3rd sec

Analytic Solid Geometry

sheet ①

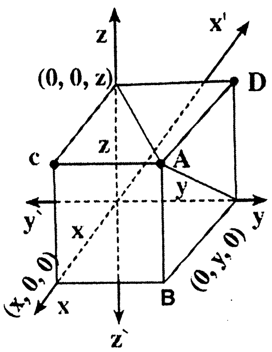
Unit Ⅱ lesson ①

Geometry and Measurement in two and three dimensions

The three - dimensional orthogonal coordinate system R^3

مهم جداً

the point on X-axis is denoted by $(x, 0, 0)$
the point on y-axis is denoted by $(0, y, 0)$
the point on Z-axis is denoted by $(0, 0, z)$



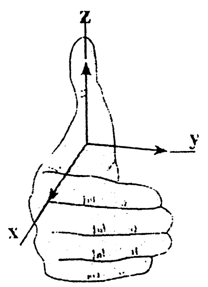
Example
1) the point $(0, -3, 0)$ lies on .... axis
2) the point $(3, 0, 0)$ lies on ..... axis
3) the point $(0, 0, -5)$ lies on ..... axis

answer
1) y-axis
2) x-axis
3) z-axis

basic definitions:

1- The right hand rule

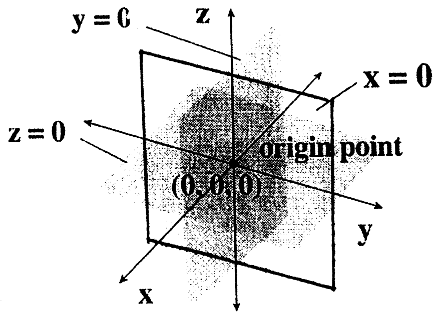
when the 3-dimensional orthogonal coordinate system is formed, we should follow the right hand rule.
where the curved fingers refer from the +ve direction of x-axis towards the positive direction of y-axis, your thumb points at the direction of positive z-axis.



مهم جداً

2- Cartisian planes

- All points in space with coordinates $(x, y, 0)$ are located in x y plane whose equation is $Z = 0$
- All points in space with coordinates $(x, 0, z)$ are located in x z plane whose equation is $y = 0$
- All points in space with coordinates $(0, y, z)$ are located in y z plane whose equation is $x = 0$.



Example what is the equation of the plane passing through
① $(3, 0, 4)$, $(-5, 0, 1)$, $(7, 0, -3)$
② $(0, 3, 5)$, $(0, -1, -2)$, $(0, 5, -2)$
③ $(6, 3, 0)$, $(-5, 0, 0)$, $(0, 0, 0)$

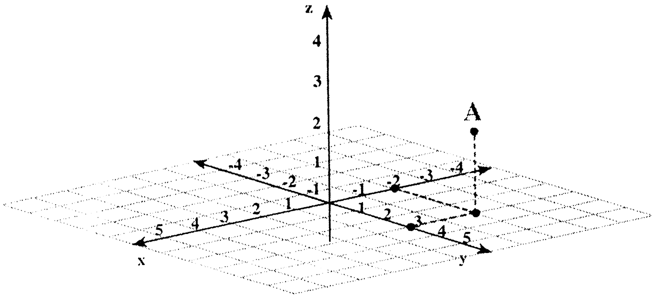
answer
① $y = 0$ ② $x = 0$ ③ $z = 0$

Example (Identifying the position of a point in space)

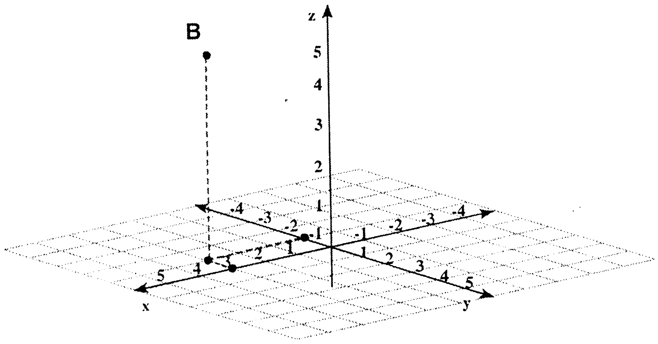
1. Identify the position of each of the following points using 3D-orthogonal coordinate system:
   a A (-2, 3, 2) **b** B (3, -1, 5) **c** C (4, 0, -1)

Solution

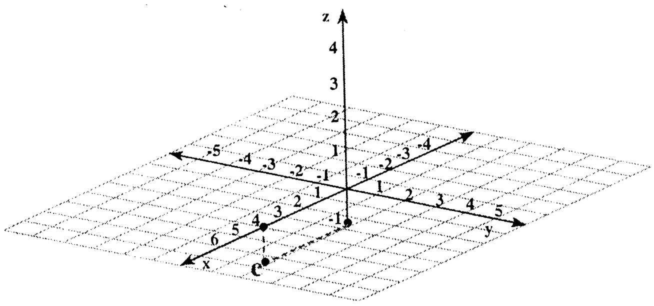
a To identify the point A (-2, 3, 2), we identify the point (-2, 3) in the x y plane, then move 2 units in the +ve direction of z-axis to get point A (-2, 3 , 2)



b To identify the point B (3, -1, 5), we identify the point (3, -1) in xy plane then move 5 units in the +ve direction of Z-axis to get point B.



c To identify the point C (4, 0, -1), we identify point (4, 0) on x-axis, then move one unit in the -ve direction of z-axis.



Try to solve

1. **a** Identify the position of each of the following points using 3-dimensional orthogonal coordinate system:
   A (3, 2, 3) B (-1, 4, 3) C (0, 0, 4)

   b **Complete:**
   1- The distance between point A (-1, 2, 3) and the Cartesian xy-plane = ... unit length.
   2- The distance between point B (4, -2, 1) and the cartesian yz-plane = ... unit length.

Solution

1) the equation of the xy − plane is $z = 0$
$\therefore$ the distance $= |3 - 0| = 3$ or $|0 - 3| = 3$ units

2) the equation of the yz − plane is $x = 0$
$\therefore$ the distance $= |4 - 0| = 4$ or $|0 - 4| = 4$ units

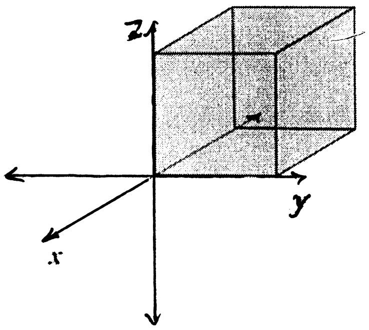
(−1, 2, 3) المسافة من xy − plane هي أخذ الـ Z coordinate

(4, −2, 1) المسافة من yz − plane هي أخذ الـ x coordinate

بالعربي كده

3-Dimensional Coordinate System

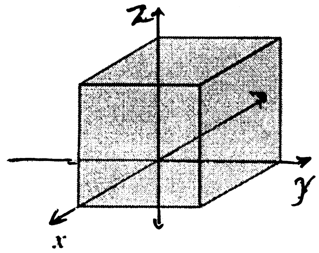
2nd octant



	(x, y, z)
I	(+, +, +)
II	(−, +, +)

