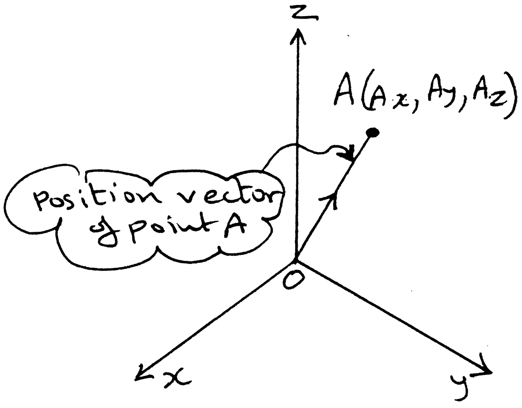
Analytic Solid Geometry 3sec

Sheet ②

Vectors in space



Position vector in space

The position vector of point A (A_x, A_y, A_z) with respect to the origin O (0, 0, 0) is defined as the directed line segment whose starting point is O and end point is A.

- The position vector of point A is denoted by $\overrightarrow{A}$ i.e. $\overrightarrow{A} = (A_x, A_y, A_z)$
- A_x is called the component of $\overrightarrow{A}$ in direction of x axis.
- A_y is called the component of $\overrightarrow{A}$ in direction of y axis.
- A_z is called the component of $\overrightarrow{A}$ in direction of z axis .

The norm of a vector

It is the length of the directed line segment which represents the vector.

If $\overrightarrow{A} = (A_x, A_y, A_z)$, then from the distance between two points rule

$$\|\overrightarrow{A}\| = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$$

Example

1 If $\overrightarrow{A} = (2, -3, 1)$, $\overrightarrow{B} = (0, 4, -3)$ then (B_x, B_y, B_z)

- The component of the vector $\overrightarrow{A}$ in the direction of x-axis is 2 $= A_x$
- The component of the vector $\overrightarrow{B}$ in the direction of Z-axis is -3 $= B_z$

$\|\overrightarrow{A}\| = \sqrt{2^2 + (-1)^2 + (3)^2} = \sqrt{14}$

$\|\overrightarrow{B}\| = \sqrt{(0)^2 + (4)^2 + (-3)^2} = 5$

The vector $\overrightarrow{B}$ lies in the y z -plane because $B_x = 0$

Example ⑪ If $\overrightarrow{A} = (-1, 4, 2)$, $\overrightarrow{B} = (3, 1, 0)$ find

a) $A_x + B_y$ b) $\|\overrightarrow{A}\| + \|\overrightarrow{B}\|$ c) $\overrightarrow{AB}$ d) $\overrightarrow{U}_{AB}$

Solution

$\overrightarrow{A} = (a_x, a_y, a_z) = (-1, 4, 2)$, $\overrightarrow{B} = (B_x, B_y, B_z) = (3, 1, 0)$

a) $A_x + B_y = -1 + 1 = 0$

b) $\|\overrightarrow{A}\| = \sqrt{(-1)^2 + 4^2 + 2^2} = \sqrt{21}$, $\|\overrightarrow{B}\| = \sqrt{3^2 + 1^2 + (0)^2} = \sqrt{10}$

$\|\overrightarrow{A}\| + \|\overrightarrow{B}\| = \sqrt{21} + \sqrt{10}$

c) $\overrightarrow{AB} = B - A = (3, 1, 0) - (-1, 4, 2) = (4, -3, -2)$

d) $\overrightarrow{U}_{AB} = \dfrac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|} = \dfrac{(4, -3, -2)}{\sqrt{(4)^2 + (-3)^2 + (-2)^2}} = \dfrac{(4, -3, -2)}{\sqrt{29}}$

$\overrightarrow{U}_{AB}$ is called unit vector in the direction of $\overrightarrow{AB}$ $= \left(\dfrac{4}{\sqrt{29}}, \dfrac{-3}{\sqrt{29}}, \dfrac{-2}{\sqrt{29}}\right)$

