Algebra

Answers of choose **by steps**

Exercise (6)

Trigonometric form of a complex number.

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[5] choose the correct answer :

Q1

Solution (b)

$z = 3 - 4i \to (x + yi)$

$(x , y) = (3 , -4)$

Q2

Solution (d)

$(\sqrt{3} , -1)$

$\tan\theta = \frac{y}{x}$, $\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{-1}{\sqrt{3}} = -30 = \frac{-\pi}{6}$

$r = \sqrt{x^2 + y^2} = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{4} = 2$

$(r , \theta) \to (2 , \frac{-\pi}{6})$

Q3

Solution (c)

$z = -1 + \sqrt{3}i \to x = -1$, $y = \sqrt{3}$

$|z| = |\bar{z}| = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$

Q4

Solution (b)

$z = -2i$

$r = \sqrt{(-2)^2} = 2$

$\theta = \tan^{-1}\frac{-2}{0} = -90 = \frac{-\pi}{2}$

Trig form

$r\,(\cos\theta + i\sin\theta) \rightarrow 2\,(\cos -90 + i\sin -90)$

Q5

Solution (b)

$z = 1 - i$

$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{-1}{1} = -45 = \frac{-\pi}{4}$

Q6

Solution (b)

$2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 2 \times \frac{-1}{2} + 2i \times \frac{\sqrt{3}}{2} = -1 + \sqrt{3}i$

Q7

$$z = x + yi$$

$$\bar{z} = x - yi$$

- *First* : $z \rightarrow \theta \;, \bar{z} \rightarrow -\theta$ *Solution (b)*

- *Second* : $z \rightarrow \theta = \tan^{-1}\frac{y}{x}$

$2z \rightarrow \theta = \tan^{-1}\frac{2y}{2x} = \tan^{-1}\frac{y}{x}$

same amlitude *Solution (a)*

- *Third* : $z \rightarrow \theta\,,\ \frac{1}{z} \rightarrow -\theta$ *Solution (b)*

Q8

Solution (b)

$z = \frac{1}{\bar{z}} \rightarrow z \times \bar{z} = 1$

$r^2 = |z|^2 = |\bar{z}|^2 = 1 \rightarrow |z| = 1 \ , |\bar{z}| =- 1 \ (refused)$

Q9

Solution (b)

$|z| = 10$

$z \times \bar{z} = |z|^2 = 10^2 = 100$

Q10

Solution (a)

$|z| = 6$

$z = x + yi \rightarrow |z| = \sqrt{x^2 + y^2} = r$

$\bar{z} = x - yi \ , \overline{-z} =- x + yi$

$|\overline{-z}| = \sqrt{(-x)^2 + y^2} = \sqrt{x^2 + y^2} = r$

$|\overline{-z}| = |z| = 6$

Q11

Solution (a)

$z = 6\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$z = r(\cos\theta + i\sin\theta)$

$r = |z| = |\bar{z}| = 6$

Q12

Solution (b)

$2\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right) \rightarrow not\ the\ trig\ form$

$r\,(\cos\theta + i\sin\theta\,)$

$2(\cos\frac{-\pi}{4} + i\sin\frac{-\pi}{4})$

$the\ amplitude\ is\ \frac{-\pi}{4}$

Q13

Solution (c)

$|z| + |\bar{z}| = 2r = 12 \to r = 6$

$let\ z = x + yi$

$zi = (x + yi)i = xi - y$

$|zi| = \sqrt{x^2 + y^2} = |z| = r = 6$

Q14

Solution (a)

$z + \bar{z} = \theta + -\theta = zero$

Q15

Solution (c)

$z \to \theta\,, \frac{i}{z} \to\to (90 - \theta)$

$\bar{z} \to -\theta\,, \frac{i}{\bar{z}} \to (90 - (-\theta))$

$90 - (-\theta) = 90 + \theta = 90 + 28 = 118$

Q16

$z \times \bar{z} = |z|^2 = |\bar{z}|^2 = r^2 = x^2 + y^2$

Solution (c)

$z \times \overline{z} = |z|^2 \rightarrow a$ ✓

$z \times \overline{z} = |\overline{z}|^2 \rightarrow b$ ✓

$z \rightarrow \theta, \overline{z} \rightarrow -\theta \rightarrow c$ ✗

$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2} \rightarrow d$ ✓

Q17

Solution (d)