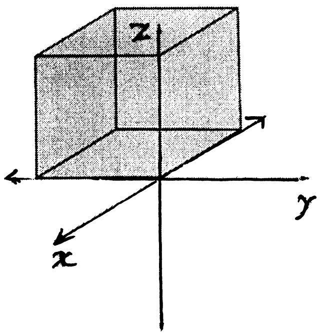
3-Dimensional Coordinate System

1st octant



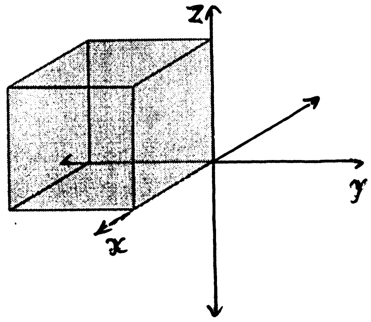
	(x, y, z)
I	(+, +, +)

3rd octant



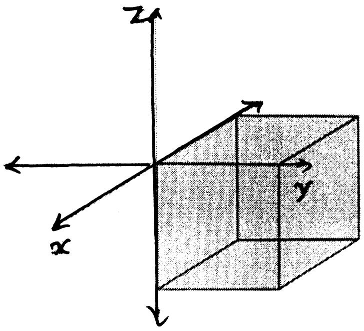
	(x, y, z)
I	(+, +, +)
II	(−, +, +)
III	(−, −, +)

4th octant



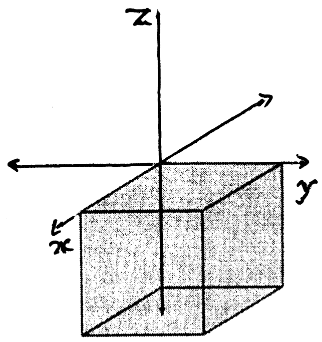
	(x, y, z)
I	(+, +, +)
II	(−, +, +)
III	(−, −, +)
IV	(+, −, +)

6th octant



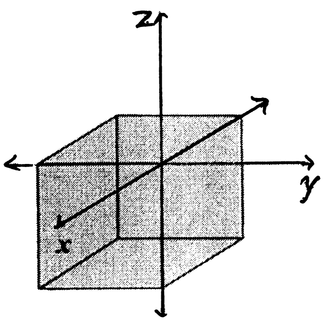
	(x, y, z)
I	(+, +, +)
II	(−, +, +)
III	(−, −, +)
IV	(+, −, +)
V	(+, +, −)
VI	(−, +, −)

5th octant



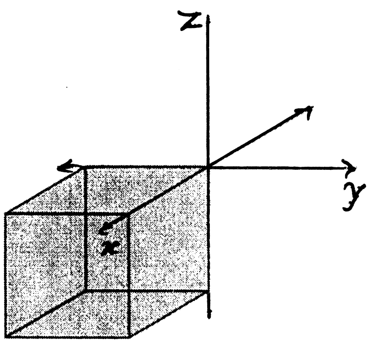
	(x, y, z)
I	(+, +, +)
II	(−, +, +)
III	(−, −, +)
IV	(+, −, +)
V	(+, +, −)

7th octant



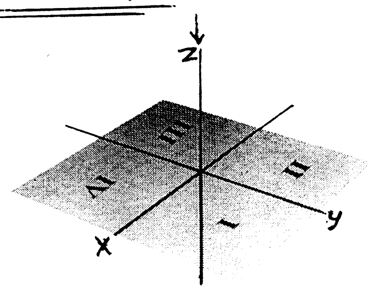
	(x, y, z)
I	(+, +, +)
II	(−, +, +)
III	(−, −, +)
IV	(+, −, +)
V	(+, +, −)
VI	(−, +, −)
VII	(−, −, −)

8th octant



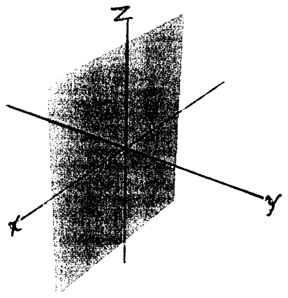
	(x, y, z)
I	(+, +, +)
II	(−, +, +)
III	(−, −, +)
IV	(+, −, +)
V	(+, +, −)
VI	(−, +, −)
VII	(−, −, −)
VIII	(+, −, −)