Example

(2) If $\vec{C} = (2,-3,1)$, $\vec{D} = (0,2,-2)$

a) find $5\vec{C} - 2\vec{D}$

b) If $3\vec{A} - 4\vec{D} = \vec{C}$, then find $\vec{A}$

Solution

a) $5\vec{C} - 2\vec{D} = 5(2,-3,1) - 2(0,2,-2)$

$= (10,-15,5) + (0,-4,4)$

$= (10,-19,9)$

b) $3\vec{A} = \vec{C} + 4\vec{D}$

$= (2,-3,1) + 4(0,2,-2)$

$= (2,-3,1) + (0,8,-8)$

$3\vec{A} = (2,5,-7) \rightarrow \vec{A} = \left(\frac{2}{3}, \frac{5}{3}, \frac{-7}{3}\right)$

(3) If $(2x+1, 5, K+4) = (-1, y^2-4, x+1)$ then find the value of x, y, K?

Solution

$2x+1 = -1$	$y^2-4=5$	$K+4 = x+1$
$2x = -2$	$y^2 = 9$	$K+4 = -1+1$
$x = -1$	$y = \pm 3$	$K = -4$

(4) show which of the following vectors is a unit vector

a) $\vec{A} = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$, b) $\vec{B} = \left(\frac{1}{5}, \frac{4}{5}, \frac{-\sqrt{5}}{5}\right)$

Solution

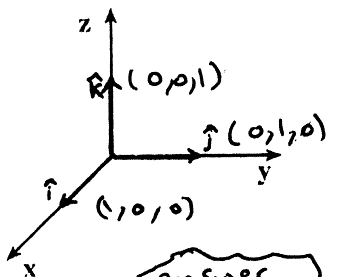
$\|\vec{A}\| = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = 1$ $\therefore \vec{A}$ represents a unit vector

$\|\vec{B}\| = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{4}{5}\right)^2 + \left(\frac{-\sqrt{5}}{5}\right)^2} = \frac{\sqrt{22}}{5} \neq 1$ $\therefore B$ is not unit vector

Fundamental unit vectors ($\hat{i}$, $\hat{j}$, $\hat{k}$)

It is a directed segments whose starting point is the origin point and its norm is the unit length and its direction is the positive direction of x, y and z axes respectively:

$\hat{i} = (1,0,0)$, $\hat{j} = (0,1,0)$, $\hat{k} = (0,0,1)$



Critical thinking

Express the vectors (-1, 0, 0) , (0, -1, 0), (0, 0, -1) interms of the fundamental unit vectors.

answer $-\hat{i}, -\hat{j}, -\hat{k}$

5) If $\vec{A} = -3\hat{j} - \hat{k} + 5\hat{i}$, $\vec{B} = -2\hat{k} + 3\hat{i}$. find

a) $3\vec{A} - 5\vec{B}$ b) $\|\vec{A} - \vec{B}\|$, $\|\vec{A}\| - \|\vec{B}\|$ What do you notice?

Solution

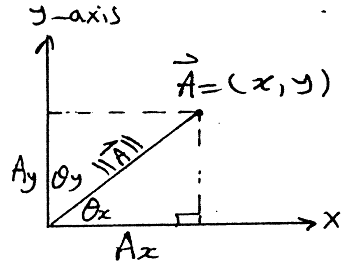
$$3\vec{A} - 5\vec{B} = 3(-3\hat{j} - \hat{k} + 5\hat{i}) - 5(-2\hat{k} + 3\hat{i})$$
$$= -9\hat{j} - 3\hat{k} + 15\hat{i} + 10\hat{k} - 15\hat{i}$$
$$= -9\hat{j} + 7\hat{k}$$
$$\vec{A} - \vec{B} = -3\hat{j} - \hat{k} + 5\hat{i} - (-2\hat{k} + 3\hat{i})$$
$$= -3\hat{j} - \hat{k} + 5\hat{i} + 2\hat{k} - 3\hat{i} = 2\hat{i} + \hat{k} - 3\hat{j}$$
$$\|\vec{A} - \vec{B}\| = \sqrt{(2)^2 + (1)^2 + (-3)^2} = \sqrt{14}$$
$$\|\vec{A}\| - \|\vec{B}\| = \sqrt{(-3)^2 + (-1)^2 + (5)^2} - \sqrt{(-2)^2 + (3)^2} = \sqrt{35} + \sqrt{13}$$

