(40) sphere touches → xy-plane ✓
→ yz-plane ✓
→ xz-plane ✓

$C = (r, -r, r)$, $A = (1, -4, 5)$

$r = \sqrt{(r-1)^2 + (-r+4)^2 + (-r-5)^2}$

$r^2 = r^2 - 2r + 1 + r^2 - 8r + 16 + r^2 - 10r + 25$

$r^2 = 3r^2 - 20r + 42$

$2r^2 - 20r + 42 = 0 \rightarrow \boxed{r = 7}$

(b)

ليها أكثر من مرا احتمال لكن

ال point عندي $(1, -4, 5)$

$\downarrow \quad \downarrow \quad \downarrow$

$+ \quad - \quad +$

$c = (r, -r, r)$

(5) $\vec{A} = (1,-1,2)$, $\vec{B} = (0,2,-3)$, $\vec{C} = (-2,1,0)$

$\| 3\vec{A} - \vec{B} + \vec{C} \| = ?$

$3\vec{A} - \vec{B} + \vec{C}$

$= 3(1,-1,2) - (0,2,-3) + (-2,1,0)$

$= (1, -4, 9)$

norm $= \sqrt{1^2 + 4^2 + 9^2} = 7\sqrt{2}$ (d)

---

(6) $\vec{A} = (2, 3\sqrt{3}, 0)$ $\quad$ $\vec{B} = (4, 2\sqrt{3}, \sqrt{6})$

$\vec{AB} = \vec{B} - \vec{A}$

$(4, 2\sqrt{3}, \sqrt{6}) - (2, 3\sqrt{3}, 0)$

$\vec{AB} = (2, -\sqrt{3}, \sqrt{6})$

$\|\vec{AB}\| = \sqrt{4 + 3 + 6} = \sqrt{13}$ (c)

---

(7) $\vec{AB} = \vec{B} - \vec{A} = (-3, 3, 7)$

$+$ $\vec{BC} = \vec{C} - \vec{B} = (0, 1, 5)$ } by addition

$\vec{AC} = \vec{C} - \vec{A} = (-3, 4, 12)$

$\|\vec{AC}\| = \sqrt{3^2 + 4^2 + 12^2} = 13$ (a)

(8) $\vec{A} = (-2, 4, 6)$ $\quad \vec{B} = (0, K, 3)$ $\quad , K \in Z^+$

$\|\vec{AB}\| = 7$

$\vec{B} - \vec{A} = (2, K-4, -3)$

$\|\vec{AB}\| = \sqrt{2^2 + (K-4)^2 + 3^2} = 7$

$13 + (K-4)^2 = 49$

$(K-4)^2 = 36$

$K - 4 = 6 \quad$ or $\quad K - 4 = -6$

$\underline{\underline{K = 10}} \quad$ (a) $\quad K = -2$

refused

why?

---

(9) $\|K(3, -3\sqrt{3}, 0)\| = 1$

$= |K| \cdot \|(3, -3\sqrt{3}, 0)\| = 1$

$= |K| \cdot \sqrt{3^2 + (3\sqrt{3})^2} = 1$

$|K| = \dfrac{1}{6} \rightarrow K = \pm \dfrac{1}{6}$ (b)

---

(10) $\vec{A} = (-9, 1, 8)$, $B = (-6, -3, -4)$

$\vec{AB} = \vec{B} - \vec{A} = (3, -4, -12)$

$\|\vec{AB}\| = \sqrt{3^2 + 4^2 + 12^2} = 13$

$U_{\vec{AB}} = \dfrac{\vec{AB}}{\|\vec{AB}\|} = \dfrac{(3, -4, -12)}{13}$

$= \left(\dfrac{3}{13}, \dfrac{-4}{13}, \dfrac{-12}{13}\right)$ (b)

(11) "unit vector" $\rightarrow (A_x, A_y, A_z)$

$$\sqrt{A_x^2 + A_y^2 + A_z^2} = 1$$

- a $\rightarrow \sqrt{(-3)^2 + 2^2 + 2^2} \neq 1$
- b $\rightarrow \sqrt{(\frac{1}{2})^2 + (\frac{1}{3})^2 + (\frac{1}{5})^2} \neq 1$
- c $\rightarrow \sqrt{(\frac{3}{5})^2 + (\frac{4}{5})^2 + 0^2} = 1$ c
- d $\rightarrow \sqrt{(\frac{-1}{12})^2 + (\frac{3}{12})^2 + (\frac{2}{12})^2} \neq 1$