Equations in R^3



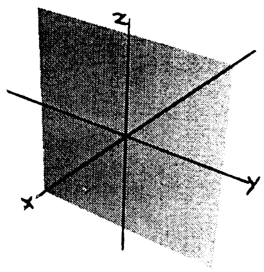
xy plane

$z = 0$



xz plane

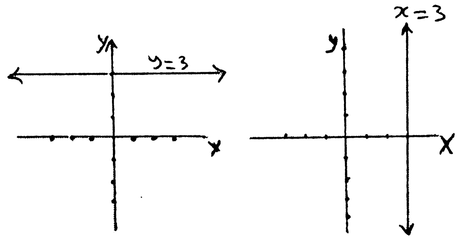
$y = 0$



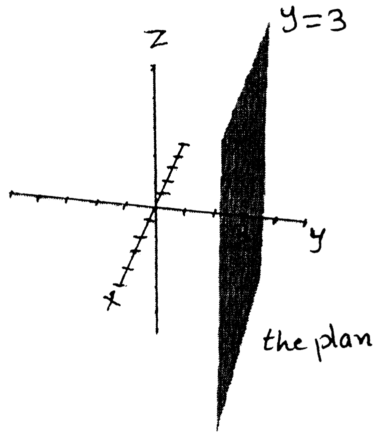
yz plane

$x = 0$

R^2

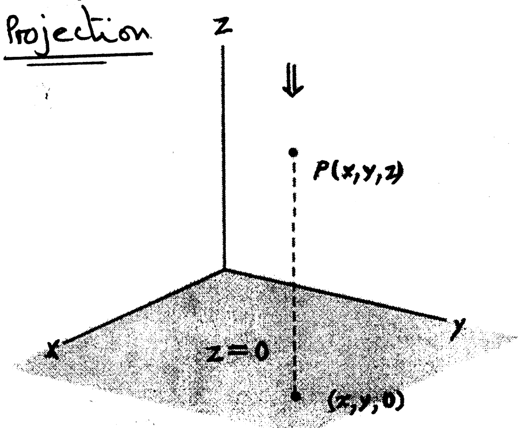


R^3

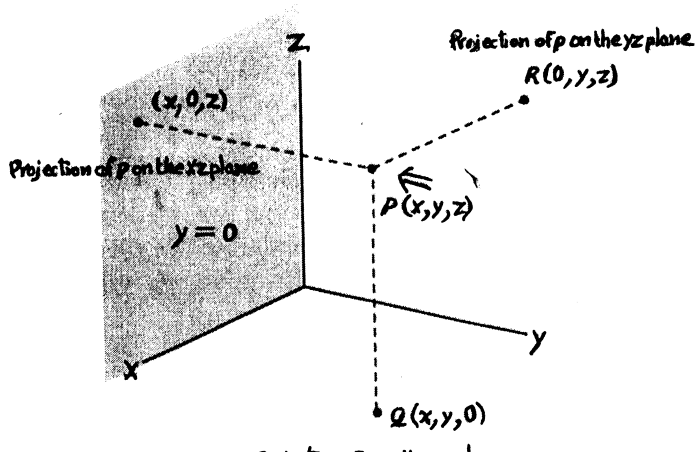


the plane $y = 3 \,//\, xz$ plane

Projection



Projection of p on the xy plane

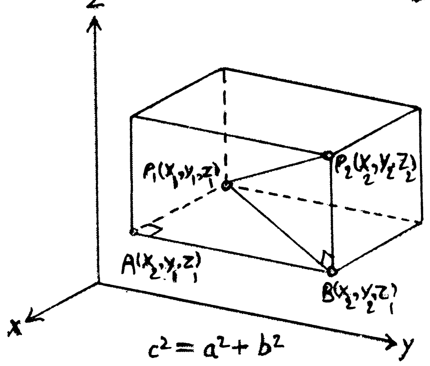


Projection of p on the yz plane $R(0,y,z)$

Projection of p on the xy plane

Distance Formula

$$D = |P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



$c^2 = a^2 + b^2$

Pythagorean theorem

The distance between two points in space

If $A = (x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ are two points in space, then the distance between A and B is given from:

$$AB = \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Example [1] Find the distance between the two points $A(3, -2, 5)$, $B(5, 2, 3)$

Solution

$$AB = \sqrt{(5-3)^2 + (2--2)^2 + (3-5)^2} = 2\sqrt{6}$$

Example [2]

(2) Prove that the triangle ABC where A (2, -1, 3), B(-4, 4, 2) and C (-2, 5, 1) is right angled at C.

Solution

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ the distance rule

$= \sqrt{(2+4)^2 + (-1-4)^2 + (3-2)^2} = \sqrt{62}$

$BC = \sqrt{(-4+2)^2 + (4-5)^2 + (2-1)^2} = \sqrt{6}$

$AC = \sqrt{(2+2)^2 + (-1-5)^2 + (3-1)^2} = \sqrt{56}$

$\because (AB)^2 = (\sqrt{62})^2 = 62$, $(BC)^2 + (AC)^2 = (\sqrt{6})^2 + (\sqrt{65})^2 = 6 + 56 = 62$

$\therefore (AB)^2 = (BC)^2 + (AC)^2$ $\therefore m(\hat{C}) = 90°$

Try to solve

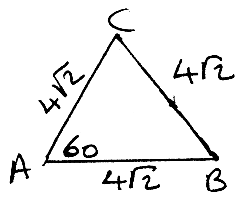
(2) Prove that the points A(4, 4, 0), B(4, 0, 4), C(0, 4, 4) are the vertices of an equilateral triangle and find its area.

Solution

$AB = \sqrt{(4-4)^2 + (0-4)^2 + (4-0)^2} = 4\sqrt{2}$

$BC = \sqrt{(0-4)^2 + (4-0)^2 + (4-4)^2} = 4\sqrt{2}$

$AC = \sqrt{(0-4)^2 + (4-4)^2 + (4-0)^2} = 4\sqrt{2}$



$\therefore AB = BC = AC \rightarrow \therefore \triangle ABC$ is equilateral $\triangle$

area of equilateral $\triangle = \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(4\sqrt{2})^2 = 8\sqrt{3}\ \text{Cm}^2$

or area of equilateral triangle $= \frac{1}{2} AB \times AC \sin \angle A$

$= \frac{1}{2} \times (4\sqrt{2})(4\sqrt{2}) \sin 60$

$= 8\sqrt{3}\ \text{Cm}^2$

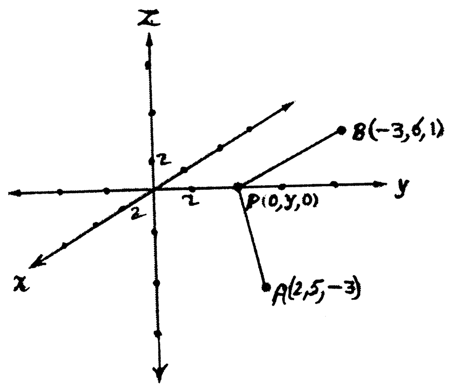
Example 3 Determine wheter the points $P(2,-1,5)$, $Q=(6,0,6)$, $R(14,2,8)$ lie on a st. line (collinear)

Solution

$$PQ=\sqrt{(6-2)^2+(0--1)^2+(6-5)^2}=3\sqrt{2}$$
$$QR=\sqrt{(14-6)^2+(2-0)^2+(8-6)^2}=6\sqrt{2}$$
$$PR=\sqrt{(14-2)^2+(2--1)^2+(8-5)^2}=9\sqrt{2}$$
$$\because PR=PQ+QR \quad \therefore P, Q, R \text{ are collinear}$$

Example 4 Find the point on the y-axis equidistant from the point $(2,5,-3)$ and $(-3,6,1)$

Solution



let the point $P=(0,y,0)$ then

$PA=PB$, $A=(2,5,-3)$, $B=(-3,6,1)$

$$PA=\sqrt{(2-0)^2+(5-y)^2+(-3-0)^2}$$
$$\therefore PA=\sqrt{4+(5-y)^2+9}=\sqrt{13+(5-y)^2}$$

$$PB=\sqrt{(-3-0)^2+(6-y)^2+(1-0)^2}$$
$$\sqrt{9+(6-y)^2+1}=\sqrt{10+(6-y)^2}$$

$$\therefore \sqrt{10+(6-y)^2}=\sqrt{13+(5-y)^2} \quad \text{by squaring}$$
$$10+(6-y)^2=13+(5-y)^2$$
$$10+36-12y+y^2=13+25-10y+y^2$$
$$46-12y=38-10y$$
$$-12y+10y=-8$$
$$-2y=-8 \quad \rightarrow \div -2$$
$$\therefore y=4$$

$\therefore$ the point $P=(0,4,0)$

Rules and Ideas

If $A=(x_1, y_1, z_1)$, $B=(x_2, y_2, z_2)$ then
$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

special cases:

→ 1) the distance between the point (x, y, z) and the origin point $=\sqrt{x^2+y^2+z^2}$

→ 2) the distance between the two points (x_1, y_1, z_1), (x_1, y_1, z_2) is $|z_2-z_1|$

eg: $A=(1, 2, 5)$, $B=(1, 2, 8)$ then $AB=|5-8|=3$

→ 3) the distance between the point (x, y, z) and xy-plane (whose equation $z=0$) $=|z|$

eg: $A=(1, 2, 5)$ then the distance between A and xy-plane $=|-5|=5$ units

– the distance between the point $(2, -3, 5)$ and the xz-plane $=|-3|=3$ units

→ 4) – the distance between the point (x, y, z) with x-axis equals $\sqrt{y^2+z^2}$

– the length of the perpendicular from the point (x, y, z) to y-axis equals $\sqrt{x^2+z^2}$

→ 5) the distance between the two planes $z-5=0$ and $z+3=0$ equals- - - -

Solution

$z=5$, $z=-3$

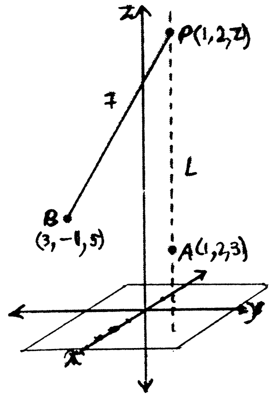
∴ the distance $=|5--3|=8$ units

→ 6) If a st. line perpendicular to xy-plane and passes through the point $(2, 3, 7)$ then any point on it will be $(2, 3, z)$

x, y are constants

Example (5) If the st. line L passes through point $A(1,2,3)$ and is perpendicular to the xy-plane, what are the coordinates of the points that lie on line L that are at a distance of 7 from point $B(3,-1,5)$?