we notice that $\|\vec{A} - \vec{B}\| \neq \|\vec{A}\| - \|\vec{B}\|$

also $\|\vec{A} + \vec{B}\| \neq \|\vec{A}\| + \|\vec{B}\|$

Direction angles and Direction cosines of a vector in space

2 Dimensions



θ_x is the angle between $\vec{A}$ and x-axis

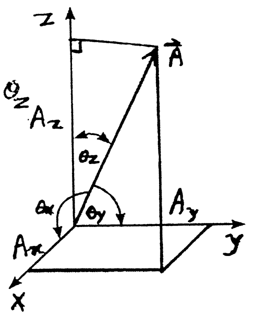
θ_y is the angle between $\vec{A}$ and y-axis

$\cos\theta_x = \frac{A_x}{\|\vec{A}\|} \longrightarrow A_x = \|\vec{A}\| \cos\theta_x$

$\cos\theta_y = \frac{A_y}{\|\vec{A}\|} \longrightarrow A_y = \|\vec{A}\| \cos\theta_y$

$\therefore \vec{A} = (x, y) = (\|\vec{A}\| \cos\theta_x , \|\vec{A}\| \cos\theta_y)$

3 Dimensions



$\vec{A} = (A_x , A_y , A_z)$

$A_x = \|\vec{A}\| \cos\theta_x$, $A_y = \|\vec{A}\| \cos\theta_y$, $A_y = \|\vec{A}\| \cos\theta_z$

$\vec{A} = (\|\vec{A}\| \cos\theta_x , \|\vec{A}\| \cos\theta_y , \|\vec{A}\| \cos\theta_z)$

$\vec{A} = \|\vec{A}\| (\cos\theta_x , \cos\theta_y , \cos\theta_z) \rightarrow \div \|\vec{A}\|$

$\therefore \frac{\vec{A}}{\|\vec{A}\|} = (\cos\theta_x , \cos\theta_y , \cos\theta_z) = \vec{u}_A$

$\therefore (\cos\theta_x , \cos\theta_y , \cos\theta_z)$ represents unit vector in the direction of vector $\vec{A}$

$\rightarrow \therefore \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\rightarrow (\theta_x , \theta_y , \theta_z)$ are called the direction angles of vector $\vec{A}$

$\rightarrow \cos\theta_x , \cos\theta_y , \cos\theta_z$ are called the direction cosines of vector $\vec{A}$

$\rightarrow \cos\theta_x = \frac{A_x}{\|A\|}$, $\cos\theta_y = \frac{A_y}{\|A\|}$, $\cos\theta_z = \frac{A_z}{\|A\|}$

Example If $\vec{A} = 2\hat{i} - 3\hat{j} + \sqrt{3}\hat{k}$ then find:

① the direction cosines of $\vec{A}$

② the direction angles of $\vec{A}$

Try by yourself (HW)

① $\vec{B} = 2\hat{i} - \hat{j} + 2\hat{k}$

② $\vec{c} = 2\hat{i} - 4\hat{j} + 4\hat{k}$

Solution

$\vec{A} = 2\hat{i} - 3\hat{j} + \sqrt{3}\hat{k} \rightarrow \therefore \|\vec{A}\| = \sqrt{(2)^2 + (-3)^2 + (\sqrt{3})^2} = 4$

$\cos\theta_x = \frac{A_x}{\|\vec{A}\|} = \frac{2}{4} = \frac{1}{2} \Rightarrow \theta_x = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$

$\cos\theta_y = \frac{A_y}{\|\vec{A}\|} = \frac{-3}{4} \Rightarrow \theta_y = \cos^{-1}\left(\frac{-3}{4}\right) = 138^\circ 35' 25''$

$\cos\theta_z = \frac{A_z}{\|\vec{A}\|} = \frac{\sqrt{3}}{4} \Rightarrow \theta_z = \cos^{-1}\left(\frac{\sqrt{3}}{4}\right) = 64^\circ 20' 28''$

