Algebra

EXPANSION & THE TERMS OF THE BINOMIAL THEOREM

- ☐ $(x+a)^5 = x^n + {}^{n}C_1\, x^{n-1}a + {}^{n}C_2\, x^{n-2}a^2 + \dots\dots\dots\dots + a^n$
  $(x-a)^5 = x^n - {}^{n}C_1\, x^{n-1}a + {}^{n}C_2\, x^{n-2}a^2 - \dots\dots\dots\dots + (-a)^n$

- ☐ number of terms =n+1

- ☐ the middle term
  "n is odd" → M.T= $\frac{n+1}{2}$ *and* $\frac{n-1}{2}$ → Coff= ${}^{n}C_{\frac{n-1}{2}} = {}^{n}C_{\frac{n+1}{2}}$
  "n is even" → M.T= $\frac{n}{2}+1$ → Coff = ${}^{n}C_{\frac{n}{2}}$

- ☐ $T_{r+1} = {}^{n}C_r \quad a^r \quad x^{n-r}$ for $(x+a)^n$ [GENERAL TERM]

- ☐ Coff. Of $T_{r+1} = {}^{n}C_r\ (coff.\ of\ second)^r\ (coff.\ of\ first)^{n-r}$

- ☐ $(x+a)^n \oplus (x \ominus a)^n = 2\,(T_1 + T_3 + T_5 + \dots\dots)$ odd
  $(x+a)^n \ominus (x \ominus a)^n = 2\,(T_2 + T_4 + T_6 + \dots\dots)$ even

- ☐ T_x from the end =no. of terms − order +1= n−x+2

- ☐ $X^c(X^a + X^b)^n$ r=$\frac{an-k+c}{a-b}$ find X^k →
  - X^3 k=3
  - X^{-4} k=-4
  - Term free of X k=0

QUESTIONS

Q1

If the sum of the terms coefficients in the expansion of $(a^2 x^2 - 2ax + 1)^9$ equals zero , then a = ...

(a) 2 (b) – 2 (c) 1 (d) – 1

Q2

In the expansion of a binomial , it has 7 positive terms and 6 negative terms , then the expression is in the form of : ....

(a) $(a - b)^{12}$ (b) $(a + b)^{13}$ (c) $(a + b)^{12}$ (d) $(a - b)^{13}$

Q3

In the expansion of $(1 + x)^{17}$ according to the ascending powers of x , if the coefficient of T_{r+4} is equal to the coefficient of T_{2r+3} , then r = ...

(a) 3 (b) 4 (c) 17 (d) 7

Q4

The set of solution of the equation :

$1 - 6x + \frac{6 \times 5}{2 \times 1} x^2 - \frac{6 \times 5 \times 4}{3 \times 2 \times 1} x^3 + \cdots + x^6 = 64$ in $\mathbb{R}$ is ....

(a) $\{-3, 1\}$ (b) $\{1, 3\}$ (c) $\{-1, 3\}$ (d) $\{-1, -3\}$

Q5

If $(1 + a x)^n = 1 + 8x + 24x^2 + \cdots + a^n x^n$ then $\frac{a - n}{a + n} =$ ....

(a) 3 (b) -3 (c) $-\frac{1}{3}$ (d) $\frac{1}{3}$

Q6

(Trial 2023) In the expansion of : $x^3 (1 + x)^7$ the coefficient of the term containing x^4 is ....

(a) ${}^7C_7$ (b) ${}^7C_4$ (c) ${}^7C_2$ (d) ${}^7C_1$

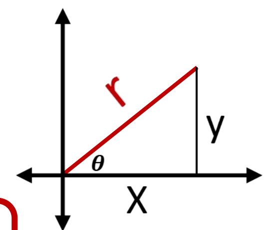
Q7

(*1st Session 2017*) In the expansion of : $(1 + X)^n$ according to the ascending powers of X , if $T_3 = 17$, $3\ T_2 \times T_4 = 544$, then find the value of each : n , X « 1

Complex numbers

Real number ← ② + ④ i → imaginary number

Complex number(Z)

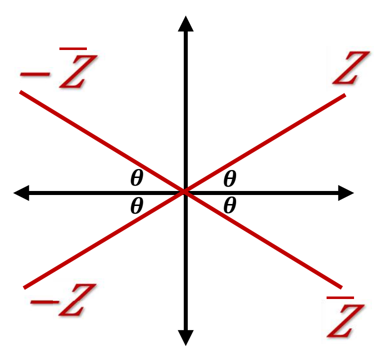


- i^2=-1
- $i^3 = -\ i$
- $i^4 = 1$
- $i + i^2 + i^3 + i^4$=0
- Z= $x + yi$ → $the\ modulus\ (r) = \sqrt{x^2 + y^2}$
  → $the\ argument(\boldsymbol{\theta}) = \tan^{-1}\frac{y}{x}$ $\boldsymbol{\theta} \in\ ]-\boldsymbol{\pi}, \boldsymbol{\pi}[$
- Z= $x + yi$ → $\overline{Z} = x - yi$ $(called\ conjugate)$

