Algebra

Answers of choose **by steps**

Exercise (8)

On De Moivre's theorem.

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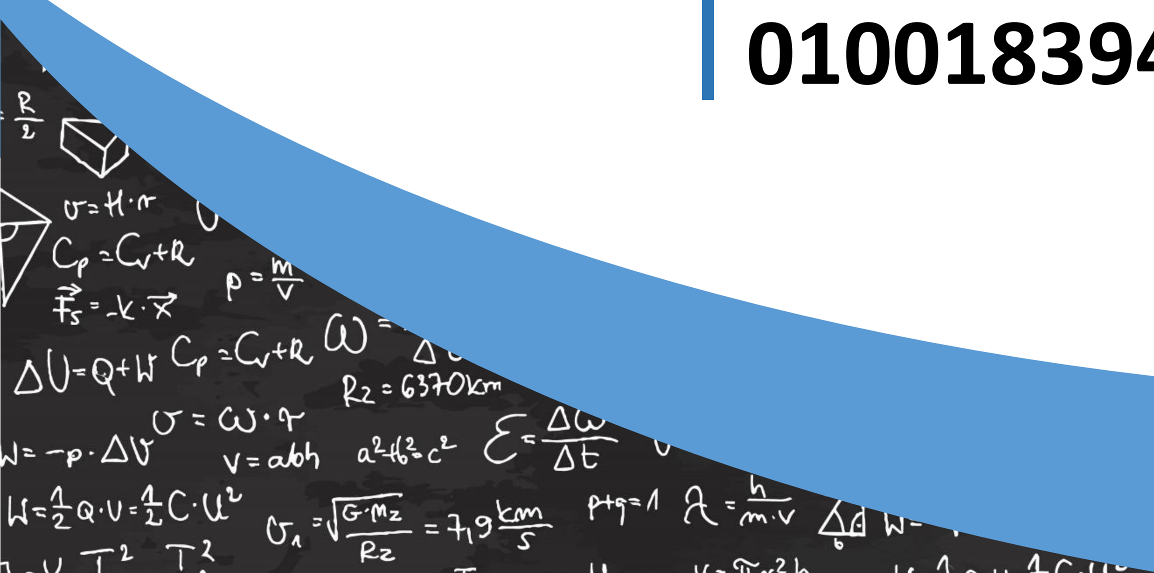

[33] Choose :

Q1

Solution (c)

$z^{\frac{1}{3}} \rightarrow equilateral\ triangle$

$z^{\frac{1}{4}} \rightarrow square$

$z^{\frac{1}{5}} \rightarrow regular\ pentagon$

Q2

Solution (b)

$z^{\frac{1}{6}} \rightarrow regular\ hexagon$

$\theta = \frac{360}{n} = \frac{360}{6} = 60$

Q3

Solution (c)

$\sqrt{7 + 24i} = x + yi \rightarrow r = \sqrt{7^2 + 24^2} = 25$

$\sqrt{7 + 24i} = \pm \left(\sqrt{\frac{25+7}{2}} + \sqrt{\frac{25+7}{2}}i\right) = \pm \left(\sqrt{16}, \sqrt{9}i\right) = \pm (4, 3i)$

$(x + y)^2 = (4 + 3)^2 = 49$

Q4

Solution (b)

$\sqrt{5 + 12i} = \pm \left(\sqrt{\frac{r+x}{2}} + \sqrt{\frac{r-x}{2}}i\right)$

$r = \sqrt{5^2 + 12^2} = 13$

$\sqrt{5 + 12i} = \pm \left(\sqrt{\frac{13+5}{2}} + \sqrt{\frac{13-5}{2}} i \right) = \pm (3 + 2i)$

Q5

Solution (b)

$z_1 = 2(\cos 120 + i \sin 120)$

$\bar{z_1} \rightarrow r = 2 , \theta = - 120$

$\bar{z_1} = 2(\cos(-120) + i \sin(-120))$

$z = - 2iz_1$

$z = 4(- i \cos(-120) + i \sin(-120))$

$z = 4(\sin(-120) - i \cos(-120)) = 4(\cos 150 + i \sin 150)$

$z^{\frac{1}{2}} = \sqrt{4} \left(\cos \frac{150+2\pi n}{2} + i \sin \frac{150+2\pi n}{2} \right) , where\ n = 0 , 1$

$at\ n = 0 \rightarrow z^{\frac{1}{2}} = 2(\cos 75 + i \sin 75) , 75 = \frac{5\pi}{12}$

$at\ n = 1 \rightarrow z^{\frac{1}{2}} = 2(\cos 255 + i \sin 255), 255 = \frac{17\pi}{12}$

$z^{\frac{1}{2}} = re^{\theta i} = 2e^{\frac{5}{12}\pi}$

Q6

Solution (b)