$0 \leq \theta_x \leq 180$

$0 \leq \theta_y \leq 180$

$0 \leq \theta_z \leq 180$

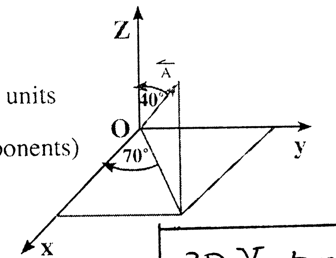
To be sure: $\left(\frac{1}{2}\right)^2 + \left(\frac{-3}{4}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2 = 1$ ✓

Example

8. The opposite figure represents a vector $\vec{A}$ whose norm is 10 units

a. Express the vector $\vec{A}$ in algebraic form (Cartesian components)

b. Find the measure of the direction angles of $\vec{A}$



Solution

First resolve $\vec{A}$ into two components; the first in the direction of $\overrightarrow{OZ}$ with magnitude A_z

$A_z = \|\vec{A}\| \cos\theta_z = 10 \cos 40 = 7.66$

The second in xy-plane

$A_{xy} = \|\vec{A}\| \sin\theta_z = 10 \sin 40 = 6.428$

Now, resolve the component A_{xy} into two components; the first is in the direction of $\overrightarrow{OX}$ with magnitude A_x

$A_x = A_{xy} \cos 70 = 6.428 \cos 70 = 2.199$

the second is in the direction of $\overrightarrow{OY}$ with magnitude

$A_y = A_{xy} \sin 70 = 6.428 \sin 70 = 6.04$

$\therefore \vec{A} = (2.199, 6.04, 7.66)$

the cartesian form of the vector $\vec{A}$ is

$\vec{A} = 2.199\hat{i} + 6.04\hat{j} + 7.66\hat{k}$

Second

$\|\vec{A}\| = \sqrt{(2.199)^2 + (6.04)^2 + (7.66)^2} = 10$

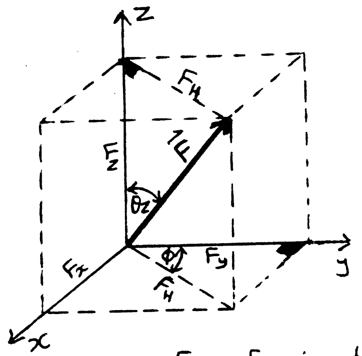
$\cos\theta_x = \frac{A_x}{\|\vec{A}\|} = \frac{2.199}{10} = 0.2199 \rightarrow \theta_x = 77.3^\circ$

$\cos\theta_y = \frac{A_y}{\|\vec{A}\|} = \frac{6.04}{10} = 0.604 \rightarrow \theta_y = 52.84^\circ$

$\cos\theta_z = \frac{A_z}{\|\vec{A}\|} = \frac{7.66}{10} = 0.766 \rightarrow \theta_z = 40^\circ$

3D Vectors described by 2 angles

ϕ is the swing angle between $\vec{oy}$ and the bottom of the door



$F_x = F_H \sin\phi$

$F_y = F_H \cos\phi$

$F_z = F \cos\theta_z$

$F_H = F \sin\theta_z$

$\therefore F_x = F \sin\theta_z \sin\phi$

$F_y = F \sin\theta_z \cos\phi$

$F_z = F \cos\theta_z$

$\|\vec{A}\| = 10, \quad \theta_z = 40^\circ, \quad \phi = 90 - 70 = 20$

$\therefore F_x = 10 \sin 40 \sin 20 = 2.199$

$F_y = 10 \sin 40 \cos 20 = 6.04$

$F_z = 10 \cos 40 = 7.66$

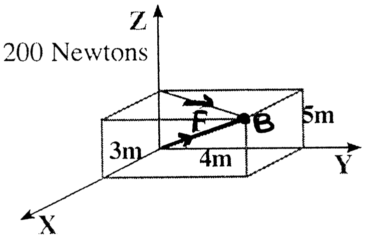
$\therefore \vec{F} = 2.199\hat{i} + 6.04\hat{j} + 7.66\hat{k}$

Try to solve

9. The opposite figure represents the force $\overrightarrow{F}$ with a magnitude 200 Newtons

a. Express the force $\overrightarrow{F}$ in an algebraic form.

b. Find the measures of the direction angles of the force $\overrightarrow{F}$.



