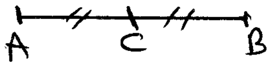
Rules of S. Geo

1 Distance between 2 points

$$AB=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

2 mid point $=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1-z_2}{2}\right)$



$c=\frac{A+B}{2}$, $B=2C-A$, $A=2C-B$

3 If $(x,y,z)\in X$-axis then $y=0$, $z=0$

$(x,y,z)\in Z$-axis then $x=0$, $y=0$

4 the distance between the point (x,y,z) and

$\sqrt{(\text{اللى ماتقالش})^2}$ → x-axis → $\sqrt{y^2+z^2}$
→ y-axis → $\sqrt{x^2+z^2}$
→ z-axis → $\sqrt{x^2+y^2}$

the distance of (x,y,z) with → plane $xy=\sqrt{z^2}=|z|$
→ plane $yz=\sqrt{x^2}=|x|$
→ plane $xz=\sqrt{y^2}=|y|$

5 the image of the point (x,y,z) by reflection in
→ X-axis $(a,-b,-c)$
→ y-axis $(-a,b,-c)$
→ X-axis $(a,-b,c)$

the image of the point (x,y,z) by reflection in
→ plane xy $(a,b,-c)$
→ plane yz $(-a,b,c)$
→ plane xz $(a,-b,c)$

Equation of sphere

1 standard form: $(x-\ell)^2+(y-K)^2+(z-n)^2=r^2$

the center (ℓ,K,n)

2 General form: $x^2+y^2+z^2+\underset{\downarrow}{2\ell x}+2Ky+2nZ=d$

نغير اشارة وبدون تقسيم على 2

the center $H=(-\ell,-K,-n)$

$r=\sqrt{\ell^2+K^2+n^2-d}>0$ لازم

3 sphere its center $M=(x,y,z)$

1) touches plane xy: $r=\sqrt{z^2}=|z|$
2) touches plane xz: $r=\sqrt{y^2}=|y|$
3) touches plan yz: $r=\sqrt{x^2}=|x|$

4 Center of sphere touches 3 coordinate planes and its radius r is $M=(\pm r,\pm r,\pm r)$

5 center of sphere touches axis and its radius $=r$

$\therefore$ the center $M=\left(\pm\frac{r}{\sqrt2}, \pm\frac{r}{\sqrt2}, \pm\frac{r}{\sqrt2}\right)$

6 Center of sphere passes through

$A=(a,0,0)$, $B=(0,a,0)$, $c=(0,0,a)$

or $A=(a,a,0)$, $B=(0,a,a)$, $c=(a,0,a)$

$M=\frac{A+B+C}{3}$, $r=\frac{|a|\sqrt6}{3}$

7 sphere touches X-axis → $r=\sqrt{y^2+z^2}$ $\sqrt{(\text{اللى ماتقالش})^2}$

sphere touches xy plane → $r=\sqrt{z^2}=|z|$

8 0 one inside the other — $MN=r_1-r_2$ touch int — intersecting — $MN=r_1+r_2$ touch externally — distant → ∞

Vector in space

1 Unit vector in dir of $\vec A = \vec{A^*} = \vec{U_A} = \frac{\vec A}{\|\vec A\|}$ → $\vec A = \|\vec A\|\,\vec{U_A}$

2

$\vec{AB}+\vec{BC}=\vec{AC}$, $\vec{AB}=B-A$

$\vec{AB}+\vec{AC}=2\vec{AD}$ (median rule)

3 Direction angles with axes $\theta_x, \theta_y, \theta_z$

→ $\cos^2\theta_x+\cos^2\theta_y+\cos^2\theta_z=1=$ unit vector in dir of $\vec A$

→ $\sin^2\theta_x+\sin^2\theta_y+\sin^2\theta_z=2$

→ $\cos 2\theta_x+\cos 2\theta_y+\cos 2\theta_z=-1$

$*\ \vec A=(A_x, A_y, A_z)=\|\vec A\|(\cos\theta_x, \cos\theta_y, \cos\theta_z)$

1 $*\ \vec F=\|\vec F\|\times\frac{\vec{AB}}{\|\vec{AB}\|}$, $*\ \|\vec A\|+\|\vec B\|\geq\|\vec A+\vec B\|$ triangle inequality

