Algebra

Answers of choose **by steps**

Exercise (7)

On exponential form of complex number.

MR. Hatem Mahran
01001839459

Q1

Solution (c)

$e^{2\pi i} \to re^{\theta i} \to r = 1\,,\theta = 2\pi$

$z = r(\cos\theta + i\sin\theta) = \cos 2\pi + i\sin 2\pi = 1$

Q2

Solution (b)

$3i \to r = 3\,,\theta = \frac{\pi}{2}$

$re^{\theta i} = 3e^{\frac{\pi}{2}i}$

Q3

Solution (a)

$z = \sqrt{2}e^{\frac{\pi}{4}i} \to r = \sqrt{2}\,,\theta = \frac{\pi}{4}$

$z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 1 + i$

Q4

Solution (a)

$$\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)^4 = \left(\cos\left(\frac{5\pi}{3}\times 4\right) + i\sin\left(\frac{5\pi}{3}\times 4\right)\right) = \cos\frac{20\pi}{3} + i\sin\frac{20\pi}{3}$$
$$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$$

$re^{\theta i} = e^{\frac{2\pi}{3}i}$

Q5

Solution (b)

$z = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$z + 1 = \frac{1}{2} + 1 + \frac{\sqrt{3}}{2}i = \frac{3}{2} + \frac{\sqrt{3}}{2}i \rightarrow r = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{3} \rightarrow \theta = \tan^{-1}\frac{\sqrt{3}}{3} =$

$\rightarrow \theta = \tan^{-1}\frac{\sqrt{3}}{3} = \frac{\pi}{6}$

$re^{\theta i} = \sqrt{3}e^{\frac{\pi}{6}i}$

Q6

Solution (d)

$\theta = 50 - 180 = -130 \rightarrow \frac{-13}{18}\pi \ , r = 3$

$re^{\theta i} = 3e^{\frac{-13\pi}{18}i}$

Q7

Solution (a)

$z_1 = 2(\cos 150 - i\sin 150)$

$z_1 = 2(\cos 210 + i\sin 210)$

$z_2 = 3(\sin 150 + i\cos 150)$

$z_2 = 3(\cos 300 + i\sin 300)$

$z_1 z_2 = 2 \times 3(\cos(210 + 300) + i\sin(210 + 300))$

$= 6(\cos 510 + i\sin 510) = 6(\cos 150 + i\sin 150)$

$re^{\theta i} = 6e^{\frac{5}{6}\theta i}$

another solution

$z_1 = 2(\cos 150 - i\sin 150) = 2\left(\frac{-\sqrt{3}}{2} - \frac{1}{2}i\right)$

$z_1 = -\sqrt{3} - i \rightarrow r = 2 \ , \theta = \tan^{-1}\frac{-\sqrt{3}}{-1} = 210$

$$z_2 = 3(\sin 150 + i\cos 150) = 3\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$z_2 = \frac{3}{2} - \frac{3\sqrt{3}}{2}i \rightarrow r = 3\ ,\ \theta = \tan^{-1}\frac{3\sqrt{3}}{3} = 300$$

$$z_1z_2 \rightarrow r_1r_2 = 2\times 3 = 6 \rightarrow \theta_1 + \theta_2 = 210 + 300 = 510 - 360 = 150$$

$$re^{\theta i} = 6e^{\frac{5}{6}\theta i}$$

Q8

Solution (b)

$$z = 2 + 2\sqrt{3}i \rightarrow r = \sqrt{2^2 + \left(2\sqrt{3}\right)^2} = 4\ ,\ \theta = \tan^{-1}\frac{2\sqrt{3}}{2} = 60 = \frac{\pi}{3}$$

$$re^{\theta i} = 4e^{\frac{\pi}{3}i}$$

Q9

Solution (c)

$$z_1 = 8(\cos\pi + i\sin\pi)$$

$$z_2 = 4e^{\frac{3\pi}{2}i} \rightarrow r = 4\ ,\ \theta = \frac{3}{2}\pi$$

$$z_2 = 4(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi)$$

$$\frac{z_1}{z_2} = \frac{8}{4}\left(\cos\left(\pi - \frac{3}{2}\pi\right) + i\sin\left(\pi - \frac{3}{2}\pi\right)\right) = 2(\cos -90 + i\sin -90) =- 2i$$

Q10

$$e^{\theta i} + e^{-\theta i} = 2\cos\theta$$

$$e^{\theta i} - e^{-\theta i} = 2i\sin\theta$$

$$e^{-\theta i} - e^{\theta i} =- 2i\sin\theta$$

Solution (b)

$$e^{\theta i} \rightarrow \cos\theta + i\sin\theta$$

$$e^{-\theta i} \rightarrow \cos(-\theta) + i\sin(-\theta) = \cos\theta - \sin\theta$$

$by\ summation = 2\cos\theta$

Q11

Solution (b)

$e^{\pi i} - e^{-\pi i} \rightarrow 2i\sin\pi = zero$

Q12

Solution (a)

$z = x + yi$

$e^{z} = e^{x+yi} = e^{x}.e^{yi}$

$e^{yi} \rightarrow r = 1\ ,\theta = y \rightarrow \cos y + i\sin y$

$e^{x}.e^{yi} = e^{x}(\cos y + i\sin y)$

$the\ real\ part\ is\ e^{x}\cos y$

Q13

Solution (b)

$z_1 = e^{\frac{\pi}{3}-\frac{\pi}{3}i} \rightarrow e^{\frac{\pi}{3}}.e^{\frac{\pi}{3}i} \rightarrow r_1 = e^{\frac{\pi}{3}}, \theta_1 = \frac{\pi}{3}$

