Algebra

Answers of choose by steps

Exercise (9)

On Cubic roots of unity.

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[1] Choose :

Q1

Solution (c)

$w^{22}+w^{32}=w+w^2=-1$

Q2

Solution (c)

$w^{-13}+\frac{1}{w}=\frac{1}{w^{13}}+w^2=\frac{1}{w}+w^2=w^2+w^2=2w^2$

Q3

$w^n+w^{n+1}+w^{n+2}=0$

Solution (c)

$w^{2020}+w^{2021}+w^{2022}=zero$

Q4

Solution (d)

$(w-w^2)^4=(\pm\sqrt{3}i)^4=(\sqrt{3})^4 i^4=9$

Q5

Solution (b)

$\left(w+\frac{1}{w}\right)^2\left(w^2+\frac{1}{w^2}\right)^2=(w+w^2)^2(w^2+w)^2=(-1)^2(-1)^2=1$

Q6

Solution (a)

$$(1+w)^4+(1+w^2)^4+(w+w^2)^4=(-w^2)^4+(-w)^4+(-1)^4$$
$$=w^8+w^4+1=w^2+w+1=zero$$

Q7

Solution (a)

$$\left(\frac{1}{w+1}\right)\left(1+w-\frac{3}{w}\right)=\left(\frac{1}{-w^2}\right)(-w^2-3w^2)=(-w)(-4w^2)=4w^3=4$$

Q8

Solution (d)

$$\frac{-1+\sqrt{3}i}{2}\rightarrow w$$

$$w^5+w^4+5=w^2+w+5=-1+5=4$$

Q9

Solution (d)

$$w\rightarrow by\ squaring\rightarrow w^2$$
$$w^2\rightarrow by\ squaring\rightarrow w^4=w$$
$$w\ ,w^2$$

Q10

Solution (a)

$$w^a+w^b+w^c\rightarrow a\ ,b\ ,c\ are\ three\ consecutive\ integers=zero$$

Q11

Solution (b)

$conj.\ of\ w\ is\ w^2$

Q12

Solution (c)

$conj.\ of\ \ 1 + w\ is\ 1 + w^2$

Q13

Solution (c)

$conj.\ of\ i - w^2 =- i - w$

Q14

Solution (b)

$conj.\ of\ i + w + 2023 \to - i + w^2 + 2023$

Q15

Solution (b)

$conj.\ of\ 2w + 3w^2 \to 2w^2 + 3w$

Q16

Solution (b)

$\frac{3}{2+w} = x \to 3 = x(2 + w) \to$ نجرب الاختيارات $\to x = 2 + w^2$

Q17

Solution (a)

$$\frac{a+bw+cw^2}{c+aw+bw^2} + \frac{a+bw+cw^2}{b+cw+aw^2} = \frac{a+bw+cw^2}{cw^2+aw+bw^2} + \frac{aw^3+bw+cw^2}{b+cw+aw^2}$$

$\frac{a+bw+cw^2}{w(cw^2+a+bw)} + \frac{w(aw^2+b+cw)}{b+cw+aw^2} = \frac{1}{w} + w = w^2 + w =- 1$

Q18

Solution (c)

$x^3 = 8$

$\rightarrow x = \{2\} \; in \; R$

$\rightarrow x = \{ 2 , 2w , 2w^2\}$

Q19

Solution (c)

$1 + w + w^2 + w^3 + \; .... + w^{99} + w^{100} = 1 + w^{100} = 1 + w =- w^2$

Q20

Solution (a)

$\left(w^2 + \frac{1}{w}\right)\left(1 + \frac{1}{w^2}\right)^2 = (w^2 + w^2)(1 + w)^2 = (2w^2)(- w^2)^{\,2} = (2w^2)(w^4)$

$(2w^2)(w) = 2w^3 = 2$

Q21

Solution (c)

$a \rightarrow \frac{1+w}{1+w^2} = \frac{-w^2}{-w} = w \rightarrow complex$

$b \rightarrow (1 + w)(2 + w^2) =- w^2(2 + w^2) =- 2w^2 - w^4 =- 2w^2 - w \rightarrow cmplx$