Solution.



$\because$ point P lies on st. line $\perp$ xy-plane

$\therefore$ the x, y coordinates are fixed as the x, y coordinates of the point $(\overset{x}{1}, \overset{y}{2}, 3)$

$\therefore$ Point $P = (1, 2, Z)$, $B = (3, -1, 5)$

$\therefore$ $PB = 7$

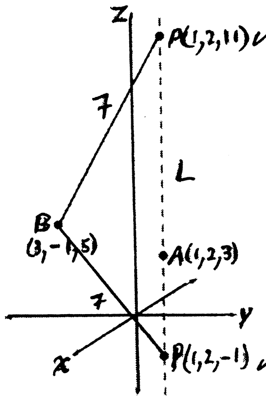
$\therefore \sqrt{(1-3)^2 + (2--1)^2 + (Z-5)^2} = 7$

$\sqrt{4 + 9 + (Z-5)^2} = 7$ by squaring

$\left(\sqrt{13 + (Z-5)^2}\right)^2 = 7^2$

$13 + (Z-5)^2 = 49$

$(Z-5)^2 = 36$



$\therefore (Z-5)^2 = 36 \quad (\pm\sqrt{\ })$

$Z - 5 = \pm 6$

$Z - 5 = 6 \quad \Big\} \quad Z - 5 = -6$

$Z = 11 \quad \Big\{ \quad Z = -1$

$\therefore$ there are two points on $\overleftrightarrow{L}$

$(1, 2, 11)$, $(1, 2, -1)$

The coordinates of a midpoint of a line segment

If $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ are two points in space, then the coordinates of point C which lies at the midpoint of $\overline{AB}$ is :

$$C = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

Example

3 If A(1, -3, 2), B (4, -1, 4), find the coordinates of the midpoint of $\overline{AB}$

Solution

The coordinates of the midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

$= \left(\frac{1+4}{2}, \frac{-3-1}{2}, \frac{2+4}{2}\right)$

$= \left(\frac{5}{2}, -2, 3\right)$

Try to solve

3 Find the coordinates of the midpoint of $\overline{CD}$ where C (0, 4, -2), D(-6, 3, 4)

Critical thinking: If C (2, 2, 6) is the midpoint of $\overline{AB}$ where A (1, -4 , 0), find coordinates of the point B

Solution

let $B = (x, y, z)$, $C = \frac{A+B}{2}$

$\therefore C(2, 2, 6) = \left(\frac{x+1}{2}, \frac{y-4}{2}, \frac{z+0}{2}\right)$

$\therefore \frac{x+1}{2} = 2 \longrightarrow x+1 = 4 \longrightarrow x = 3$

$\frac{y-4}{2} = 2 \longrightarrow y-4 = 4 \longrightarrow y = 8$

$\frac{z+0}{2} = 6 \longrightarrow z = 12$ $\therefore B = (3, 8, 12)$

or

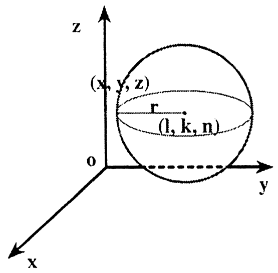
$B = 2C - A$, $A = 2C - B$

$= 2(2, 2, 6) - (1, -4, 0)$

$= (4, 4, 12) - (1, -4, 0) = (3, 8, 12)$

Equation of sphere in space

The sphere is identified by a set of points in space equidistant from a fixed point called (the center of the sphere) and the distance called (the radius length of the sphere).



If point (x, y, z) lies on the sphere of center (ℓ, k, n) and radius length r according to the rule of the distance between two points,

then $r = \sqrt{(x - L)^2 + (y - k)^2 + (z - n)^2}$ by squaring both sides, we get the standard form of the equation of the sphere $(x - L)^2 + (y - k)^2 + (z - n)^2 = r^2$

The general equation of the sphere

$$x^2 + y^2 + z^2 + 2\ell x + 2Ky + 2nz + d = 0$$

the center is $(-\ell, -K, -n)$

$$\ell^2 + K^2 + n^2 = r^2 + d \longrightarrow d = \ell^2 + K^2 + n^2 - r^2$$

$$r^2 = \ell^2 + K^2 + n^2 - d \longrightarrow r = \sqrt{\ell^2 + K^2 + n^2 - d}\ ,\ \ell^2 + K^2 + n^2 - d > 0$$

the center $= (-\frac{1}{2}$ coeff of x, $-\frac{1}{2}$ coeff of y, $-\frac{1}{2}$ coeff of $z)$

Example

(4) Find the standard form of the equation of the sphere where center is (2, -1, 4)and radius length is 3 units.

Solution

the equation of the sphere is $(x - 2)^2 + (y + 1)^2 + (z - 4)^2 = 9$

Try to solve

(4) Find the equation of the sphere whose center is the origin and its radius length is 5 units.

Example

(5) Find the equation of the sphere in which A(-1, 5 ,4) , B(5, 1, -2) are the end points of its diameter.

Solution

The center of the sphere is the midpoint of $\overline{AB} = (\frac{-1+5}{2}, \frac{5+1}{2}, \frac{4-2}{2}) = (2, 3, 1)$

The radius length is the distance between the center and point A

$\therefore r = \sqrt{(2+1)^2 + (3-5)^2 + (1-4)^2} = \sqrt{22}$

$\therefore$ The equation of the sphere is: $(x - 2)^2 + (y - 3)^2 + (z - 1)^2 = 22$

Try to solve

(5) Find equation of the sphere whose diameter is $\overline{AB}$ where A (-1, 4 , 2), B (3, -2, 6)

Example

(6) Identify the center and the radius length of the sphere whose equation is
$x^2 + y^2 + z^2 + 4x - 2y - 6z + 11 = 0$

Solution

The coordinates of the center $= (-\frac{1}{2}$ coeff. of x, - coff. of y, $-\frac{1}{2}$coeff. of z$) = (-2, 1, 3)$

$\therefore r = \sqrt{\ell^2 + k^2 + n^2 - d} = \sqrt{(-2)^2 + (1)^2 + (3)^2 - 11} = \sqrt{3}$ unit length

