مس الآخر

Complex numbers

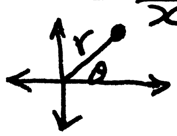
1) imaginary number i

2) Complex number $x+yi=Z$

3) The modulus $|Z| = r = \sqrt{x^2+y^2}$

4) the argument $\tan\theta = \frac{y}{x}$

5) Argand's plane



6) Trig-form $Z = r(\cos\theta + i\sin\theta)$

7) Exponential form

$$Z = re^{\theta i}$$

8) Multiplying 2 numbers Z_1, Z_2

$$Z_1 Z_2 = r_1 r_2 (\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

9) Division of 2 numbers $\frac{Z_1}{Z_2}$

$$\frac{Z_1}{Z_2} = \frac{r_1}{r_2}[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)]$$

10) the integer power

$$Z^n = r^n(\cos n\theta + i\sin n\theta)$$

11) the rational power

$$Z^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\frac{1}{n}(\theta+360m) + i\sin\frac{1}{n}(\theta+360m)\right]$$

12) Conjugate number → same x opp y
→ same r opp θ

$$Z \cdot \bar{Z} = r^2$$

13) change the Trig form

$180-\theta$	θ
$180+\theta$	$360-\theta$

,

$90+\theta$	$90-\theta$
$270-\theta$	$270+\theta$

or put $\theta=1$

14) $1+\cos\theta + i\sin\theta$ → مطلوب تغيرها

$$1 + (2\cos^2\frac{\theta}{2} - 1) + i(2\sin\frac{\theta}{2}\cos\frac{\theta}{2})$$

$$2\cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\, i$$

$$\cos\theta = 2\cos^2\frac{\theta}{2} - 1$$
$$= 1 - 2\sin^2\frac{\theta}{2}$$
$$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$i =$

$i^2 =$

$i^3 =$

$i^4 =$

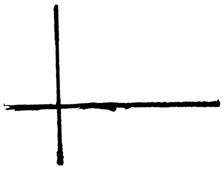
$i + i^2 + i^3 + i^4 =$

$\dots + \dots + \dots + \dots =$

$i^5 + i^6 + i^7 + i^8 =$

$i + i^2 + i^3 + \dots + i^{100} =$

① $z = \sqrt{3} + i$

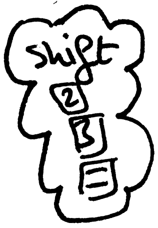


$x = \quad , y =$

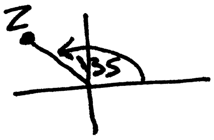
$r = \sqrt{\quad} =$

$\tan\theta = \frac{y}{x} =$

$\theta =$



② $z = -1 + i$



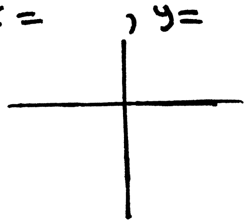
$r = \sqrt{\quad}$

$\tan\theta = \frac{y}{x} = \quad \to \theta =$

③ $z = -2i \quad , x = \quad , y =$

$r =$

$\theta =$



④ $z = 3$

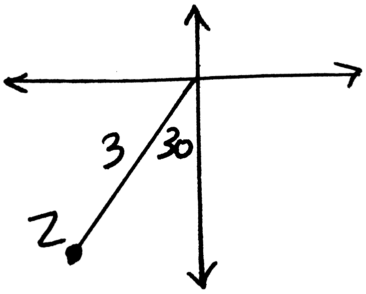
⑤ $z = \frac{4i}{\sqrt{3} + i} \times \frac{\quad}{\quad}$

$=$

The diff. forms of the complex numbers

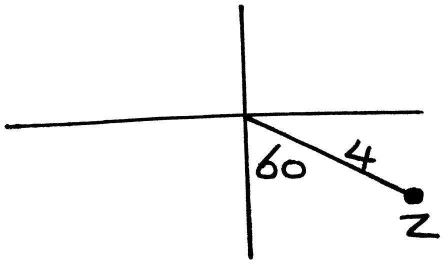
$r =$

$\theta =$



$r =$

$\theta =$



Algebraic form

$z = \sqrt{3} - i$

$r = \qquad \theta =$

Trig form

$z = r(\cos\theta + i\sin\theta)$

$z =$

Exponential Form (Euler's)