$let\ z = x + yi$

$z + 2 = i(z - 2)$

$x + yi + 2 = i(x + yi - 2)$

$x + 2 + yi = xi - y - 2i$

$x + 2 + yi = -y + (x - 2)i$

$x + 2 = -y \rightarrow x + y = -2$

$y = x - 2 \rightarrow x - y = 2$

$\therefore x = 0\ , y = -2$

$z = x + yi = -2i$

$r - \sqrt{(-2)^2} = 2 \quad , \theta = \tan^{-1}\frac{-2}{0} = -90$

$z = 2(\cos -90 + i \sin -90)$

Q18

Solution (c)

$z = \frac{2-i}{2+i} \times \frac{2-i}{2-i} = \frac{4-2i-2i-1}{2^2+1^2} = \frac{3-4i}{5} = \frac{3}{5} - \frac{4}{5}i$

$r=\sqrt{\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2}=1$

Q19

Solution (b)

$x=\cos 17+i\sin 17$

$x^3=1^3(\cos(3\times 17)+i\sin(3\times 17))=\cos 51+i\sin 51$

$y=\cos 11+i\sin 11$

$y^9=\cos(11\times 9)+i\sin(11\times 9)=\cos 99+i\sin 99$

$x^3y^9=\cos(51+99)+i\sin(51+99)=\cos 150+i\sin 150$

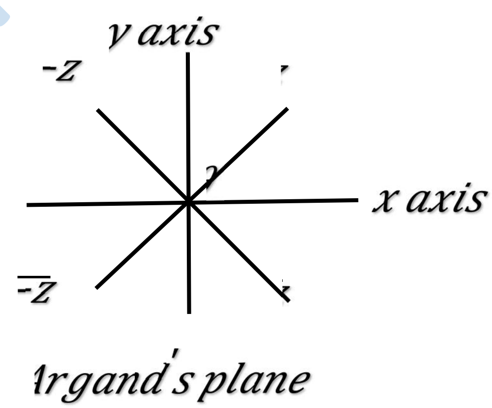
Q20

Solution (b)

$z\to\theta$

$\bar{z}\to-\theta$

$reflection\ about\ x\ axis$



Q21

Solution (c)

$z_1=2\sqrt{3}-i\to\overline{z_1}=2\sqrt{3}+i$

$z_2=2(\cos 30+i\sin 30)$

$z_2=2(\frac{\sqrt{3}}{2}+\frac{1}{2}i)$

$z_2=\sqrt{3}+i\to\overline{z_2}=\sqrt{3}-i$

$\overline{z_1}-\overline{z_2}=2\sqrt{3}+i-(\sqrt{3}-i)=2\sqrt{3}+i-\sqrt{3}+i=\sqrt{3}+2i$

Q22

Solution (a)

$z_1 \to \theta_1 \ , z_2 \to \theta_2$

$z_1 z_2 \to \theta_1 + \theta_2$

Q23

Solution (c)

$3(\cos 30 + i \sin 30) \times 6(\cos 70 + i \sin 70)$

$= 3 \times 7(\cos(30+70) + i \sin(30+70))$

$= 18(\cos 100 + i \sin 100)$

Q24

Solution (d)

$6(\cos 210 + i \sin 210) \div 3(\cos 70 + i \sin 70)$

$\frac{6}{3}(\cos(210-70) + i \sin(210-70))$

$2(\cos 140 + i \sin 140)$

Q25

Solution (b)

$(5(\cos 10 + i \sin 10)^2$

$= 5^2(\cos(2 \times 10) + i \sin(2 \times 10))$

$= 25(\cos 20 + i \sin 20)$

Q26

Solution (b)

$r = \sqrt{2} \ , \theta = \frac{7\pi}{6}$

$z=\sqrt{2}(\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6})$

$z=\sqrt{2}\left(\frac{-\sqrt{3}}{2}-\frac{1}{2}i\right)=\frac{-\sqrt{6}}{2}-\frac{\sqrt{2}}{2}i$

$\therefore\ the\ real\ part\ is\ \frac{-\sqrt{6}}{2}$

Q27

Solution (d)

$\left(i^{25}\right)^{3}=i^{75}=i^{3}$

$z=-i\rightarrow r=\sqrt{0^2+(-1)^2}=1$

$\rightarrow\theta=-90=\frac{-\pi}{2}$

$z=r\,(\cos\theta+i\sin\theta)=1(\cos-90+i\sin-90)=\cos\frac{-\pi}{2}+i\sin\frac{-\pi}{2}$

Q28

Solution (d)

$z=i^{22}+i^{24n-13}=i^2+\left(i^{24}\right)^n\times i^{-13}=-1+1^n\times\frac{1}{i^{13}}=-1+1\times\frac{1}{i}$