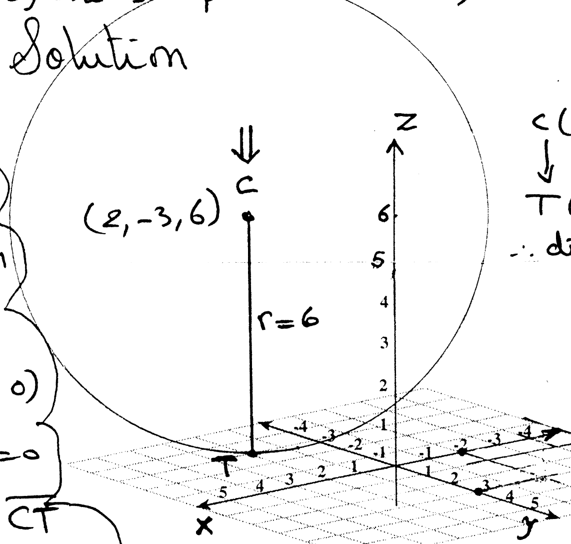
Try to solve

(6) Identify the center and the radius length of the sphere whose equation is
$x^2 + y^2 + z^2 + 6x - 8y + 4z + 1 = 0$

Example find the equation of the sphere with center $(2,-3,6)$ that touch

a) the xy plane b) the yz plane c) the xz plane

Solution



fixed
$C(2,-3,6)$
$T(2,-3,0)$
$\therefore$ distance $= 6$

$\because$ the sphere touch xy plane
$\therefore$ point of tangency is point T which is the projection of C on xy plane

$C(2,-3,6)$ $\xrightarrow[\text{on } xy \text{ place}]{\text{its projection}}$ $T(2,-3,0)$

we put $z=0$

$\therefore$ the radius is the the length of $\overline{CT}$
$=$ distance between $(2,-3,6),(2,-3,0) = |6-0| = 6$ units $=$ Z coordinate

Directely

a) the sphere touch xy plane, the center is $(2,-3,6)$ (6 → radius)

$\therefore r = 6$ units $=$ the Z coordinate of the center

$\therefore$ the equation is $(x-2)^2+(y+3)^2+(z-6)^2=36$

b) the sphere touch yz plane, the center $(2,-3,6)$

$\therefore r = 2 =$ the x coordinate

$\therefore$ the equation is: $(x-2)^2+(y+3)^2+(z-6)^2=4$

c) the sphere touch xz plane, the center $(2,-3,6)$

$\therefore r = |-3| = 3$

$\therefore$ the equation is $(x-2)^2+(y+3)^2+(z-6)^2=9$

Exercises 1 - 1

Complete the following:

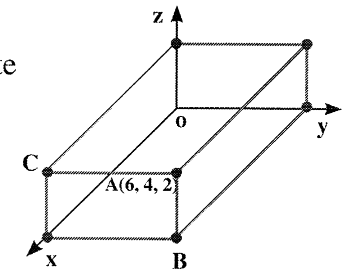
1. If point (x, y, z) lies in x y - plane, then z = ...

2. The two straight lines $\overleftrightarrow{xx`}$ and $\overleftrightarrow{zz`}$ form the coordinate plane .... whose equation is ....

3. The opposite figure represents a cuboid in an orthogonal coordinate system. One of it's vertices is the origion O(0, 0, 0) ,then

   the coordinates of the point B is ....

   Coordinates of the point C is ....



4. If A (1, -1, 4) , B (0, -3, 2), then the coordinates of the midpoint of $\overline{AB}$ is ....

5. The equation of the sphere of center (2, -1, 4) and radius length 5 units is ....

Choose the correct answer from the following:

6. The distance between point (3, -1, 2) and the Cartesian xz plane is .... unit length
   - a 3
   - b -1
   - c 2
   - d 1

7. The length of the perpendicular drawn from point (-2, 3, 4) to the x-axis is .... unit length.
   - a 2
   - b 3
   - c 5
   - d 4

8. The coordinates of the midpoint of the line segment whose terminals (-3, 2, 4), (5, 1, 8) is
   - a $(1, \frac{3}{2}, 6)$
   - b (2, -1, 4)
   - c (8, -1, 4)
   - d $(1, -\frac{3}{2}, 2)$

9. The equation of the sphere whose center is the origin and radius length 5 units is
   - a $x^2 + y^2 + z^2 = 5$
   - b $x^2 + y^2 + z^2 = 0$
   - c $(x - 5)^2 + (y - 5)^2 + (z - 5)^2 = 25$
   - d $x^2 + y^2 + z^2 = 25$

10. The equation of a sphere with center (2, -3, 4) and touches xy-plane is
    - a $(x - 2)^2 + (y + 3)^2 + (z - 4)^2 = 4$
    - b $(x - 2)^2 + (y + 3)^2 + (z - 4)^2 = 9$
    - c $(x - 2)^2 + (y + 3)^2 + (z - 4)^2 = 16$
    - d $(x + 2)^2 + (y - 3)^2 + (z + 4)^2 = 16$

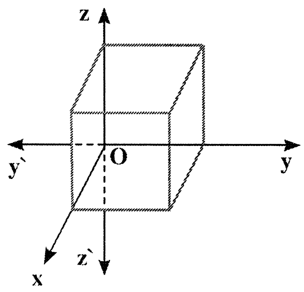
Answer the following questions :

11. Find the distance between the two points A and B in each of the following :
    - a A (7 , 0, 4) , B (1 , 0 , 0)
    - b A (4, 1, 9) , B (2 , 1 , 6)
    - c A (1 , 1 , -7) , B (-2 , -3 , -7)

12. Prove that the triangle whose vertices are the given points is a right angled triangle and find its Area:

   a. (-2 , 5 , 2), (0 , 0 , 2), (0 , 4 , 0) b. (-4 , 4 , 1), (2 , -1 , 2), (-2 , 5 , 0)

13. The opposite figure represents a cube whose volume is 27 cubic unit and one of its vertices is the origin, find coordinates of the other vertices .



14. If the points (7, 1, 3) , (5, 3, k), (3, 5, 3) are the vertices of a triangle , prove that this triangle is isosceles, then find the value (values) of k which make (s) the triangle equilateral

15. Find the coordinates of the midpoint of $\overline{AB}$ in each of the following:

   a. A(3, -1, 4) , B (2, 0, -1) b. A(-3, 5, 5) , B (-6, 4, 8)

16. If C (-1, 4, 0) is the midpoint of $\overline{AB}$ where B (4, -2, 1), find coordinates of point A.

17. Find the equation of the sphere if:

   a. the center is point (3, -1, 2) and its radius length $\sqrt{7}$

   b. (3, 4, -3), (0, 2, 1) are terminals of a diameter.

   c. the center is point (1, -6, 1) and passes through point (2, -1, 5)

18. Find the center and the radius length of the sphere in each of the following:

   a. $x^2 + y^2 + z^2 = 9$

   b. $x^2 + y^2 + z^2 - 2x + 4y = 0$

   c. $2x^2 + 2y^2 + 2z^2 - 2x - 6y - 4z + 5 = 0$

19. Find the equation of the sphere whose radius is 3 units and touches the Cartesian planes (given that three coordinates of the center are positive).