Solution

$\because \|\overrightarrow{F}\| = 200N,\ \overrightarrow{B} = (3,4,5) \rightarrow \therefore \|\overrightarrow{B}\| = \sqrt{3^2+4^2+5^2} = 5\sqrt{2}$

$\therefore \frac{\|\overrightarrow{F}\|}{\|\overrightarrow{B}\|} = \frac{200}{5\sqrt{2}} = 20\sqrt{2}$

$\therefore \|\overrightarrow{F}\| = 20\sqrt{2}\,\|\overrightarrow{B}\|$

$\therefore \overrightarrow{F} = 20\sqrt{2}(3,4,5)$

$\therefore \overrightarrow{F} = (60\sqrt{2}, 80\sqrt{2}, 100\sqrt{2})$

a) $\rightarrow \overrightarrow{F} = 60\sqrt{2}\,\hat{i} + 80\sqrt{2}\,\hat{j} + 100\sqrt{2}\,\hat{K}$

$\cos\theta_x = \frac{F_x}{\|\overrightarrow{F}\|} = \frac{60\sqrt{2}}{200} \rightarrow \theta_x = 64^\circ 53' 45''$

$\cos\theta_y = \frac{F_y}{\|\overrightarrow{F}\|} = \frac{80\sqrt{2}}{200} \rightarrow \theta_y = 55^\circ 33'$

$\cos\theta_z = \frac{F_z}{\|\overrightarrow{F}\|} = \frac{100\sqrt{2}}{200} \rightarrow \theta_z = 45^\circ$

Example Find the direction cosines and direction angles of the vector $\overrightarrow{v} = (c,c,c)$, where $c > 0$. Round direction angles to the nearest degree

Solution

$\overrightarrow{v} = (c,c,c) \rightarrow \|\overrightarrow{v}\| = \sqrt{c^2+c^2+c^2} = \sqrt{3c^2} = |c|\sqrt{3} = c\sqrt{3}$

$\cos\theta_x = \frac{c}{c\sqrt{3}} = \frac{1}{\sqrt{3}} \rightarrow \theta_x = 55^\circ$

$\cos\theta_y = \frac{c}{c\sqrt{3}} = \frac{1}{\sqrt{3}} \rightarrow \theta_y = 55,\quad \theta_z = 55^\circ$

Example If a vector has direction angles $\frac{\pi}{4}$, $\frac{\pi}{3}$ then find the third direction angle

Solution

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\frac{\pi}{4} + \cos^2\frac{\pi}{3} + \cos^2\gamma = 1$

$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\gamma = 1$

$\therefore \cos^2\gamma = \frac{1}{4}$

$\cos\gamma = \pm\frac{1}{2}$

$\cos\gamma = \frac{1}{2} \rightarrow \gamma = 60^\circ = \frac{\pi}{3}$

$\cos\gamma = \frac{-1}{2} \rightarrow \gamma = 120^\circ = \frac{2\pi}{3}$

Rules and Ideas

[1] $\overrightarrow{AB} = \vec{B} - \vec{A}$

[2] $\vec{A} = (2, -4, 4) \rightarrow \|\vec{A}\| = \sqrt{(2)^2 + (-4)^2 + (4)^2} = 6$ units

- A_x, A_y, A_z are the components of $\vec{A}$ in the directions of the cartesian axes.
- the component of $\vec{A}$ vanishes in the direction of X-axis means 1) $A_x = 0$ 2) $\vec{A}$ lies in the yz plane
- $\vec{A}$ parallel to xy plane means $A_z = C$ (constant) $\rightarrow A = (x, y, c)$
- the algebraic form of vector $\vec{A}$ is $\vec{A} = 2\hat{i} - 4\hat{j} + 4\hat{k}$
- the component of $\vec{A}$ in the direction of X-axis = 2 (number)

لاحظ الفرق the projection of $\vec{A}$ in the direction of x-axis = $2\hat{i}$ (vector)

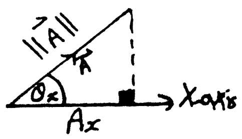
- $\theta_x, \theta_y, \theta_z$ are the direction angles of $\vec{A}$