4 If vector makes equal angles with the Coordinate axis

$\theta_x = \theta_y = \theta_z \therefore \cos\theta = \pm\frac{1}{\sqrt{3}}$, $\vec{U}_A = \left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$

$\theta_x + \theta_y \geq 90^\circ$ and if $\theta_x + \theta_y = 90$ then $\theta_z = 90^\circ$

5 If $\vec{A}$ makes angles $(\theta_x, \theta_y, \theta_z)$ then $K\vec{A}$ makes angles

→ $(\theta_x, \theta_y, \theta_z)$ if $K > 0$

→ $(180 - \theta_x, 180 - \theta_y, 180 - \theta_z)$ if $K < 0$

6 If $\vec{A} \parallel \vec{B}$ then

→ $\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2}$

→ $\vec{A} \times \vec{B} = \vec{0}$

→ $\vec{A} = K\vec{B}$

7 If $\vec{A} \perp \vec{B}$ then $\vec{A} \cdot \vec{B} = 0$

8 $\vec{A} \cdot \vec{B} = \|\vec{A}\|\|\vec{B}\|\cos\theta = x_1x_2 + y_1y_2 + z_1z_2$

9 $\vec{A} \times \vec{B} = (\|\vec{A}\|\|\vec{B}\|\sin\theta)\vec{C} = \begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$

Scalar

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

$\vec{A} \cdot \vec{A} = \|\vec{A}\|^2$

$\vec{A} \cdot \vec{B} = 0$ then $\vec{A} \perp \vec{B}$

$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

work $= \vec{F} \cdot \vec{S}$

Alg. component of $\vec{A}$ in the dir of $\vec{B}$

لأهل الفوق $= \frac{\vec{A} \cdot \vec{B}}{\|\vec{B}\|}$

Vector comp of $\vec{A}$ in the dir of $\vec{B}$

$= \frac{\vec{A} \cdot \vec{B}}{\|\vec{B}\|} \times \frac{\vec{B}}{\|\vec{B}\|}$

Vector

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

$\vec{A} \times \vec{A} = \vec{0}$

if $\vec{A} \times \vec{B} = \vec{0}$ then $\vec{A} \parallel \vec{B}$

$\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$

$\hat{j} \times \hat{i} = -\hat{k}$, $\hat{k} \times \hat{j} = -\hat{i}$

Area of parallelogram $= \|\vec{AB} \times \vec{AD}\|$

Area of $\Delta = \frac{1}{2}\|\vec{A} \times \vec{B}\|$

$\vec{A} \cdot \vec{B} \times \vec{C} = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$ = Volume of parallelepiped

Mode 6 1 1 , AC , shift 5 4 7
shift 5 4 3

Note If $\vec{A} \cdot \vec{B} \times \vec{C} = 0$ then A, B and C are in the same plane → Coplanar.

* If $\vec{A} + \vec{B} = \vec{A} \times \vec{B}$ then $\vec{A}$ and $\vec{B}$ are additive inverse to each other.

Diff. forms of equation of a st. line:

if $A(x_1, y_1, z_1) \in$ st. line, $\vec{d} = (a, b, c)$ is direction vector

then ① $\vec{r} = \vec{A} + t\vec{d} = (x_1, y_1, z_1) + t(a, b, c)$

② $x = x_1 + ta$, $y = y_1 + tb$, $z = z_1 + tc$

③ $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

If a st. line passes through 2 points A, B then $\vec{d} = \vec{AB} = B - A$ or $\vec{d} = \vec{BA} = A - B$

Equation of plane , A is point , $\vec{n}$ = normal vector

① $\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}$

② $ax + by + cz + d = 0 \rightarrow \vec{n} = (a, b, c)$

③ $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, a, b, c are parts of axis