$c \rightarrow (3 + w)(3 + w^2) = 9 + 3w^2 + 3w + w^3 = 9 + 3(w^2 + w) + 1 = 7 \rightarrow$ ✓

$d \rightarrow (w^2 - w)^3 = (\pm \sqrt{3}i)^3 =\pm 3\sqrt{3}i \rightarrow imaginary$

Q22

Solution (c)

z_1 , z_2 *are two conj.*

$z_1 = (c+d)w^2 + (c-d)w$

$conj.\ of\ z_1 \rightarrow (c+d)w + (c-d)w^2 \ = z_2 = 5w + 7w^2$

$c + d = 5 \ , c - d = 7 \rightarrow \therefore \ c = 6 \ , d =- 1$

$c \times d = 6 \times - 1 =- 6$

Q23

Solution (a)

$x = a \ , y = bw \ , z = cw^2$

$\frac{x}{a} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{a}{a} + \frac{b^2w^2}{b^2} + \frac{c^2w^4}{c^2} = 1 + w^2 + w = zero$

Q24

Solution (b)

$\cos\left(\left(w^{10} + w^{23}\right)\pi + \frac{\pi}{4}\right) = \cos\left(\left(w \ + w^2\right)\pi + \frac{\pi}{4}\right) = \cos\left(-\pi + \frac{\pi}{4}\right) =$

$= \cos\left(\frac{-3\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

Q25

Solution (c)

$z = 2 + 3w^2 \ \ , \bar{z} = 2 + 3w$

$z\bar{z} = \left(2 + 3w^2\right)(2 + 3w) = 4 + 6w + 6w^2 + 9w^3 = 4 + 6\left(w + w^2\right) + 9$

$= 4 - 6 + 9 = 7$

Q26

Solution (a)

$$\left(\frac{a}{w}-\frac{a}{w^2}+\frac{3a}{w^4}-\frac{3a}{w^5}\right)^2=\left(aw^2-aw+\frac{3a}{w}-\frac{3a}{w^2}\right)^2$$

$$=\left(aw^2-aw+3aw^2-3aw\right)^2=\left(4aw^2-4aw\right)^2$$

$$=\left(4a(w^2-w)\right)^2=16a^2\left(\pm\sqrt{3}i\right)^2=16a^2\times 3i^2=-48a^2$$

Q27

Solution (a)

$$x-y=\frac{1}{1+wi}-\frac{w+i}{1+w^2i}=\frac{(1+w^2i)-(1+wi)(w+i)}{(1+wi)(1+w^2i)}$$

$$=\frac{(1+w^2i)-(w+i+w^2i-w)}{1+wi+w^2i-1}=\frac{1+w^2i-i-w^2i}{(w^2+w)i}=\frac{1-i}{-i}=\frac{-1}{i}+1$$

$$=i+1$$

[15] Choose :

Q1

Solution (d)

$$\left(2+\frac{3}{w}\right)\left(2+\frac{3}{w^2}\right)\left(3-\frac{2}{w}\right)\left(3-\frac{2}{w^2}\right)$$

$$=\left((2+3w^2)(2+3w)\right)\left((3-2w^2)(3-2w)\right)$$

$$=(4-6+9)\left(9-6w^2-6w+4w^3\right)=7\times(9+6+4)=7\times 19=133$$

Q2

Solution (a)

$$\left(1-\frac{1}{w}\right)\left(1-\frac{1}{w^2}\right)\left(1-\frac{1}{w^4}\right)\left(1-\frac{1}{w^8}\right)\ ......to\ 10\ factors$$

$$=(1-w^2)(1-w)\left(1-\frac{1}{w}\right)\left(1-\frac{1}{w^2}\right)......to\ 10\ factors$$

$$=(1-w^2)(1-w)(1-w^2)(1-w).....to\ 10\ factors$$

$$\rightarrow (1-w^2)(1-w) \rightarrow let\ 2n=10 \rightarrow n=5$$

$$(1+1+1)=3$$

$$3^n=3^5=243$$

Q3

Solution (c)

$$\left(\frac{3+5w}{5+3w^2}-\frac{5+3w^2}{3+5w}\right)^8=\left(\frac{3w^3+5w}{5+3w^2}-\frac{5+3w^2}{3w^3+5w}\right)^8$$

$$=\left(\frac{w(3w^2+5)}{5+3w^2}-\frac{5+3w^2}{w(3w^2+5)}\right)^8=\left(w-\frac{1}{w}\right)^8=\left(w-w^2\right)^8=(\pm\sqrt{3})^8=81$$