$0 \le \theta_x \le \pi$
$0 \le \theta_y \le \pi$
$0 \le \theta_z \le \pi$

- $\cos\theta_x = \dfrac{A_x}{\|\vec{A}\|} \rightarrow A_x = \|\vec{A}\| \cos\theta_x$

$\cos\theta_y = \dfrac{A_y}{\|\vec{A}\|} \rightarrow A_y = \|\vec{A}\| \cos\theta_y$

$\cos\theta_z = \dfrac{A_z}{\|\vec{A}\|} \rightarrow A_z = \|A\| \cos\theta_z$



- $\cos\theta_x, \cos\theta_y, \cos\theta_z$ are the direction cosines of $\vec{A}$

شرط مهم جداً $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

تستخدم عندما يعطي 2 angles ويطلب الثالثة

- $\vec{u}_A$ is a unit vector in the direction of $\vec{A} \Rightarrow \|\vec{u}_A\| = 1$ unit

$\vec{u}_A = \dfrac{\vec{A}}{\|\vec{A}\|} = \dfrac{(A_x, A_y, A_z)}{\|\vec{A}\|} = \left(\dfrac{A_x}{\|\vec{A}\|}, \dfrac{A_y}{\|\vec{A}\|}, \dfrac{A_z}{\|\vec{A}\|}\right) = (\cos\theta_x, \cos\theta_y, \cos\theta_z)$

$\because \|\vec{u}_A\| = 1 \rightarrow \therefore \sqrt{\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z} = 1$ by squaring

$\therefore \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

[3] $\hat{i} = (1,0,0)$, $\hat{j} = (0,1,0)$, $\hat{k} = (0,0,1)$, $\|\hat{i}\| = \|\hat{j}\| = \|\hat{k}\| = 1$

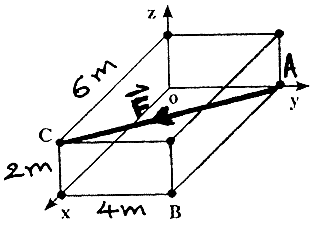
If $\vec{A} = (K, 0, 0) = K(1,0,0) = K\hat{i} \rightarrow \therefore \|\vec{A}\| = \|K\hat{i}\|$

$\therefore \|\vec{A}\| = |K| \, \|\hat{i}\| = |K|$

[4] $\vec{F} = 14N$, $C = (6,0,2)$, $A = (0,4,0)$

$\overrightarrow{AC} = C - A = (6,0,2) - (0,4,0) = (6,-4,2)$

$\vec{F} = K\,\overrightarrow{AC} = K(6,-4,2) = (6K, -4K, 2K)$



$\|\vec{F}\| = \sqrt{(6K)^2 + (-4K)^2 + (2K)^2}$

$14 = \sqrt{56K^2} \rightarrow 196 = 56K^2 \rightarrow K = \dfrac{\sqrt{14}}{2}$

$\therefore \vec{F} = \dfrac{\sqrt{14}}{2}(6,-4,2) = (3\sqrt{14}, -2\sqrt{14}, \sqrt{14}) = 3\sqrt{14}\,\hat{i} - 2\sqrt{14}\,\hat{j} + \sqrt{14}\,\hat{k}$

Exercises 1 - 2

P.122

Complete the following:

(1) If $\overrightarrow{A} = (-3, 4, 2)$, then $\| \overrightarrow{A} \| =$ ....

(2) If $\overrightarrow{A} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\overrightarrow{B} = 3\hat{i} - \hat{k}$, then $\overrightarrow{A} - \overrightarrow{B} =$ ....

(3) The unit vector in the direction of $\overrightarrow{AB}$ where A (-1, 2, 0) , B (3, -1, 2) is ....

(4) The vector $\overrightarrow{A} = 3\hat{i} + \hat{j} - 2\hat{k}$ makes an angle of measure .... with the +ve direction of x-axis.

(5) The vector $\overrightarrow{B} = \hat{i} + 2\hat{j}$ makes an angle of measure .... with the +ve direction of z-axis.