* If a plane passes through the point $(2, 3, 7)$ and

P // → Xy plane → $z = 7$ (ثابت) ← ما يتقابلش

equ. of plane // yz plane → $x = 2$

" " " // xz plane → $y = 3$

* equation of xy plane → $z = 0$

yz plane → $x = 0$

* equ. of plane // X-axis → $by + cz + d = 0$

" " // contains X-axis → $by + cz = 0$

equ. of plane by

* using 3 non collinear points

$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$

* using the Centroid (p, q, r)

$\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$

Angle between

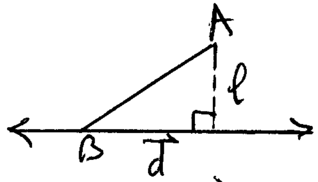
→ 2 vectors → $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{\|\vec{A}\|\,\|\vec{B}\|}$

→ 2 lines → $\cos\theta = \dfrac{|\vec{d_1}\cdot\vec{d_2}|}{\|\vec{d_1}\|\,\|\vec{d_2}\|}$

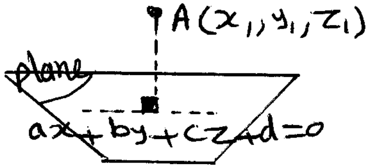
→ 2 planes → $\cos\theta = \dfrac{|\vec{n_1}\cdot\vec{n_2}|}{\|\vec{n_1}\|\,\|\vec{n_2}\|}$

the angle between line and plane

$$\sin\theta = \frac{|\vec{n}\cdot\vec{d}|}{\|\vec{n}\|\,\|\vec{d}\|}$$



L of ⊥ from a given point



$$\text{L of } \perp = \frac{|\vec{BA}\times\vec{d}|}{\|\vec{d}\|}$$

$$l = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

Dividing line segment ratio of division $= -\left(\dfrac{\text{diff in } x(s)}{\text{diff in } x(s)}\right)$

[1] xy plane divides the line joining the 2 points $(2,4,5)$, $(-4,3,-2)$ in the ratio = ...

Solution

xy plane → $z=0$ $(x, y, 0)$ point of division

ratio of division $= -\left(\dfrac{\text{diff in } z(s)}{\text{diff in } z(s)}\right) = -\left(\dfrac{5-0}{-2-0}\right) = +\dfrac{5}{2}$

dividing internally

[2] the plane $ax+by+cz+d=0$ divides the line joining (x_1, y_1, z_1), (x_2, y_2, z_2) in the ratio $\left(-\dfrac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}\right) =$ (+) → internally, (−) → externally

Note the perpendicular distance between 2 parallel planes $P_1: ax+by+cz+d_1=0$

$P_2: ax+by+cz+d_2=0$

$$\text{L of } \perp = \frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$$

in condition of $a=a$, $b=b$, $c=c$

the intersection point between

[1] L_1 and L_2 → put $\vec{r_1} = \vec{r_2}$ → find $t_1 = ..$, $t_2 = \dots$

(x, y, z) check in the third equation satisfy

[2] P_1 and P_2 → remove y from the 2 equations to get $x = lz$

then remove z from the 2 equations to get $x = ky$ → $x = ky = lz$ → st. line

[3] L and P → Put $x = x_1 + at$, $y = y_1 + bt$, $z = z_1 + ct$

sub in the equ. of plane to get $t = ..$

then sub in $x = \dots$, $y = \dots$, $z = \dots$

relations between 2 planes $P_1: a_1x+b_1y+c_1z+d_1=0$

$P_2: a_2x+b_2y+c_2z+d_2=0$

[1] $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2} = \dfrac{d_1}{d_2}$ → congruent or coincident

[2] $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2} \neq \dfrac{d_1}{d_2}$ → parallel

[3] $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$ → intersecting at a st. line

Trick — equations → plane / st.line

ثابتة

[1] $z=5 \longrightarrow$ plane

اللى ماتقالش

① parallel to xy plane
② $\perp$ z-axis
③ $\vec{n}=(0,0,1)=\hat{k}$
④ To find point on it $(x,y,5)$

Special Case

$z=0 \to$ ① xy plane نفسه $\longrightarrow$ ④ passes through the origin $(0,0,0)$
② $\perp$ z axis
③ $\vec{n}=(0,0,1)=\hat{k}$