20. **Creative thinking:**

   If A ∈ x -axis, B ∈ y -axis, C ∈ z-axis and if point(1, -1, 0) is the midpoint of $\overline{AB}$ and point (0, -1, 2) is the midpoint of $\overline{BC}$, find the coordinates of the midpoint of $\overline{AC}$

21 **Creative thinking:**
If x-axis cuts the sphere $(x - 2)^2 + (y + 3)^2 + (z - 1)^2 = 14$ at the two points A and B, find the length of $\overline{AB}$

22 **Writing in math:** If all the points in the space in the form of (x, y, 0) lie in x y-plane whose equation z = 0, find the equation of the plane in which all of its points in the form (x, y, 2)

23 **Discover the error:** If point B (-1, 4, 2) is the midpoint of $\overline{AC}$ where A(1 , 0 , 2), find C

Ashraf's solution

$$C = ((\frac{x_1 + x_2}{2}), (\frac{y_1 + x_2}{2}), (\frac{z_1 + z_2}{2}))$$
$$= ((\frac{-1 + 1}{2}), (\frac{4 + 0}{2}), (\frac{2 + 2}{2}))$$
$$= (0, 2, 2)$$

Zeiad's solution

let C (x, y, z)

$\therefore \frac{1 + x}{2} = -1 \longrightarrow x = -3$

$\frac{0 + y}{2} = 4 \longrightarrow y = 8$

$\frac{2 + z}{2} = 2 \longrightarrow z = 2$

$\therefore$ c (-3, 8, 2)

Which answer is correct? Why?

Answer of Exercise (1-1) P. 111

Complete

① 0 ② xz , $y=0$

③ $\overline{CB} \perp$ xy plane , $C=(6,4,2)$

∴ x, y coordinates are fixed as C

∴ $B=(6,4,z)$

∵ $B \in$ xy plane ∴ $z=0$

∴ $B=(6,4,0)$

∵ $\overline{AC} \perp$ xz plane, $C=(6,4,2)$

∴ x, y coordinate are fixed as C

∴ $C=(6,y,2)$

∵ $C \in$ xz plane ∴ $y=0$

∴ $C=(6,0,2)$

④ mid point of $\overline{AB} = \left(\frac{1+0}{2}, \frac{-1+-3}{2}, \frac{4+2}{2}\right)$

$= \left(\frac{1}{2}, -2, 3\right)$

⑤ $(x-2)^2+(y+1)^2+(z-4)^2=25$

⑥ distance with xz plane = y coordinate

$= |-1| = 1$ (d)

⑦ $\sqrt{3^2+4^2}=5$

⑧ $\left(\frac{-3+5}{2}, \frac{2+1}{2}, \frac{4+8}{2}\right) = \left(1, \frac{3}{2}, 6\right)$

⑨ $(x-0)^2+(y-0)^2+(z-0)^2=5^2$

$x^2+y^2+z^2=25$

⑩ touch xy plane , r = z coordinate of the center $(2,-3,4)$ ∴ $r=4$

∴ $(x-2)^2+(y+3)^2+(z-4)^2=16$

⑪ (a) $\sqrt{(1-7)^2+(0-0)^2+(0-4)^2}=2\sqrt{13}$

(b) $\sqrt{13}$ (c) 5

⑫ $A=(-2,5,2)$, $B(0,0,2)$, $C(0,4,0)$

$AB=\sqrt{(0+2)^2+(0-5)^2+(2-2)^2}=\sqrt{29}$

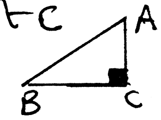
$BC=2\sqrt{5}$ $AC=3$

$(AB)^2=(\sqrt{29})^2=29$

$(BC)^2+(AC)^2=(2\sqrt{5})^2+3^2=29$

∴ $(AB)^2=(BC)^2+(AC)^2$

∴ $\triangle ABC$ right angled $\triangle$ at C



∴ area of $\triangle = \frac{1}{2}\, BC \times AC$

$=\frac{1}{2}\times 2\sqrt{5}\times 3 = 3\sqrt{5}$ sq. unit

(b) area $= 2\sqrt{21}$ squared unit

⑬ side length $= \sqrt[3]{\text{volume}} = \sqrt[3]{27}$

$= 3$ units

the vertices are

$(3,0,0)$, $(3,3,0)$, $(3,0,3)$

$(0,3,0)$, $(0,3,3)$, $(0,0,3)$

$(3,3,3)$, $(0,0,0)$

⑭ *****

$A(7,1,3)$, $B(5,3,K)$, $C(3,5,3)$

$AB=\sqrt{(5-7)^2+(3-1)^2+(K-3)^2}$

$AB=\sqrt{8+(K-3)^2} \longrightarrow$ ①

$BC=\sqrt{(3-5)^2+(5-3)^2+(3-K)^2}$

$BC=\sqrt{8+(K-3)^2} \longrightarrow$ ②

$AC=\sqrt{(3-7)^2+(5-1)^2+(3-3)^2}=\sqrt{32} \rightarrow$ ③

from ①, ②, ③ $AB=BC \neq AC$

∴ $\triangle ABC$ is equilateral

∴ $\sqrt{8+(K-3)^2}=\sqrt{32}$ by squaring

$8+(K-3)^2=32$

∴ $(K-3)^2=24$ $(\pm\sqrt{\ })$

$K-3=\pm 2\sqrt{6}$

$K-3=2\sqrt{6}$ or $K-3=-2\sqrt{6}$

$K=3+2\sqrt{6}$ $K=3-2\sqrt{6}$

(15) a) $\left(\frac{5}{2}, \frac{-1}{2}, \frac{3}{2}\right)$ b) $\left(\frac{-9}{2}, \frac{9}{2}, \frac{13}{2}\right)$

(16) $A = 2C - B$

$A = 2(-1, 4, 0) - (4, -2, 1)$

$A = (-2, 8, 0) - (4, -2, 1)$

$A = (-6, 10, -1)$

or let $A = (x, y, z)$

$(-1, 4, 0) = \left(\frac{4+x}{2}, \frac{-2+y}{2}, \frac{1+z}{2}\right)$

$\therefore \frac{4+x}{2} = -1 \rightarrow 4+x = -2 \rightarrow x = -6$

$\frac{-2+y}{2} = 4 \rightarrow -2+y = 8 \rightarrow y = 10$

$\frac{1+z}{2} = 0 \rightarrow 1+z = 0 \rightarrow z = -1$

$\therefore A = (-6, 10, -1)$

(17) a) $(x-3)^2 + (y+1)^2 + (z-2)^2 = 7$

b) the center $= \left(\frac{3+0}{2}, \frac{4+2}{2}, \frac{-3+1}{2}\right)$

$= \left(\frac{3}{2}, 3, -1\right)$

diameter $= \sqrt{(0-3)^2 + (2-4)^2 + (1+3)^2} = \sqrt{29}$

$\therefore r = \sqrt{29} \div 2 = \frac{\sqrt{29}}{2}$

$\left(x - \frac{3}{2}\right)^2 + (y-3)^2 + (z+1)^2 = \left(\frac{\sqrt{29}}{2}\right)^2$

$\left(x - \frac{3}{2}\right)^2 + (y-3)^2 + (z+1)^2 = \frac{29}{4}$

c) $r = \sqrt{(2-1)^2 + (-1+6)^2 + (5-1)^2} = \sqrt{42}$

$(x-1)^2 + (y+6)^2 + (z-1)^2 = 42$

(18) $(x-\ell)^2 + (y-k)^2 + (z-n)^2 = r^2$

a) the center $(0, 0, 0)$, $r = 3$

b) the center

$= \left(\frac{-1}{2}\text{coeff of } x, \frac{-1}{2}\text{coeff of } y, \frac{-1}{2}\text{coeff of } z\right)$