Choose the correct answer from the following:

(6) If $\overrightarrow{A} = (-2, k, 1)$ and $\| \overrightarrow{A} \| = 3$ unit, then k = ...

a) 4 b) -4 c) ± 2 d) $\sqrt{14}$

(7) If 30°, 70°, θ are the direction angles of a vector, then one of the values of $\theta =$ ....

a) 100° b) 80° c) 260° d) 68,61°

(8) If $\overrightarrow{A} = (-1, 5, -2)$, $\overrightarrow{B} = (3, 1, 1)$ and if $\overrightarrow{A} + \overrightarrow{B} + \overrightarrow{C} = \hat{i}$, then $\overrightarrow{C} =$ ....

a) $\hat{i} + 6\hat{j} - \hat{k}$ b) $-\hat{i} - 6\hat{j} + \hat{k}$

c) $\hat{i} + 4\hat{j} - 3\hat{k}$ d) $\hat{i} + 4\hat{j} - \hat{k}$

(9) The direction cosines of the vector $\overrightarrow{A} = (-2, 1, 2)$ is ....

a) (-2, 1, 2) b) $(\frac{-2}{3}, \frac{1}{3}, \frac{2}{3})$ c) $(\frac{5}{-2}, 5, \frac{5}{2})$ d) (-1, 1, 1)

Answer the following :

(10) If $\overrightarrow{A} = (2, -3, 1)$, $\overrightarrow{B} = (4, -2, 0)$, $\overrightarrow{C} = (-6, 0, 3)$, find each of the following vectors:

a) $\overrightarrow{A} + \overrightarrow{B}$ b) $3\overrightarrow{A} - \frac{1}{3}\overrightarrow{C}$ c) $\frac{3}{2}\overrightarrow{B} + \frac{2}{3}\overrightarrow{C}$

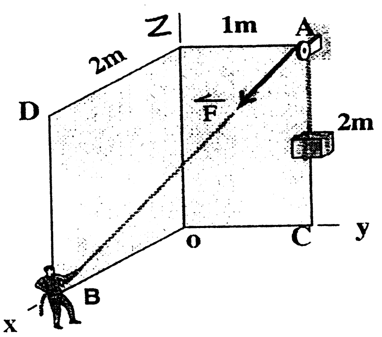
(11) If $\overrightarrow{A} = 2\hat{i} - 3\hat{j} + 5\hat{k}$, $\overrightarrow{B} = 4\hat{j} - 2\hat{k}$, $\overrightarrow{C} = 4\hat{i} + 5\hat{j} - 6\hat{k}$, find each of the following vectors :

a) $2\overrightarrow{A} + \overrightarrow{B}$ b) $\frac{1}{2}\overrightarrow{B} - \overrightarrow{C}$ c) $3\overrightarrow{A} - 2\overrightarrow{C}$

(12) Find the norm of each of the following vectors:

a) $\overrightarrow{A} = (2, -1, 0)$ b) $\overrightarrow{B} = (1, 2, -2)$ c) $\overrightarrow{C} = \hat{j}$ d) $\overrightarrow{D} = \hat{i} - 4\hat{j}$

(13) If $\overrightarrow{A} = (k, 0, 0)$, $\hat{i} = (1, 0, 0)$, prove that $\| \overrightarrow{A} \| = |k| \, \| \hat{i} \|$



(14) If the tension force in a stringe equals 21 newtons , find the algebraic components of the force $\overrightarrow{F}$ in the directions of the Cartesian axes

(15) **Open question:** What can you say about the coordinates of the vector $\overrightarrow{A}$ if the vector $\overrightarrow{A}$ is parallel to y z-plane ?

(16) **Open question:** If $\overrightarrow{A}$ and $\overrightarrow{B}$ are two vectors in R^3. Is $\| \overrightarrow{A} + \overrightarrow{B} \| = \| \overrightarrow{A} \| + \| \overrightarrow{B} \|$? Which side is greater if both sides are unequal ?

(17) **Creative thinking:** Find the Algebric form of the vector $\overrightarrow{A}$ if its norm is 5 units and makes equal angles with the +ve directions of the Cartesian axes .