Same x $opposite$ y

Same r $opposite$ $\boldsymbol{\theta}$

- $modulus = |Z| = |-Z| = |\overline{Z}| = |-\ \overline{Z}| = r = \sqrt{x^2 + y^2}$
- $Z + \overline{Z} = 2x$ 'pure real'
- $Z - \overline{Z} = 2yi$ 'pure imaginary'
- $Z.\overline{Z} = |Z|^2 = |\overline{Z}|^2 = r^2 = x^2 + y^2$
- $(Z)^2 = (\overline{Z})^2$
- $\frac{|Z|}{|\overline{Z}|} = 1$
- $|Z_1.Z_2| = |Z_1|.|Z_2|$
- $\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}$
- $(x + yi)(x - yi) = x^2 - y^2$



Amplitude = argument= $\theta=\tan^{-1}\frac{y}{x}$

○ $Amplitude(Z_1.Z_2)=\theta_1+\theta_2$ ○ $Amplitude\left(\frac{Z_1}{Z_2}\right)=\theta_1-\theta_2$

○ $Amp\left(\frac{|Z|}{|\overline{Z}|}\right)=2\theta$ ○ $Amp(Z.\overline{Z})=zero$

○ $Amp(Z^n)=n\times\theta$ → $eg.\ Amp(Z^3)=3\theta$

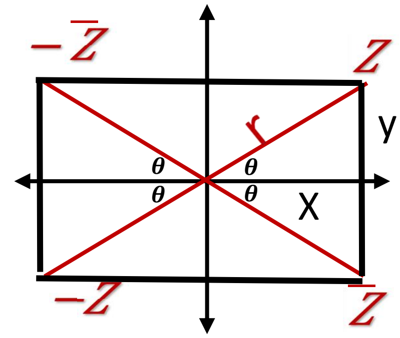
○ $Amp(Z)=\theta\,,Amp(KZ)=\theta \quad where\ K\in R^+$

○ $Amp(-KZ)=180-\theta$

○ $Z\,,\overline{Z}\,,-Z\,,-\overline{Z}$ are vertices of a rectangle

Area = $4|x||y|$

perimeter = $4(|x|+|y|)$



$$x+yi=r(\cos\theta+i\sin\theta)=re^{\theta i}$$

Algebraic form — trig form (polar form) — exponential form — (in rad)

De moivre's with positive rational powers

○ $Z^{\frac{1}{n}}=r^{\frac{1}{n}}\left(\cos\left(\frac{\theta+360m}{n}\right)+isin\left(\frac{\theta+360m}{n}\right)\right)$ where m=0 ,1, -1 n=3

○ $Z=x\pm yi \qquad \sqrt{Z}=\sqrt{x\pm yi}=\pm\left(\sqrt{\frac{r+x}{2}}\pm\sqrt{\frac{r-x}{2}}\,i\right)$

○ $the\ n^{th}\ roots\ for\ a\ complex\ number\ represented\ by\ a\ regular\ polygon\ lie\ on\ one\ circle$

- Its center is (0,0)
- Its radius=$\sqrt[n]{r}=r_1=r_2=r_3$
- No of sides of the polygon is n

- the measure of the angle between two adjacent sides= $\frac{(n-2)\times180}{n}$
- Area of the polygon $= \frac{n}{2} r_1 \sin\left(\frac{360}{n}\right)$
- Side length (L)= $r_1 . \sqrt{2} . \sqrt{1-\cos\left(\frac{360}{n}\right)}$
- Perimeter of polygon =n× side length
- Number of roots = number of diagonals = n
- Sum of roots =zero

Cubic roots of unity

○ $1,\ \frac{-1}{2}+\frac{\sqrt{3}}{2}i,\ \frac{-1}{2}-\frac{\sqrt{3}}{2}i$

$1=\omega^3,\quad \omega\quad,\quad \omega^2$

○ $1+\omega+\omega^2 = zero$ → $1+\omega=-\omega^2$

→ $1+\omega^2=-\omega$

→ $\omega+\omega^2=-1$

○ The conjugate of ω is $\omega^2$and so the conjugate of $(1+\omega)$ is $(1+\omega^2)$ and the conjugate of $(2023+\omega)$ is $(2023+\omega^2)$ and the conjugate of(a ω − bω^2) is (a ω^2 − b ω) for every a &b ∈ R^+

○Modulus always for $(1, \omega, \omega^2, \omega^3, \omega^4, ...etc)$ =r=1

○$\omega-\omega^2=\omega-\omega^2=\pm\sqrt{3}i$

○difference between amplitudes of each two consecutive cubic roots is 120°

QUESTIONS

Q1 If $z_1 = 1 - \sqrt{3}\, i$, $z_2 = 1 + i$ Find each of the following in trigonometric & exponential form

(1) $z_1 z_2$ (2) $\frac{z_2}{z_1}$ (3) $(z_1)^8$ (4) $(z_2)^6$

Q2

If z is a complex number , its principle amplitude is θ , then

First : Amplitude $(\bar{z})$ = ...