$z = ze^{\theta i} \qquad \theta$ in rad

$z =$

$\boxed{1}$ $z = \frac{1+i}{1-i}$ find the diff. forms

Solution

$z = \frac{1+i}{1-i} \times \frac{\quad}{\quad}$

[2] $Z = 3e^{-\frac{\pi}{4}i}$ find the diff. forms

Solution

$r =$ $\theta =$

Trig form

$Z =$

Algebraic form

$Z =$

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[3] $Z = e^{\theta i}$

Trig → $=$

$Z = e^{-\theta i}$

Trig → $=$

$e^{\theta i} + e^{-\theta i} = \dots$

$=$

$e^{\theta i} - e^{-\theta i}$

$=$

$Z_1 = 4(\cos 60 + i \sin 60)$

$Z_2 = 2(\cos 150 + i \sin 150)$

$Z_1 Z_2 =$

$=$

$\frac{Z_1}{Z_2} =$

$Z_2^4 =$

$=$

$Z_1^{\frac{1}{2}} =$

$m = \dots, \dots$

$Z_1 =$

$Z_2 =$

2019

Put the number $Z = \frac{8}{1+\sqrt{3}i}$ in trig-form then find the two roots in Exp. forms

Solution

$\boxed{2018}$

$Z = \frac{16}{1-\sqrt{3}i}$ find Z in the Trig form then find the Cubic roots of Z in the exponential form.

Solution

Z its argument θ

Complete

① the argument of iZ is ....

$\cdots + \cdots$

② the argument of $2Z$ is ....

$\cdots + \cdots$

③ the arg of $\frac{-i}{Z}$ is ....

④ $\arg(Z_1) = 30°$, $\arg(Z_2) = 60$

→ then $\arg(Z_1^3 Z_2^2) = \cdots$

$\cdots + \cdots$

→ $\arg\left(\frac{Z_1^4}{Z_2^2}\right) =$

⑤ Z_1, Z_2 have modulus r_1, r_2

$\theta_1 + \theta_2 = \pi$

$Z_1 Z_2 = \cdots$

Solution

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⑥ $Z = (1+\sqrt{3}i)^n$, $|Z| = 8$

the arg = ...

Solution

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change ① $r(\cos\theta - i\sin\theta)$

Solution

Put $\theta = 1$ → Cal → $\angle$ → $-\theta$

$Z = r(\cos(-\theta) + i\sin(-\theta))$

$\angle 1 \to \theta$

$\angle -1 \to -\theta$

$\angle 179 \to 180 - \theta$

$\angle -179 \to -180 + \theta$

$\angle 89 \to 90 - \theta$

$\angle -89 \to -90 + \theta$

$\angle 91 \to 90 + \theta$

$\angle -91 \to -90 - \theta$

Examples

1. If $z = \frac{-1+\sqrt{3}i}{2}$, $z_1 = \frac{1+z}{1-z}$ then find z_1 and the square roots in the trig form

2. find the exp. form of $z = \frac{2+6i}{3-i}$ then find z^{-1}, $\bar{z}$, $\sqrt{z}$ in the Trig form

3. If $|z_1| = |z_2| = 1$, $\arg(z_1 z_2^3) = 81°$, $\arg\left(\frac{z_1}{z_2}\right) = 33$ then find the number $(z_1^{15} + z_2^{15})$ in the form of $x+yi$

4. find the S.S of the equation $z^4 = 2+2\sqrt{3}i$. write the solution in exp. form

5. If $z = \sin\frac{\pi}{9} + i\cos\frac{\pi}{9}$ then find $(\bar{z})$ in the Trig. form then find the cubic roots of $(\bar{z})^9$

6. $z_1 = \frac{6+4i}{1+i}$, $z_2 = \frac{26}{5-i}$, if $z = 4(z_1 - z_2)$ find in Exp form the cubic roots of z

7. If $z_1 = \left(\frac{\sqrt{3}+i}{2}\right)^4$, $z_2 = \sin\frac{\pi}{3} + i\cos\frac{\pi}{3}$, $i^2 = -1$, $z = \frac{z_1}{z_2}$ find in trig form the square roots of z

8. find the square roots of the number $3+4i$

9. If $z = e^{\theta i}$ then find the modulus and the argument of $\frac{1+z}{1-z}$