(14) $A = (0, 1, 2)$, $B = (2, 0, 0)$

$\overrightarrow{AB} = B - A = (2, 0, 0) - (0, 1, 2)$

$\overrightarrow{AB} = (2, -1, -2)$

$\|\overrightarrow{AB}\| = \sqrt{(-2)^2 + (-1)^2 + (-2)^2} = 3.$

$\cos\theta_x = \frac{A_x}{\|\vec{A}\|} = \frac{2}{3}$

$\cos\theta_y = \frac{A_y}{\|\vec{A}\|} = \frac{-1}{3}$

$\cos\theta_z = \frac{A_z}{\|\vec{A}\|} = \frac{-2}{3}$

$F_x = F\cos\theta_x = 21 \times \frac{2}{3} = 14$

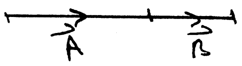
$F_y = F\cos\theta_y = 21 \times \frac{-1}{3} = -7$

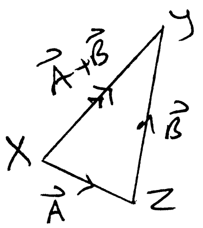
$F_z = F\cos\theta_z = 21 \times \frac{-2}{3} = -14$

$F = 14\hat{i} - 7\hat{j} - 14\hat{k}$

(15) $A_x = K$ (constant) $\rightarrow \vec{A} = (K, y, z)$

(16) No





$\|\vec{A} + \vec{B}\| = \|\vec{A}\| + \|\vec{B}\|$ only iff $\vec{A}, \vec{B}$ on the same St. line

but from triangle inequality

$XY < XZ + ZY$

$\|\vec{A} + \vec{B}\| < \|\vec{A}\| + \|\vec{B}\|$

so we say that

$\|\vec{A} + \vec{B}\| \leq \|\vec{A}\| + \|\vec{B}\|$

(17) $\|\vec{A}\| = 5$

$\theta_x = \theta_y = \theta_z$

$\therefore \cos\theta_x = \cos\theta_y = \cos\theta_z$

$\because \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\therefore 3\cos^2\theta_x = 1$

$\cos^2\theta_x = \frac{1}{3}$ $(\pm\sqrt{\ })$

$\therefore \cos\theta_x = \pm\frac{\sqrt{3}}{3}$

$A_x = \|A\|\cos\theta_x = 5 \times \pm\frac{\sqrt{3}}{3} = \pm\frac{5\sqrt{3}}{3}$

$A_y = A_z = \pm\frac{5\sqrt{3}}{3}$

$\therefore \vec{A} = (\pm\frac{5\sqrt{3}}{3})\hat{i} + (\pm\frac{5\sqrt{3}}{3}\hat{j}) + (\pm\frac{5\sqrt{3}}{3})\hat{k}$

$\therefore \vec{A} = \pm\frac{5\sqrt{3}}{3}(\hat{i} + \hat{j} + \hat{k})$

(13) $\vec{A} = (K, 0, 0)$

$\vec{A} = K(1, 0, 0)$

$\vec{A} = K\hat{i}$

$\|\vec{A}\| = \|K\hat{i}\|$

$\|\vec{A}\| = |K|\,\|\hat{i}\|$

Answers of exercises (1 - 2)

(1) $\sqrt{29}$ (2) $\hat{i} - \hat{j} + 4\hat{k}$

(3) $(\frac{4}{\sqrt{29}}, \frac{-3}{\sqrt{29}}, \frac{2}{\sqrt{29}})$

(4) 41' 57"36° (5) 90°

(6) ±2 (7) 82,355°

(8) $-\hat{i} - 6\hat{j} + \hat{k}$ (9) $(\frac{-2}{3}, \frac{1}{3}, \frac{2}{3})$

(10) a (6, -5, 1) b (8, -9, 2) c (2, -3,2)

(11) a (4, -2, 8) b $(-3, \frac{-13}{2}, \frac{17}{2})$ c (-2, -19- 27)

(12) a $\sqrt{5}$ b 3 c 1 d $\sqrt{17}$

(13) proof (14) (-14, 7, 14)

(15) $\vec{A} = (K, y, z)$

(16) $\|\vec{A} + \vec{B}\| \leqslant \|\vec{A} + \vec{B}\|$