$z=-1-i\rightarrow r=\sqrt{(-1)^2+(-1)^2}=\sqrt{2}$

$\rightarrow\theta=\tan^{-1}\frac{-1}{-1}=45=\frac{\pi}{4}\rightarrow\theta\ lies\ in\ 3^{rd}\ quad.\ =\frac{-3\pi}{4}$

$z=\sqrt{2}\,(\cos\left(\frac{-3\pi}{4}\right)+i\sin\left(\frac{-3\pi}{4}\right))$

Q29

Solution (a)

$z=3\,(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$

$z=\frac{3\sqrt{2}}{2}+\frac{3\sqrt{2}}{2}i=\frac{3\sqrt{2}}{2}(1+i)$

$\bar{z} = \frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2} i = \frac{3\sqrt{2}}{2}(1 - i)$

$\frac{1}{\bar{z}} = \frac{2}{3\sqrt{2} - 3\sqrt{2}i} \times \frac{3\sqrt{2} + 3\sqrt{2}i}{3\sqrt{2} + 3\sqrt{2}i}$

$\frac{1}{\bar{z}} = \frac{2(3\sqrt{2} + 3\sqrt{2}i)}{18 + 18} = \frac{3\sqrt{2}(1 + i)}{18}$

$\frac{1}{\bar{z}} = \frac{3\sqrt{2}}{2}(1 + i) \times \frac{1}{9} = z \times \frac{1}{9} = \frac{1}{9} z$

another solution

$|z| = 3$

$\frac{1}{\bar{z}} = \frac{z}{z\bar{z}} = \frac{z}{|z|^2} = \frac{z}{9} = \frac{1}{9} z$

Q31

Solution (c)

$r = 3$

$\theta = 30 + 90 = 120$

$r(\cos\theta + i\sin\theta) = 3(\cos 120 + i\sin 120)$

Q32

Solution (b)

$z_1 \to r_1 = 3$

$\to \theta_1 = 28$

$z_2 \to r_2 = 2$

$\to \theta_2 = 62 - 180 = -118$

$z_1 \times z_2 = 2 \times 3(\cos(28 - 118) + i\sin(28 - 118))$

$= 6(\cos(-90) + i\sin(-90))$

$\therefore \theta = -90 = \frac{-\pi}{2}$

Q33

Solution (d)

$-z \rightarrow wrong\ answer$

$\bar{z} \rightarrow wrong\ answer$

$\frac{1}{z} \rightarrow wrong\ answer$

$z \rightarrow \theta\ ,\ \frac{1}{\bar{z}} \rightarrow same\ \theta$

$\bar{z} \rightarrow -\theta\ ,\ \frac{1}{z} \rightarrow same - \theta$

Q34

Solution (a)

$z = -3(\cos 45 + i \sin 45)$

$z = r\ (\cos\theta + i \sin\theta)$

r always positive

$-z = 3(\cos 45 + i \sin 45)$

$-z \rightarrow \theta\ , z \rightarrow \theta - 180$

$-z \rightarrow 45\ \ , z \rightarrow 45 - 180 = -135$

Q35

Solution (a)

$z_1 \rightarrow \theta_1 = \frac{4\pi}{5}$

$z_2 \rightarrow \theta_2 = \frac{2\pi}{3}$

$\frac{z_1}{z_2} \rightarrow \theta_1 - \theta_2 = \frac{4\pi}{5} - \frac{2\pi}{3} = \frac{2}{15}\pi$

Q36

Solution (b)

$4\theta + 10 = 90 \qquad 4\theta + 10 = 90$

$4\theta = 80 \qquad 4\theta = -100$

$\theta = 20 \qquad \theta = -25$

Q37

Solution (d)

$(3\theta - 110) = -(2\theta + 50)$

$= 3\theta - 110 = -2\theta - 50$

$5\theta = 60 = \frac{\pi}{3}$

$\theta = \frac{\pi}{15}$

Q38

$z \to r = 2$

$\to \theta = \frac{\pi}{2}$

$z = r(\cos\theta + i\sin\theta)$

$z = 2(\cos 90 + i\sin 90) = 0 + 2i = 2i$

$z + 2 = 2 + 2i \to r = 2\sqrt{2}$ $\qquad first\ (d)$

$\to \theta = \tan^{-1}\frac{2}{2} = \frac{\pi}{4} or \frac{5\pi}{4}$

$x\ and\ y\ positive \to 1^{st}\ quad.$

$\theta = \frac{\pi}{4}$ $\qquad second\ (a)$

Q39

Solution (c)

$$midpoint = \frac{z_1+z_2}{2} = \frac{1}{2}(z_1 + z_2)$$

Q40

Solution (c)