---

(12) [direction of cosines = unit vector]

$$\vec{A} = (-2, 1, 2) \rightarrow \|\vec{A}\| = 3$$

$$\vec{U}_A = \frac{\vec{A}}{\|\vec{A}\|} = \frac{(-2, 1, 2)}{3} = \left(\frac{-2}{3}, \frac{1}{3}, \frac{2}{3}\right) \text{ (b)}$$

---

(13) $(45°, 45°, \theta)$

حفظها أكثر من اسمك

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos^2 45 + \cos^2 45 + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - 2\cos^2 45 = 0$$

$$\cos\theta = 0 \longrightarrow \theta = 90° \quad \text{(b)}$$

---

(14) $(30 + 70 + \theta)$

$$\cos^2 30 + \cos^2 70 + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \cos^2 30 - \cos^2 70$$

$$\cos\theta = \sqrt{1 - \cos^2 30 - \cos^2 70}$$

$$\theta = \cos^{-1}\sqrt{1 - \cos^2 30 - \cos^2 70} = 68°.61$$

d

(15) $\| 4\vec{A} \| = \| 3K\vec{A} \|$

$= |4| \cdot \|\vec{A}\| = |3K| \cdot \|\vec{A}\|$

$4 = \pm 3K \; , \; K = \pm \frac{4}{3}$ (c)

---

(16) $\vec{A} = (1, 2, -4) \; , \; \vec{B} = (1, 1, K-1)$

$\|\vec{A} + \vec{B}\| = 7$

$\vec{A} + \vec{B} = (2, 3, K-5)$

$\|\vec{A} + \vec{B}\| = \sqrt{2^2 + 3^2 + (K-5)^2} = 7$

$(K-5)^2 = 49 - 4 - 9$

$(K-5)^2 = 36 \rightarrow K-5 = \pm 6$

$K - 5 = 6 \qquad\qquad K - 5 = -6$

$K = 11 \qquad\qquad K = -1$

(c)

---

(17) $\vec{AB} = \vec{B} - \vec{A} = (2, 5, -1) \; , \; \vec{A} = (2, 3, 1)$

$\vec{B} = (2, 5, -1) + \vec{A}$

$= (2, 5, -1) + (2, 3, 1)$

$= (4, 8, 0)$ (a)

$= 4i + 8j$

---

(18) $\vec{A} = \left(\frac{-1}{2}, \frac{3}{4}, K\right) \rightarrow$ unit vector

$\left(\frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)^2 + K^2 = 1$

$K^2 = \frac{3}{16} \rightarrow K = \pm \frac{\sqrt{3}}{4}$ (d)

(19) $\vec{A} = (3, 1, -2) \rightarrow \|\vec{A}\| = \sqrt{14}$

$$\vec{U}_A = \frac{\vec{A}}{\|A\|} = \frac{(3, 1, -2)}{\sqrt{14}} = \left(\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}\right)$$

$$U_A = (\cos\theta_x, \cos\theta_y, \cos\theta_z)$$

$$\cos\theta_x = \frac{3}{\sqrt{14}}, \quad \theta_x \cong 37^\circ \quad (d)$$

---

(20) $\vec{B} = (1, 2, 0)$

$\cancel{\theta_x}$ $Z = 0 \rightarrow \theta_z = 90^\circ$ (b)

---

(21) $\vec{C} = (2, 4, K) \rightarrow \|\vec{C}\| = \sqrt{20 + K^2}$

$$\cos\theta_y = \frac{4}{\sqrt{20 + K^2}} = \cos 45$$

$$= \frac{4}{\sqrt{20 + K^2}} = \frac{1}{\sqrt{2}} = \frac{4}{4\sqrt{2}}$$

$$\sqrt{20 + K^2} = 4\sqrt{2}$$

$$20 + K^2 = 32$$

$$K^2 = 12 \rightarrow K = \pm 2\sqrt{3} \quad (c)$$

---

(22) $(30^\circ, 120^\circ, 90^\circ)$

$(\cos 30, \cos 120^\circ, \cos 90^\circ)$

$\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}, 0\right) \rightarrow Z = 0$

xy - plane (c)