Q4

Solution (a)

$$a=2w-3w^2\quad ,b=3+5w^2$$

$$b=3+3w^2+2w^2$$

$$b=-3w+2w^2=2w^2-3w$$

$$a^2+b^2=\left(2w-3w^2\right)^2+\left(2w^2-3w\right)^2$$

$$= 4w^2 - 12 + 9w + 4w - 12 + 9w^2 = -4 - 9 - 24 = -37$$

Q5

Solution (c)

$$\left(a + bw + aw^2\right)\left(a + bw^2 + aw^4\right) = \left(a + bw + aw^2\right)\left(a + bw^2 + aw\right)$$

$$= (-aw + bw)(-aw^2 + bw^2)$$

$$= w(b - a).w^2(b - a) = (b - a)^2$$

$$(b - a)^2 = (a - b)^2$$

Q6

Solution (b)

$$\left(1 + 2w^5 + \frac{1}{w^2}\right)\left(1 + 2w + \frac{1}{w^4}\right) = \left(1 + 2w^2 + w\right)\left(1 + 2w + w^2\right)$$

$$= \left(2w^2 - w^2\right)(2w - w) = w^2.w = w^3 = 1$$

Q7

Solution (b)

$$\frac{a-dw}{aw^2-d} - w^2 = \frac{aw^3-dw}{aw^2-d} - w^2 = \frac{w\left(aw^2-d\right)}{aw^2-d} - w^2 = w - w^2 = \pm\sqrt{3}i$$

Q8

Solution (a)

$$z = w^x \to r = 1\ (always)$$

$$r = |z| = 1$$

Q9

Solution (b)

$\sum_{r=1}^{5} w^r = w + w^2 + w^3 + w^4 + w^5 = w + w^2 =- 1$

Q10

Solution (b)

$\sum_{r=1}^{6} (1 + w^r) = 1 + w + 1 + w^2 + 1 + w^3 + 1 + w^4 + 1 + w^5 + 1 + w^6$

$= 6 + w + w^2 + w^3 + w^4 + w^5 + w^6 = 6 + 0 + 0 = 6$

Q11

Solution (b)

$1 + \sum_{r=1}^{6} w^r = 1 + w + w^2 + w^3 + w^4 + w^5 + w^6 = 1$

Q12

Solution (c)

$i^{12} = w^{21} \rightarrow i^4 = w^3 \rightarrow 1 = 1$

$choice\ (a) and\ (b) \rightarrow wrong\ , (c) \rightarrow z^{12} \rightarrow 12\ answers$

Q13

Solution (b)

$(1 + w)^7 = a + bw = \left(- w^2\right)^7 = -w^{14} = -w^2 = 1 + w$

$(a, b) = (1,1)$

Q14

Solution (b)

$(1+w^2)^n=(1+w)^n$

$(-w)^n=\left(-w^2\right)^n$

$\left(\frac{-w^2}{-w}\right)^n=1$

$w^n=1\rightarrow n=3$

Q15

Solution (b)

$a\,,b\ \rightarrow w\,,w^2$

$ab+a^5+b^5$

$w.w^2+w^5+w^{10}=w^3+w^2+w=zero$

Q16

Solution (a)

$3\,,2+w^2\,,x$

$3\,.\left(2+w^2\right).\,x=real\,number\,doesn't\,contain\,"i"$

$*\,a\rightarrow\left(2+w^2\right).(2+w)=4+2w^2+2w+w^3=4-2+1=3\ \rightarrow true$

$*\,b\rightarrow\left(2+w^2\right).(2-w)=4+2w^2-2w-w^3=4\pm2\sqrt{3}i-1\ \rightarrow contain\,i$

$*\,c\rightarrow\left(2+w^2\right).\left(w^2-2\right)=2w^2-4+w^4-2w^2=-4+w\ \rightarrow contain\,i$