(a) θ (b) $-\theta$ (c) $\frac{\pi}{2}-\theta$ (d) $\pi-\theta$

Second : Amplitude (2 z) = ...

(a) θ (b) $-\theta$ (c) 2θ (d) -2θ

Third : Amplitude $\left(\frac{1}{z}\right)$ = ...

(a) θ (b) $-\theta$ (c) $\pi-\theta$ (d) $-\pi+\theta$

Q3

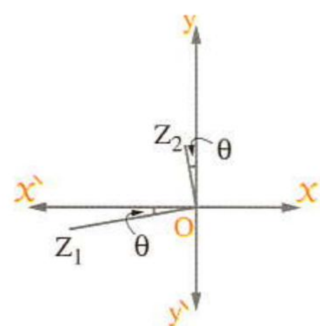
If the amplitude for the number z = 28° , then

the amplitude for the number $\left(\frac{i}{\bar{z}}\right)$ = ...

(a) 28° (b) 62° (c) 118° (d) – 28°

Q4

(2nd Session 2022) The opposite figure represents two complex numbers z_1 , z_2 on argand's plane. If $|z_1| = 2|z_2|$, then the trigonometric form of the complex numer z where $z = \frac{z_2}{z_1}$ is ....



(a) $\frac{1}{2}\left(\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right)$

(b) $2\left(\cos\left(\frac{-\pi}{2}\right)+i\sin\left(\frac{-\pi}{2}\right)\right)$

(c) $\frac{1}{2}\left(\cos\left(\frac{-\pi}{2}\right)+i\sin\left(\frac{-\pi}{2}\right)\right)$

(d) $2\left(\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right)$

Q5

If arg $(z_1 z_2) = \frac{\pi}{6}$, arg $(z_1 z_3) = \frac{2\pi}{9}$, arg $(z_2 z_3) = \frac{5\pi}{18}$, then arg $(z_1 z_2 z_3) =$ ....

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{5}$ (d) $\frac{\pi}{6}$

Q6

If z is a complex number and $|z - 3| = |z|$ and the principle amplitude for the number (z) equals $\frac{-\pi}{4}$, then $|z| =$ ....

(a) $3\sqrt{2}$ (b) $\frac{3\sqrt{2}}{2}$ (c) $2\sqrt{2}$ (d) 2

Q7

(1st Session 2022) If $z_1 = 2(\cos 150° - i \sin 150°)$, $z_2 = 3(\sin 150° + i \cos 150°)$, then the exponential form of the number $z_1 z_2$ could be equals ....

(a) $6e^{\frac{5}{6}\pi i}$ (b) $6e^{-\frac{5}{6}\pi i}$ (c) $6e^{\frac{\pi}{3}i}$ (d) $6e^{\frac{\pi}{2}i}$

Q8

If $z = x + iy$, then the real part of the number e^z is ....

(a) $e^x \cos y$ (b) $e^x \sin y$ (c) e^x (d) $\cos y$

Q9

(Trial 2023) If $z_1 = e^{\frac{\pi}{3} - \frac{\pi}{3} i}$, and $z_2 = e^{\frac{\pi}{6} - \frac{\pi}{6} i}$ then $z_1 z_2 =$ .... where $i^2 = -1$

(a) $e^{\frac{-\pi}{2} i}$ (b) $-e^{\frac{\pi}{2}} i$ (c) $-e^{\frac{\pi}{2} i}$ (d) $e^{\frac{\pi}{2}} i$

Q10

Find the two square roots of the number z for each of the following :

(1) *(2nd Session 2014)* $z = -8i$ in the exponential form.

(2) 📖 $z = 2 - 2\sqrt{3}\,i$ in the trigonometric form.

(3) *(2nd Session 2015)* $z = -3 + 4i$ without converting into trigonometric form.

Q11

The solution set of $x^3 = 8$ in $\mathbb{C}$ is ....

(a) $\{2\}$

(b) $\{2, 2\omega, 4\omega^2\}$

(c) $\{2, 2\omega, 2\omega^2\}$

(d) $\{8, 8+\omega, 8+\omega^2\}$

Q12

If $a = 2\omega - 3\omega^2$, $b = 3 + 5\omega^2$, then $a^2 + b^2 =$ ....

(a) -37 (b) -19 (c) 1 (d) 38

Q13

(Trial 2021) On Argand diagram , area of the triangle whose vertices are the points represent the cubic roots of one equals .... square units.

(a) $\frac{3\sqrt{3}}{4}$ (b) $\frac{3\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{\sqrt{3}}{4}$