(23) y - axis $\rightarrow$ $x=0$, $z=0$, $y \rightarrow (+)$

$\theta_x = 90$, $\theta_z = 90$

$(90°, 0°, 90°)$ (d)

---

(24) $-z$ - axis $\rightarrow$ $x=0$, $y=0$, $z \rightarrow (-)$

$\theta_x = 0$, $\theta_y = 0$

$(90°, 90°, 0)$

$\downarrow$

$(0, 0, 1)$ $\rightarrow$ $(0, 0, (-1))$

(c)

---

(25) $\vec{U}_A = \dfrac{\vec{A}}{\|\vec{A}\|}$, $\vec{A} = \|\vec{A}\| \vec{U}_A$

$\vec{A} = 6 \left(\frac{-1}{2}, 0, \frac{\sqrt{3}}{2} \right)$

$\vec{A} = (-3, 0, 3\sqrt{3})$ (a)

---

(26) $(60°, \theta_y, 45°)$

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\cos^2 60° + \cos^2\theta_y + \cos^2 45 = 1$

$\cos^2\theta_y = 1 - \cos^2 60 - \cos^2 45$

$\theta_y = \cos^{-1}\left(\pm\sqrt{1 - \cos^2 60 - \cos^2 45} \right)$

$\theta_y = 60$ (+)

$\theta_y = 120$ (−)

(c)

(27) $\left(\frac{1}{2}, \frac{-1}{2}, \frac{1}{\sqrt{2}}\right) = \left(\cos\theta_x, \cos\theta_y, \cos\theta_z\right)$

- $\cos\theta_x = \frac{1}{2} \rightarrow \theta_x = 60$
- $\cos\theta_y = \frac{-1}{2} \rightarrow \theta_y = 120$
- $\cos\theta_z = \frac{1}{\sqrt{2}} \rightarrow \theta_z = 45$

$(60^\circ, 120^\circ, 45)$ (b)

---

(28) $(45^\circ, 135^\circ, 90)$

$\downarrow$

$\vec{U}_A = (\cos 45, \cos 135, \cos 90)$

$\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)$ (c)

---

(29) $\vec{U}_A = \frac{\vec{A}}{\|\vec{A}\|}$

$\vec{A} = \|\vec{A}\| \vec{U}_A$

$\vec{A} = 12\sqrt{2}\left(\cos 45^\circ, \cos 120^\circ, \cos 60^\circ\right)$

$\vec{A} = (12, -6\sqrt{2}, 6\sqrt{2})$ (b)

---

(30) $A = (2\sqrt{2}, 2, -2)$

$\frac{2\sqrt{2}}{x} = \frac{2}{y} = \frac{-2}{z} = K$

$(x, y, z) = (3\sqrt{2}, 3, -3)$ (c)

(31) parallel $\rightarrow$ have the same unit vector

$\vec{A} = yz\text{-plane} \rightarrow (0, 1, 1)$

$\vec{U}_A = (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ (b)

---

(32) $\vec{n} = (a, 4, c)$ // yz-plane

$(a, 4, c) \parallel (0, y, z)$

$a = 0$

$\vec{n} = (0, 4, c)$, $\|\vec{n}\| = 5$

$\vec{U}_n = (0, \frac{4}{5}, \frac{c}{5})$

$\vec{U}_n = (\frac{4}{5})^2 + (\frac{c}{5})^2 = 1$

$\frac{c^2}{25} = \frac{9}{25}$, $\underline{c^2 = 9}$, $c = \pm 3$

(b)

---

(33) $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\theta = \theta_x = \theta_y = \theta_z \rightarrow$ equal angles with (+) direction

$3\cos^2\theta = 1$

$\cos^2\theta = \frac{1}{3}$, $\cos\theta = \pm\frac{1}{\sqrt{3}}$

$\cos\theta = \frac{1}{\sqrt{3}}$, $\cos\theta = \frac{-1}{\sqrt{3}}$ refused

$(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

(34) $\vec{AC} = \vec{C} - \vec{A} = (3, -2,5)$ $\qquad \vec{C} = \dfrac{\vec{A}+\vec{B}}{2}$

$$= \frac{\vec{A}}{2} + \frac{\vec{B}}{2} - \vec{A} = (3, -2,5)$$

$$= \frac{\vec{B}}{2} - \frac{\vec{A}}{2} = (3, -2,5)$$

$$\vec{CB} = \vec{B} - \vec{C}$$

$$= \vec{B} - \frac{\vec{A}}{2} - \frac{\vec{B}}{2}$$

$$= \frac{\vec{B}}{2} - \frac{\vec{A}}{2} = (3, -2,5) \quad (b)$$

---

(35) $C = \dfrac{\vec{A}}{\|\vec{A}\|}$

$\vec{A} \parallel \vec{C}$ شرط أساسي (d)