* $(x,y,0)$ any point on it

اللى ماتقالش

[2] $4x-3y-13=0 \to$ plane
① $//$ z-axis
② $\perp$ xy plane
③ $\vec{n}=(4,-3,0)$
④ To find point $\in$ the plane
Put $x=0 \to y=\frac{-13}{3} \to (0,\frac{-13}{3},z)$

special case
$4x-3y=0$
① passes through z-axis
② passes through the $(0,0,0)$

[3] $2x+3y-5z-3=0 \to$ plane
① not parallel to any axis or planes (xy, yz, xz)
② not $\perp$
③ $\vec{n}=(2,3,-5)$
④ To find point on it put $x=0,\ y=0 \to z=\frac{-3}{5} \to (0,0,\frac{-3}{5})$

ملحوظة
$\because$ no constant
$\therefore$ passes through the origin $(0,0,0)$

Special Case
$2x+3y-5z=0 \to$ plane passes through the origin $(0,0,0)$

اللى ماتقالش

[4] $\underbrace{y=5}_{\text{plane}}\ \overset{,}{\cap}\ \underbrace{z=3}_{\text{plane}}$ $\Big\} \to$ st.line
① $//$ x-axis
② $\perp$ yz plane
③ $\vec{d}=(1,0,0)=\hat{i}$
④ $(x,5,3)$ point on it

special cases

* $y=0,\ z=3 \to$ st. line passes through $(0,0,0)$
① $//$ x-axis
② $\subset$ xz plane
③ $\vec{d}=(1,0,0)=\hat{i}$
④ $(x,0,3)$ any point on it

** $y=0,\ z=0 \to$ st.line passes through $(0,0,0)$
① x-axis نفسه
② $\subset$ xz plane, $\subset$ xy plane
③ $\vec{d}=(1,0,0)=\hat{i}$
④ $(x,0,0)$ any point on it.

5 $Z = 5$ ثابتة , $\frac{x-2}{3} = \frac{y-7}{4}$ or $4x - 3y - 13 = 0$ → st. line ⊂ plane $z = 5$

plane ∩ plane

① parallel to xy plane
② ⊥ z-axis
③ $\vec{d} = (3, 4, 0)$
④ To find point on it put
$x - 2 = 0$, $y - 7 = 0$
$x = 2$, $y = 7$ ثابتة
the point $(2, 7, 5)$

دايما نبدأ نقرأ ال equ من الأصغر في الكتابة

$4x - 3y - 13 = 0$
$\text{slope} = \frac{-\text{Coeff of } x}{\text{Coeff of } y} = \frac{4}{3}$
$\vec{d} = (3, 4, 0)$

Special case

$Z = 0$, $\frac{x-2}{3} = \frac{y-7}{4}$ → st. line
① ⊂ the xy plane (inside)
② ⊥ Z-axis
③ $\vec{d} = (3, 4, 0)$
④ passes through $(0, 0, 0)$

6 $\frac{x-2}{3} = \frac{y-7}{4} = \frac{z-3}{8}$ → st. line
① not // } any axis
② not ⊥ } any planes (xy, xz, yz)
③ $\vec{d} = (3, 4, 8)$
④ $(2, 7, 3)$ point on it

Notice that

* $\frac{3x+1}{2} = \frac{y-1}{2} = \frac{5-z}{3}$ ⇒ equation of a st. line
① $\vec{d} = \left(\frac{2}{3}, \frac{2}{1}, \frac{3}{-1}\right)$
$= \left(\frac{2}{3}, 2, -3\right)$
② $3x + 1 = 0 \to x = \frac{-1}{3}$
$y - 1 = 0 \to y = 1$
$5 - z = 0 \to z = 5$
the point $\left(\frac{-1}{3}, 1, 5\right) \in$ st. line

* $2x = 3y = -z$ ⇒ equation of a st. line
① $\vec{d} = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{-1}\right) \to \times 6$
$\vec{d} = (3, 2, -6)$
② $(0, 0, 0)$ point ∈ st. line