$z = 6 + 8i$

$z^{\frac{1}{3}} = z_1 , z_2 , z_3$

$(z_1)^3 = z \ , (z_2)^6 = z^2 \ , (z_3)^{12} = z^4$

$\frac{25{z_1}^3 \times 4{z_2}^6}{{z_3}^{12}} = \frac{25z \times 4z^2}{z^4} = \frac{100z^3}{z^4} = \frac{100}{z}$

$$\frac{100}{z} = \frac{6-8i}{6-8i} = \frac{100(6-8i)}{6^2+8^2} = \frac{100(6-8i)}{100} = 6-8i$$

Q7

Solution (b)

$z \to r = 16$

$z_1 = \sqrt{z}$

$r_1 = \sqrt{r} = \sqrt{16} = 4 \to 4+4 = 8$

Q8

Solution (a)

$z^3 = 4 + 4\sqrt{3}i$

$\to r = 8 , \theta = 60$

$z_1 \to r_1 = \sqrt[3]{8} = 2$

$area\ of\ z_1z_2z_3 = 3 \times \frac{1}{2}ab \times \sin\theta = 3 \times \frac{1}{2} \times 2 \times 2 \sin 120 = 3\sqrt{3}$

Q9

Solution (c)

$x^4 - 1 = 0$

$x^4 = 1$

$x^4 = \left(\cos\frac{0+360n}{4}\right) + i\sin\left(\frac{0+360n}{4}\right)$, $where\ n = 0 , 1, 2 , 3$

$at\ n = 0 \to x = \cos 0 + i\sin 0$

$at\ n = 1 \to x = \cos 90 + i\sin 90$

$at\ n = 2 \to x = \cos 180 + i\sin 180$

$at\ n = 3 \to x = \cos(-90) + i\sin(-90)$

$the\ product\ of\ the\ roots = 1 \times i \times -1 \times -i = -1$

Q10

Solution (c)

$z^3 = a\ ,\ \frac{360}{3} = 120$

$z_1 \to \theta$

$z_2 \to \theta + 120$

$z_3 \to \theta + 240$

$\to \theta + \frac{2\pi}{3}\ ,\ \theta + \frac{4\pi}{3}$

Q11

Solution (a)

$complex\ number\ z\ and\ its\ square\ roots\ lie\ in\ one\ circle\ \to z \to r$

$\to z_1 \to r_1$

$r_1 = r$

$r_1 = \sqrt{r}$

$r_1 = r$

$\sqrt{r} = r \to r = 1\ ,\ 0\ (refused)$

$r = |z|$

Q12

Solution (b)

$r = r^2 = r^3 \to r = 1\ ,\ 0\ (refused)$

$r = |z| = 1$

Q13

Solution (d)

$z = r(\cos\pi + i\sin\pi) =- r$

$\sqrt{z} = \sqrt{r}\left(\cos\frac{\pi+2\pi n}{2} + i\sin\frac{\pi+2\pi n}{2}\right)$, *where* $n = 0 , 1$

at $n = 0$, $\sqrt{r}(\cos 90 + i\sin 90) = \sqrt{r}i$

at $n = 1$, $\sqrt{r}(\cos -90 + i\sin -90) =- \sqrt{r}i$

imaginary have different signs

Q14

Solution (c)

$z^{\frac{4}{3}} = \left(z^{\frac{1}{3}}\right)^4 \rightarrow$ *has* 3 *values*

Q15

Solution (a)

$z = 3 - 4i$

$\sqrt{z} =\pm (x + yi)$

the sum of two roots is : $(x + yi) + \big(- (x + yi)\big) = zero$

Q16

Solution (b)

$z_1 = 4e^{\frac{5\pi}{6}} \rightarrow r = 4 , \theta = \frac{5\pi}{6}$

$z_2 = i \rightarrow r = 1 , \theta = 90 = \frac{\pi}{2}$

$z_1 = 4(\cos 150 + i \sin 150)$

$z_2 = \cos 90 + i \sin 90$

$z_1 z_2 = 4(\cos(150+90) + i\sin(150+90)) = 4(\cos 240 + i \sin 240)$

$= 4(\cos(-120) + i\sin(-120))$

$\sqrt{z_1 z_2} = \sqrt{4}\left(\cos\frac{-120+2\pi n}{2} + i\sin\frac{-120+2\pi n}{2}\right) , where\ n = 0 , 1$

$at\ n = 0\ , 2(\cos(-60) + i\sin(-60)) = 1 - \sqrt{3}i$

$at\ n = 1 , 2(\cos 120 + i\sin 120) =- 1 + \sqrt{3}i$

$\rightarrow \pm (1 - \sqrt{3}i)$

another solution

$\sqrt{x+yi} = \pm\left(\sqrt{\frac{r+x}{2}} + \sqrt{\frac{r-x}{2}}i\right) , where\ r = \sqrt{x^2 + y^2}$

Q17

Solution (a)

$z = x + yi = i \rightarrow x = 0 , y = 1 \rightarrow r = 1$

$\sqrt{z} = \pm\left(\sqrt{\frac{r+x}{2}} + \sqrt{\frac{r-x}{2}}i\right) = \pm\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$