---

(36) $\left\|\dfrac{\vec{A}}{\|\vec{A}\|}\right\| + 2\left\|\dfrac{\vec{B}}{\|\vec{B}\|}\right\| + 3\left\|\dfrac{\vec{C}}{\|\vec{C}\|}\right\|$

$= 1 + 2 + 3 = 6$ (c)

---

(37) $\vec{A} = (4, 0, 3) \rightarrow \|\vec{A}\| = 5$

$$U_A = \left(\frac{4}{5}, 0, \frac{3}{5}\right)$$

$$\cos\theta_x = \frac{4}{5}, \quad \theta_x = 36.8698$$

$$\tan\theta_x = \frac{3}{4} \quad (a)$$

(38) $\vec{A} = (K, 12, 4)$ $\quad \|\vec{A}\| = \sqrt{12^2 + 4^2 + K^2}$

$\cos\theta_x = \frac{3}{13} = \frac{K}{\sqrt{12^2 + 4^2 + K^2}}$ $\quad$ ($K=3$ بالنمر)

$13K = 3\sqrt{160 + K^2}$ $\quad$ by squaring

$169K^2 = 9(160 + K^2)$

$169K^2 = 1440 + 9K^2$

$160K^2 = 1440$

$K^2 = 9 \rightarrow K = \pm 3$

$K = 3$ $\quad K = -3$

refused

لما تيجي تعوض
فوق مش هتطلع
محالة

---

(39) • lies on xz-plane $\rightarrow y = 0 \rightarrow \theta_y = 90^\circ$

• $\theta_x = 30^\circ$

$(30^\circ, 90^\circ, \theta_z)$

$\cos^2 30 + \cos^2 90 + \cos^2\theta_z = 1$

$\cos\theta_z = \pm\sqrt{(1 - \cos^2 30)}$

$\therefore \cos\theta_z = \frac{1}{2}, \quad \cos\theta_z = \frac{-1}{2}$

positive coordinate of plane xz $\quad$ refused

$\left(\frac{\sqrt{3}}{2}, 0, \frac{1}{2}\right)$

(40) $\cdot$ xy - plane $\rightarrow z=0 \rightarrow \theta_z = 90^\circ$

$\cdot\ \theta_y = 60^\circ$

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos^2\theta_x + \cos^2 60 + \cos^2 90 = 1$$

$$\cos^2\theta_x = 1 - \cos^2 60$$

$$\cos\theta_x = \pm\sqrt{1-\cos^2 60}$$

$$\cos\theta_x = \pm\frac{\sqrt{3}}{2}$$

$\left(\pm\frac{\sqrt{3}}{2}, \frac{1}{2}, 0\right)$ c

---

(41) $\cdot \|\vec{A}\| = 3$ $\qquad -1 \leq K \leq 2$

$K \in [-1, 2]$

$\|K\vec{A}\| = |K| \cdot \|\vec{A}\| =$

$\hookrightarrow K \in\ 0, 2]$

---

(42) $(0, 90^\circ, 90^\circ)$

$(1, 0, 0) \rightarrow x$-axis b

(43) When $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

- a $\rightarrow$ $\cos^2 60 + \cos^2 30 + \cos^2 30 \neq 1$
- b $\rightarrow$ $\cos^2 90 + \cos^2 90 + \cos^2 60 \neq 1$
- c $\rightarrow$ $\cos^2 120 + \cos^2 150 + \cos^2 90 = 1$ (c)
- d $\rightarrow$ $\cos^2 60 + \cos^2 30 + \cos^2 0 \neq 1$