$= \left(\frac{-1}{2} \times -2, \frac{-1}{2} \times 4, \frac{-1}{2} \times 0\right)$

$= (1, -2, 0) = (\ell, k, n)$

$r = \sqrt{\ell^2 + k^2 + n^2 - d} = \sqrt{1^2 + (-2)^2 + (0)^2 - 0^2}$

$= \sqrt{5}$

c) $2x^2 + 2y^2 + 2z^2 - 2x - 6y - 4z + 5 = 0$

لازم نعمل $(\div 2)$ علشان

x^2, y^2, z^2 must have coef $= 1$

$x^2 + y^2 + z^2 - x - 3y - 2z + \frac{5}{2} = 0$

the center $= \left(\frac{-1}{2} \times -1, \frac{-1}{2} \times -3, \frac{-1}{2} \times -2\right)$

$= \left(\frac{1}{2}, \frac{3}{2}, 1\right)$

$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + (1)^2 - \left(\frac{5}{2}\right)} = 1$

(19) $\because$ the center is in the first octant

$\therefore$ the center $(3, 3, 3)$, $r = 3$

$(x-3)^2 + (y-3)^2 + (z-3)^2 = 9$

(20) $A = (x, 0, 0)$, $B = (0, y, 0)$, $Z = (0, 0, z)$

midpoint of AB $=$

$= (1, -1, 0) = \left(\frac{x+0}{2}, \frac{0+y}{2}, \frac{0+0}{2}\right)$

$(1, -1, 0) = \left(\frac{x}{2}, \frac{y}{2}, 0\right)$

$\frac{x}{2} = 1 \rightarrow x = 2$, $\frac{y}{2} = -1 \rightarrow y = -2$

$A = (2, 0, 0)$, $y = (0, -2, 0)$, $C = (0, 0, z)$

mid point of BC $=$

$= (0, -1, 2) = \left(\frac{0+0}{2}, \frac{-2+0}{2}, \frac{0+z}{2}\right)$

$\frac{z}{2} = 2 \rightarrow z = 4$

$\therefore C = (0, 0, 4)$, $A = (2, 0, 0)$

midpoint of AC $= \left(\frac{0+2}{2}, \frac{0+0}{2}, \frac{4+0}{2}\right)$

$= (1, 0, 2)$

(21) $\because A, B \in X$-axis, $A, B \in$ circle

$\therefore y, z$ coordinates $= 0$

$\therefore (x-2)^2 + (y+3)^2 + (z-1)^2 = 14$ (with $y = 0$, $z = 0$)

$(x-2)^2 + 10 = 14 \rightarrow (x-2)^2 = 4$

$x - 2 = 2$ $\quad$ $x - 2 = -2$

$x = 4$ $\quad$ $x = 0$

$\therefore A = (4, 0, 0)$, $B = (0, 0, 0)$

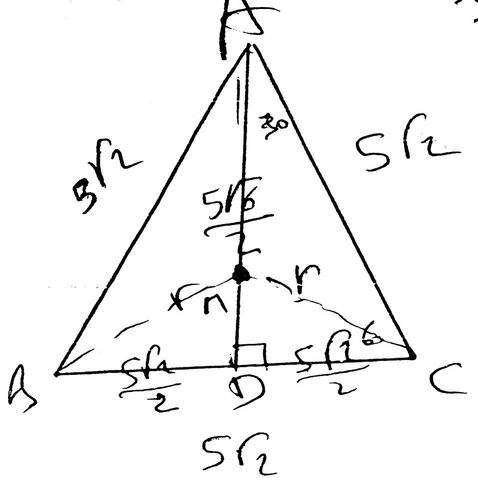
$\therefore AB = |4 - 0| = 4$ units

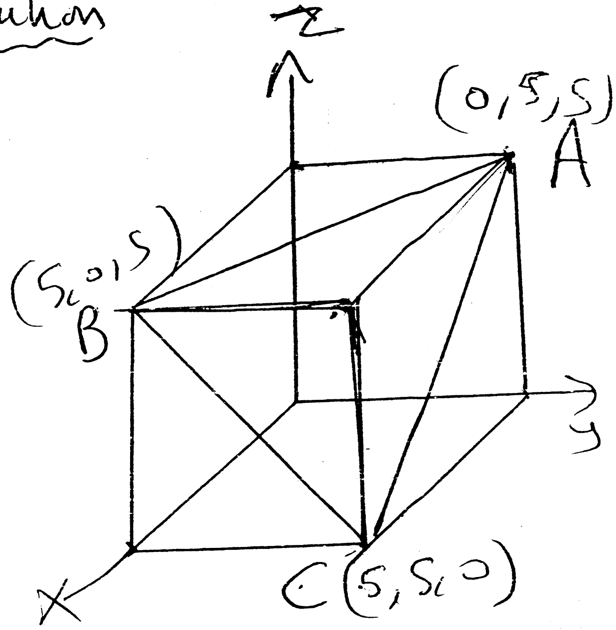
(22) $Z = 2$

(23) Zeiad Ash X

Choose the smallest sphere passes through $(5,5,0), (0,5,5), (5,0,5)$ is ......

Solution





median $AD = \frac{5\sqrt{6}}{2}$

$r = AM = \frac{2}{3} \times \frac{5\sqrt{6}}{2} = \frac{5\sqrt{6}}{3}$ ✓

Center of the sphere $= \frac{(5,5,0)+(5,0,5)+(0,5,5)}{3}$

$= \left(\frac{10}{3}, \frac{10}{3}, \frac{10}{3}\right)$

---

[1] (21) the equation of the sphere whose center $(3, m-1, 5)$ and touches the coordinate axis x and y is ....

Solution

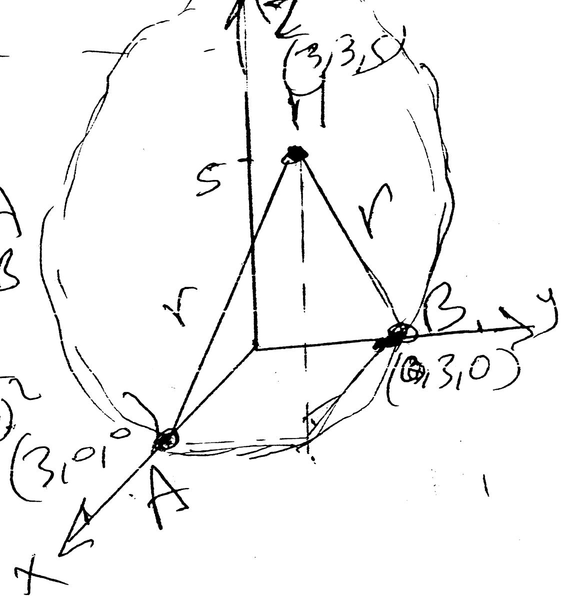
the sphere touch X-axis at $(3,0,0)$ A
" " y-axis at $(0,3,0)$ B