$z_2 = e^{\frac{\pi}{6}-\frac{\pi}{6}i} \rightarrow e^{\frac{\pi}{6}}.e^{\frac{\pi}{6}i} \rightarrow r_2 = e^{\frac{\pi}{6}}, \theta_2 = \frac{\pi}{6}$

$z_1z_2 \rightarrow r_1r_2 = e^{\frac{\pi}{3}+\frac{\pi}{6}} = e^{\frac{\pi}{2}} \rightarrow \theta_1 + \theta_2 = \frac{-\pi}{3} + \frac{-\pi}{6} = \frac{-\pi}{2}$

$z_1z_2 = re^{\theta i} = e^{\frac{\pi}{2}}.e^{\frac{-\pi}{2}i} = e^{\frac{\pi}{2}}\left(\cos\frac{-\pi}{2} + i\sin\frac{-\pi}{2}\right) = e^{\frac{\pi}{2}} \times -i = -e^{\frac{\pi}{2}}i$

Q14

Solution (c)

$z_1 = 2e^{\frac{\pi}{6}i} \rightarrow r_1 = 2 , \theta_1 = \frac{\pi}{6}$, $z_1 = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = \sqrt{3} + i$

$z_2 = 2e^{-\frac{\pi}{6}i} \rightarrow r_2 = 2 , \theta_2 = \frac{-\pi}{6}$, $z_2 = 2\left(\cos -\frac{\pi}{6} + i\sin -\frac{\pi}{6}\right) = \sqrt{3} - i$

$z_1 + z_2 = \sqrt{3} + i + \sqrt{3} - i = 2\sqrt{3}$

$z_1 z_2 \rightarrow r = 2 \times 2 = 4 , \theta = \frac{\pi}{6} + -\frac{\pi}{6} = zero$

$z_1 z_2 = 4(\cos 0 + i\sin 0) = 4$

$z_1 + z_2 + z_1 z_2 = 2\sqrt{3} + 4 = 2(\sqrt{3} + 2)$

Q15

Solution (b)

$z_1 \rightarrow 90 + \theta$

$z_2 \rightarrow \theta$

$\frac{z_1}{z_2} \rightarrow \frac{|z_1|}{|z_2|} = \frac{3|z_2|}{|z_2|} = 3 \rightarrow r$

$\rightarrow 90 + \theta - \theta = 90 \rightarrow arg$

$re^{\theta i} = 3e^{\frac{\pi}{2}i}$

Q16

Solution (c)

$z_1 = \cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6} \rightarrow \theta_1 = 210$

$z_2 \rightarrow r = 4$

$arg. of\ z_1 z_2 =- 120$

$\theta_1 + \theta_2 =- 120 \rightarrow 210 + \theta_2 =- 120 \rightarrow \theta_2 =- 330 + 360 \rightarrow \frac{\pi}{6}$

$z_2 = re^{\theta i} = 4e^{\frac{\pi}{6}i}$

Q17

Solution (c)

$z_1 = r_1 e^{\theta i} \to z_1 = r_1(\cos\theta + i\sin\theta)$

$z_2 = r_2 e^{\theta i} \to z_2 = r_2(\cos\theta + i\sin\theta)$

$z_1 + z_2 = r_1(\cos\theta + i\sin\theta) + r_2(\cos\theta + i\sin\theta) = (r_1 + r_2)(\cos\theta + i\sin\theta)$

$r_1 + r_2 \to |z_1 + z_2|$

Q18

Solution (d)

$z = re^{\theta i}$

$re^{\left(\frac{\pi}{2}+\theta\right)i} = r.e^{\frac{\pi}{2}}.e^{\theta i} = e^{\frac{\pi}{2}i}.re^{\theta i} = (\cos\theta + i\sin\theta)(z) = (0 + i)z = iz$

Q20

Solution (a)

$let\ z = x + yi, \bar{z} = x - yi$

$z + \bar{z} = 2x$

$2x = 2e^{\pi i} \to x = e^{\pi i} = \cos\pi + i\sin\pi$

$x = -1, y = 0$

$a \to e^{\pi i} \to x = -1, y = 0 \to right\ answer$

$b \to 2e^{\frac{\pi}{2}} \to x = 0 \to wrong$

$c \to e^{-\frac{\pi}{2}i} \to x = 0 \to wrong$

$d \to 2e^{\pi i} \to x = -2 \to wrong$

Q21

Solution (d)

$z_1 = e^{5+k\pi i} = e^5 . e^{k\pi i}$

$z_2 = e^{(5+ki)\pi} = e^{5\pi} . e^{k\pi i}$

$z_1 + z_2 = e^5 . e^{k\pi i} + e^{5\pi} . e^{k\pi i} = e^{k\pi i}\left(e^{5\pi} + e^5\right) \rightarrow \theta = k\pi$

$at\ k \in \ ]\frac{-1}{2} , \frac{1}{2}[\rightarrow \theta = k\pi \ \epsilon \ ]\frac{-\pi}{2} , \frac{\pi}{2}[\rightarrow -30 = \frac{-\pi}{6}$

Q22

Solution (a)

$(\sqrt{k} , -\sqrt{k})$

$r = \sqrt{\left(\sqrt{k}\right)^2 + \left(-\sqrt{k}\right)^2} = \sqrt{k+k} = \sqrt{2k}$

$x\ is\ positive , y\ is\ negative$

$\theta \ \epsilon \ ] -90 , 0 [\rightarrow \frac{-\pi}{4}$