$*\,d\rightarrow$ زيهم

Q17

Solution (d)

$w = \frac{-1+\sqrt{3}i}{2}$, $2w =- 1 + \sqrt{3}i$

$(-1 + \sqrt{3}i)^{48} = (2w)^{48} = 2^{48}. w^{48} = 2^{48}$

Q18

Solution (d)

$\left(\frac{-1-\sqrt{3}i}{2}\right)^{3n} + \left(\frac{-1+\sqrt{3}i}{2}\right)^{3n} = \left(w^2\right)^{3n} + w^{3n} = 1 + 1 = 2$

Q19

Solution (b)

$\left(\frac{1}{2} + \frac{3}{8} + \frac{9}{32} + \frac{27}{128} + \dots\right) + w + w^2 \rightarrow geomtric\ sentence$

$a_n = a + ar + ar^2$

$a = \frac{1}{2}$, $r = \frac{3}{4}$

$s_n = \frac{a(1-r^n)}{1-r}$, $n = \infty$, $r < 1$, $r^n = 0$

$s_n = \frac{a}{1-r} = \frac{0.5}{1-0.75} = 2$

$2 + w + w^2 = 2 - 1 = 1$

Q20

Solution (a)

$let\ the\ cubic\ roots\ of\ x = x_1 , x_2 , x_3$

$x_1 \,.\, x_2 \,.\, x_3 = x$

Q21

Solution (b)

$\left(\frac{-1-\sqrt{-3}}{2}\right)^{22} + \left(\frac{-1-\sqrt{-3}}{2}\right)^{17} = \left(\frac{-1-\sqrt{3}i}{2}\right)^{22} + \left(\frac{-1-\sqrt{3}i}{2}\right)^{17}$

$w^{22} + w^{17} = w + w^2 = -1$

Q22

Solution (a)

$equilateral\ triangle \rightarrow 1 + w + w^2 = zero$

Q23

Solution (a)

$x + \frac{1}{x} = -1 \rightarrow w + \frac{1}{w} = w + w^2 = -1$

$x = w$

$x^{2018} + x^{-2018} = w^{2018} + w^{-2018} = -1$

Q24

Solution (d)

$(z-2)^3 = 1$

$z - 2 = 1 \quad , z - 2 = w \quad , z - 2 = w^2$

$z = 2 + 1 \quad , z = 2 + w \quad , \ z = 2 + w^2$

$by\ summation\ of\ the\ three\ equations \ \rightarrow 2 + 1 + 2 + w + 2 + w^2 = 6$

Q25

Solution (a)

$l = aw + bw^2 \ , m = aw^2 + bw$

$l \, , m \ \ not \ multiplicative$

Q26

Solution (d)

$$\frac{14+6w+21w^2}{8w^2-7} = \frac{14+6w+21w^2-7w^2+7w^2}{8w^2-7}$$

$$= \frac{14+6w+14w^2+7w^2}{8w^2-7} = \frac{6w-14w+7w^2}{8w^2-7} = \frac{-8w+7w^2}{8w^2-7w^3}$$

$$= \frac{-(8w-7w^2)}{w(8w-7w^2)} = \frac{-1}{w} = -w^2$$

Q27

Solution (d)

$$\frac{1}{2} \times \frac{1}{2} w \times \frac{1}{2} w^2 = \frac{1}{8}$$

Q28

Solution (c)

$2 \ \rightarrow 1 + 1$

$\frac{1}{2} + \frac{\sqrt{3}}{2} i \rightarrow 1 + w$

$\frac{1}{2} - \frac{\sqrt{3}}{2} i \rightarrow 1 + w^2$

$(z-1)^{\,3} = 1$

Q29

Solution (a)

$r = 1$

$area\ of\ the\ circle = \pi r^2 = \pi(1)^2 = \pi$

Q30

Solution (a)

$area\ of\ \Delta\ z_1, z_2, z_3 = \frac{1}{2} \times r_1 \times r_2 \sin\theta = \frac{1}{2} \times 1 \times 1 \times \sin 120 \times 3 = \frac{3\sqrt{3}}{4}$