the center is $(3,3,5)$ M

$r = MA = MB = \sqrt{(3-0)^2 + (3-3)^2 + (5-0)^2}$

$= \sqrt{34}$

equation $(x-3)^2 + (y \pm 3)^2 + (z-5)^2 = 34$



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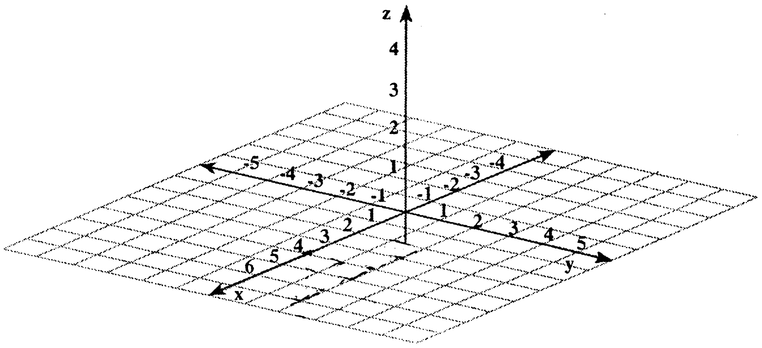
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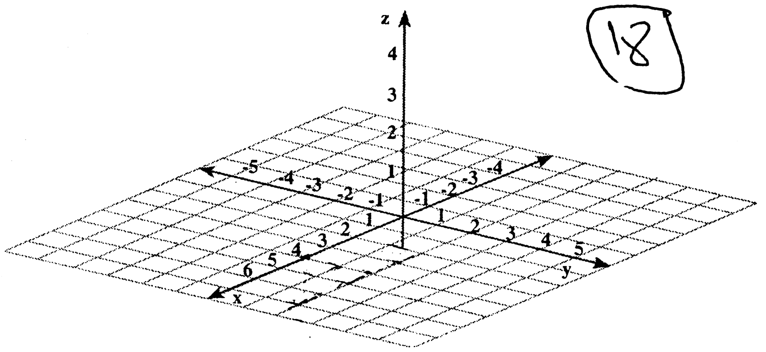
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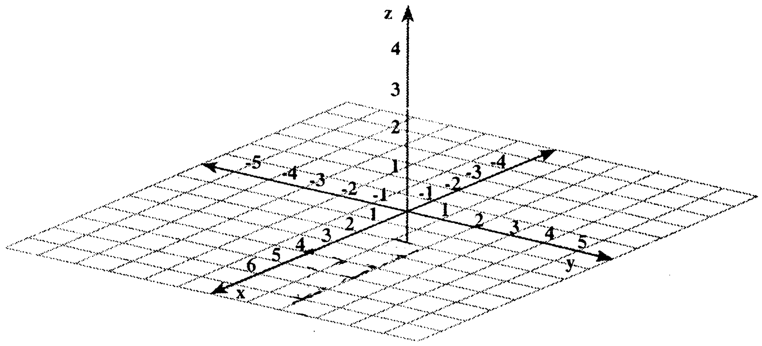
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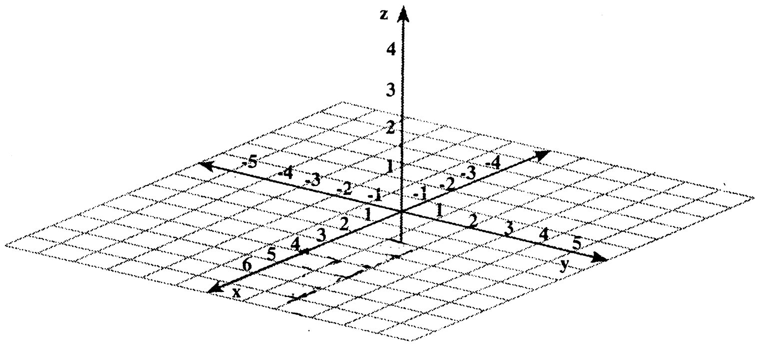
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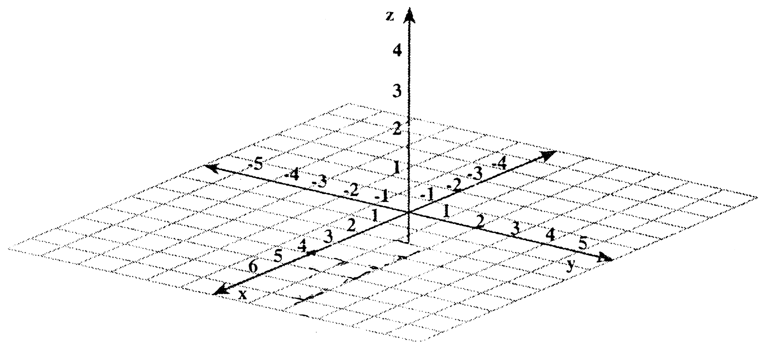
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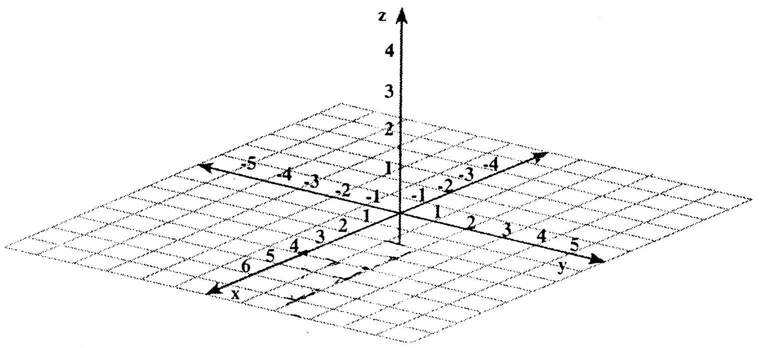
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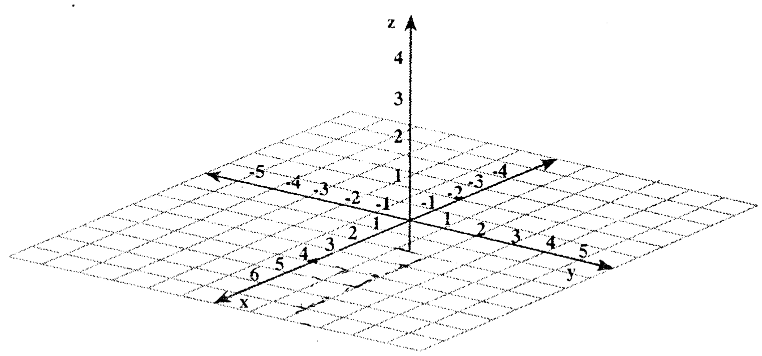
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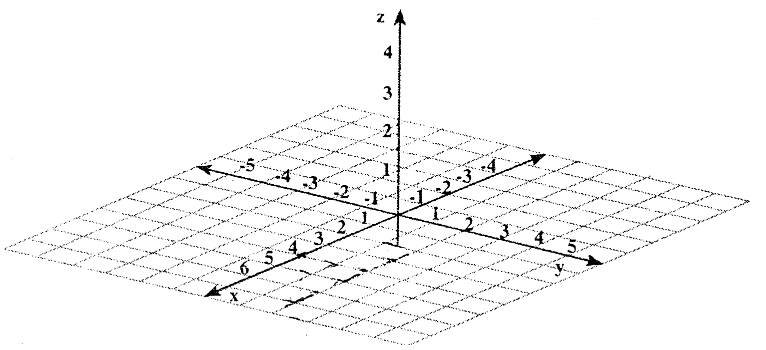
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