$\sqrt{z} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ *or* $\sqrt{z} = \frac{-1}{\sqrt{2}} + \frac{-1}{\sqrt{2}}i$

$r = 1 , \theta = 45 = \frac{\pi}{4}$ $r = 1 , \theta = 225 =- 135 = \frac{-3\pi}{4}$

$e^{\frac{\pi}{4}i}$ $e^{\frac{-3\pi}{4}i}$

Q18

Solution (b)

$let\ z = x + yi$

$z - 2 = i(z+2) = zi + 2i$

$z - zi = 2i + 2$

$x + yi - (x+yi)i = 2 + 2i$

$x + yi - xi + y = 2 + 2i \rightarrow x + y = 2$

$\rightarrow y - x = 2 \quad \therefore x = 0\,, y = 2$

$z = x + yi = 2i$

$\sqrt{z} = \pm\left(\sqrt{\frac{r+x}{2}} + \sqrt{\frac{r-x}{2}}i\right) \quad , r = 2$

$= \pm\,(1+i)$

Q19

Solution (a)

$z = \frac{-7+26i}{5-2i} \rightarrow \times \frac{5+2i}{5+2i}$

$z = \frac{(-7+26i)(5+2i)}{5^2+2^2} = \frac{-35+130i-14i-52}{29} = \frac{-87+116i}{29} = -3 + 4i$

$\sqrt{z} = \pm\left(\sqrt{\frac{r+x}{2}} + \sqrt{\frac{r-x}{2}}i\right), r = \sqrt{3^2+4^2} = 5$

$\sqrt{z} = \pm\left(\sqrt{\frac{5-3}{2}} + \sqrt{\frac{5+3}{2}}i\right) = \pm\,(1+2i)$

$a + 2b = 1 + 2 \times 2 = 5$

Q20

Solution (a)

$i + \sqrt{i}\ , let\ z = i$

$z = \cos 90 + i \sin 90$

$\sqrt{i} = \sqrt{z} = \cos\frac{90+2\pi n}{n} + i \sin\frac{90+2\pi n}{n}$ $\quad , where\ n = 0 , 1$

$at\ n = 0 , \sqrt{i} = \cos 45 + i \sin 45$

$at\ n = 1 , \sqrt{i} = \cos -135 + i \sin -135$

$\sqrt{i} = \pm\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i\right)$

$i + \sqrt{i} = i \pm \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i\right) = \frac{\sqrt{2}}{2} + \frac{2+\sqrt{2}}{2} i \ \ or\ \frac{-\sqrt{2}}{2} + \frac{2-\sqrt{2}}{2} i$

$= 1 \pm \sqrt{2}$

Q21

Solution (b)

$(1 - i)x + (1 + i)y + 2i = 0$

$x - xi + y + yi + 2i = 0$

$\rightarrow x + y = 0 , \ - x + y =- 2 \ \rightarrow x = 1 , y =- 1$

$\sqrt{3x + 4yi} = \sqrt{3 - 4i} == \pm \left(\sqrt{\frac{r+x}{2}} + \sqrt{\frac{r-x}{2}} i\right) = \pm (2 - i)$

Q22

$first : r_1 = r_2 = r_3 = r_4 \rightarrow square$ $\quad$ *solution(b)*

$second : 4 \times \frac{1}{2} ab \ \sin\theta = 4\sqrt{3}$ $\quad$ *solution(c)*

$4 \times \frac{1}{2} r_1 r_2 \times \sin\frac{360}{4} = 4\sqrt{3}$

$r^2 = 2\sqrt{3}$

$z^4 \rightarrow r^4 = \left(r^2\right)^2 = \left(2\sqrt{3}\right)^2 = 12$

Q23

Solution (b)

$z_1 = 2e^{\frac{2\pi}{3}i} \rightarrow r = 2\ , \theta = 120$

$z_1 z_2 = 8e^{\frac{11\pi}{3}} \rightarrow r_1 r_2 = 8\ , \theta_1 + \theta_2 = 660$

$z_2 \rightarrow r_2 = 4\ , \theta_2 = 660 - 120 = 540$

$z_2 = 4(\cos 540 + i \sin 540)$

$z_2 = -4 \rightarrow x = -4\ , r = 4$

$\sqrt{z} = \pm\left(\sqrt{\frac{0}{2}} + \sqrt{\frac{8}{2}}i\right) = \pm 2i$

$2i \times -2i = 4$

Q24

Solution (a)

$z \rightarrow r = 4\ , \theta = \theta$

$z_1 \rightarrow r_1 = 2\ , \theta_1 = \frac{\theta + 360n}{2}$

$at\ n = 0 \rightarrow \theta_1 = \frac{\theta}{2}$

$z_2 \rightarrow r_2 = 2\ , \theta_2 = \frac{\theta}{2} + \frac{360}{2} = \frac{\theta}{2} + 180$

$|x|, x^2 \longrightarrow$ Even functions