---

(44) (c) بالأخر

- a $\rightarrow$ $\left(\frac{1}{\sqrt{2}}\right)^2 + 0^2 + \left(\frac{-1}{\sqrt{2}}\right)^2 = 1$
- b $\rightarrow$ $\left(\frac{-6}{10}\right)^2 + \left(\frac{2\sqrt{7}}{10}\right)^2 + 0.6^2 = 1$
- c $\rightarrow$ $1^2 + 1^2 + 1^2 \neq 1$ (c)
- d $\rightarrow$ $\frac{1}{2} + \left(\frac{-\sqrt{3}}{2}\right)^2 + 0^2 \neq 1$

---

(45) $K^2 + E^2 + F^2 = 1$ (c)

---

(46) $\vec{A} + \vec{BC} = A + \vec{C} - \vec{B} = (4, 12, 9)$

$\vec{A} = (0, -1, 3)$, $B = (4, -2, 1)$

$\vec{C} = (4, 12, 9) + \vec{B} - \vec{A}$

$= (4, 12, 9) + (4, -2, 1) - (0, -1, 3)$

$\vec{C} = (8, 11, 7)$

$= 8i + 11j + 7k$ (b)

(47) $\vec{A} = (-2K, 2K, K)$

$\|\vec{A}\| = \sqrt{4K^2 + 4K^2 + K^2} = \sqrt{9K^2} = 3K$

direction of Cosines $= \left(\frac{-2K}{3K}, \frac{2K}{3K}, \frac{K}{3K}\right)$

$= \left(\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}\right)$ ⓐ

---

(48) $(3, 2, 5)$

$\rightarrow$ projection on xy-plane $\rightarrow (3, 2, 0) = B$

$\rightarrow$ projection on xz-plane $\rightarrow (3, 0, 5) = C$

$\vec{BC} = \vec{C} - \vec{B} = (3, 0, 5) - (3, 2, 0)$

$= (0, -2, 5)$

$= -2\hat{j} + 5K$ ⓒ

---

(49) $A = (4, 0, 0)$, $C' = (0, 9, 7)$

$AC' = C' - A = (-4, 9, 7)$

$\|AC'\| = \sqrt{4^2 + 9^2 + 7^2}$

$= \sqrt{146}$ ⓒ

(50) $A = (6, 8, 24)$

$\vec{O} = (0, 0, 0)$

$\vec{D} = (6, 8, 0)$

$\vec{OD} = \vec{D} = (6, 8, 0) \rightarrow \|\vec{D}\| = 10$

$\vec{U}_D = \frac{\vec{D}}{\|\vec{D}\|} = \frac{(6, 8, 0)}{10} = \left(\frac{3}{5}, \frac{4}{5}, 0\right)$

$\vec{U}_D = (\cos 53.1, \cos 36.87, \cos 90)$

$= (53.1^\circ, 36.87^\circ, 90)$ (b)

---

(51) $\boxed{\text{The SAME}}$

$(30, 60, 90)$ (a)

---

(52) $\vec{A} \rightarrow (60, 135, 60^\circ)$

$-\vec{A} \rightarrow (180-60, 180-135, 180-60)$

$-\vec{A} = (120, 45, 120)$ (d)

---

(53) perpendicular on plane xy

$\hookrightarrow$ z-axis

unit vector of z-axis $= \hat{k}$

(c)

(54) $\theta_x + \theta_y = 90 \rightarrow \theta_z = 90 \rightarrow$ قانون حفظ
أو معلم تشبه عادي

- a $\rightarrow \theta_z = 90^\circ$
- b $\rightarrow$ on xy-plane $\rightarrow z = 0 \rightarrow \theta_z = 90^\circ$
- c $\rightarrow \cos^2\theta_x + \cos^2\theta_y = 1$, $\cos^2\theta_z = 0$
- d $\rightarrow$ NO

(d)

---

(55) (1) $\theta_x + \theta_y \geq 90^\circ$

(2) $\sin^2(90 - \theta_x) + \sin^2(90 - \theta_y) + \sin^2(90 - \theta_z) = 1$

$= \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

(3) wrong

$-\vec{A} = (\pi - \theta_x, \pi - \theta_y, \pi - \theta_z)$

(4) $\cos\theta_x = \dfrac{a_x}{\|\vec{A}\|}$ wrong

$\theta_x = \cos^{-1}\dfrac{a_x}{\|\vec{A}\|}$

(1), (2) correct

(3), (4) incorrect (d)