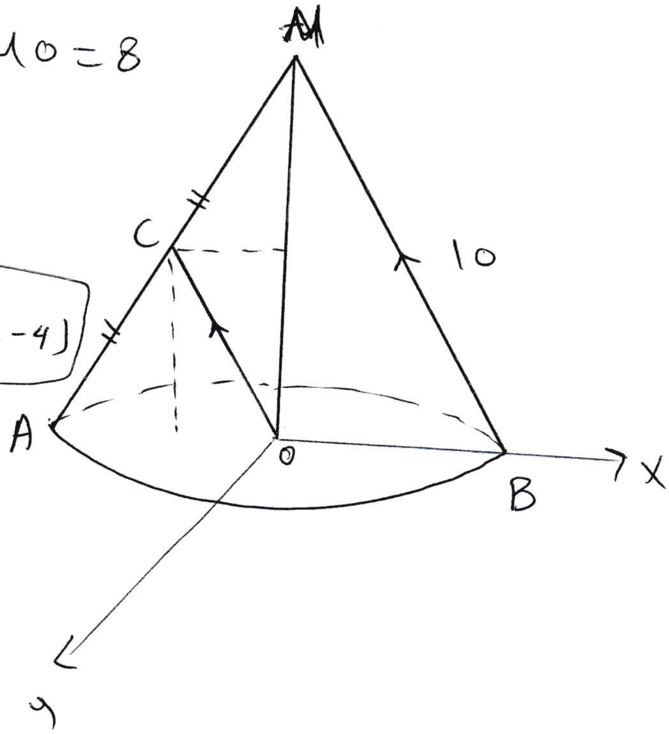
$MO = \sqrt{10^2 - 6^2}$

$MO = 8$

$B = (6, 0, 0)$

$O = (0, 0, 0)$

$C = (-3, 0, 4)$



$\vec{BC} \cdot \vec{CO}$

* $\vec{CO} = \vec{O} - \vec{C}$
  $= (3, 0, -4)$

* $\vec{BC} = \vec{C} - \vec{B}$
  $= (-9, 0, 4)$

$\vec{BC} \cdot \vec{CO} = -27 - 16$

$= -43$

(a)