$$z \to \theta \ \ , nz \to \theta \ \ , n\epsilon R$$

$$2z \to \theta$$

Q41

Solution (a)

$$z_1 = 10(\cos\theta + i\sin\theta)$$

$$z_2 = 5(\cos\propto + i\sin\propto)$$

$$\frac{z_1}{z_2} = \frac{10}{5}(\cos(\theta - \alpha) + i\sin(\theta - \alpha)) \quad , \theta - \alpha = \frac{3\pi}{4}$$

$$\frac{z_1}{z_2} = 2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = 2\left(\frac{-\sqrt{2}}{2} + i\frac{-\sqrt{2}}{2}\right) = -\sqrt{2} + \sqrt{2}i$$

Q42

Solution (a)

$$z = \cos\theta - i\sin\theta$$

$$\frac{z^2-1}{zi} \times \frac{-i}{-i} = \frac{-z^2 i + i}{z} = -zi + \frac{i}{z} = \frac{i}{z} - zi$$

$$\frac{i}{z} = \frac{i}{\cos\theta - i\sin\theta} \times \frac{\cos\theta + i\sin\theta}{\cos\theta + i\sin\theta} = \frac{i\cos\theta - \sin\theta}{(\cos\theta)^2 + (\sin\theta)^2} = \frac{i\cos\theta - \sin\theta}{1}$$

$$zi = i(\cos\theta - i\sin\theta) = i\cos\theta + \sin\theta$$

$$\frac{i}{z} - zi = i\cos\theta - \sin\theta - (i\cos\theta + \sin\theta) = -2\sin\theta$$

Q43

Solution (b)

$z_1 = \cos\theta_1 + i\sin\theta_1$

$z_1{}^{-4} = \cos(-4\theta_1) + i\sin(-4\theta_2)$

$z_2 = \cos\theta_2 + i\sin\theta_2$

$z_2{}^3 = \cos(3\theta_2) + i\sin(3\theta_2)$

$z_1{}^{-4}z_2{}^3 = \cos(-4\theta_1 + 3\theta_2) + i\sin(-4\theta_1 + 3\theta_2) = 1$

$\cos(-4\theta_1 + 3\theta_2) = 1$

$-4\theta_1 + 3\theta_2 = 0$

$\frac{\theta_1}{\theta_2} = \frac{3}{4}$

Q44

Solution (d)

$z_1 = \cos\theta + i\sin\theta$

$z_1 \to \theta$

$z_2 = \cos 2\theta + i\sin 2\theta$

$z_2 \to 2\theta$

$3z_1z_2 \to \theta + 2\theta = 3\theta$

Q45

Solution (c)

$z = k(\sin\frac{4\pi}{3} - i\cos\frac{4\pi}{3})$

$z = r(\cos\theta + i\sin\theta)$

$\sin 240 = \frac{-\sqrt{3}}{2} = \cos 150 = \cos 210$

$-\cos 240 = \frac{1}{2} = \sin 150 = \sin 30$

$z = k(\cos 150 + i \sin 150)$

$z^6 = k^6(\cos(150 \times 6) + i \sin(150 \times 6)) = k^6(-1 + 0) =- k^6$

[24] choose the correct answer :

Q1

Solution (c)

$let\ z = x + yi , \bar{z} = x - yi \ , |z| = \sqrt{x^2 + y^2}$

$multiplicative\ inverse\ of\ z \rightarrow \frac{1}{z}$

$\frac{1}{z} = \frac{1}{x+yi}$

$\frac{1}{z} = \frac{1}{x+yi} \times \frac{x-yi}{x-yi} = \frac{x-yi}{x^2+y^2} = \frac{\bar{z}}{|z|^2}$

Q2

Solution (b)

$\frac{1+2i}{1-i} \rightarrow \times \frac{1+i}{1+i} = \frac{1+i+2i-2}{1+1} = \frac{-1+3i}{2} = \frac{-1}{2} + \frac{3}{2}i$

$x \rightarrow negative , y \rightarrow positive \ \ \therefore 2^{nd}\ quad.$

Q3

Solution (c)

$z_1 = 1 + \sqrt{2}i \rightarrow r_1 = \sqrt{3} = |z_1|$

$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} = \frac{|r_1|}{|r_2|} = \frac{\sqrt{3}}{2}$

Q4

Solution (c)

$|z_1| = 2|z_2|$

$r_1 = 2r_2$

$\propto_2 = \theta + 90$

$\propto_1 = \theta - 180$

$\frac{z_2}{z_1} = \frac{r_2}{r_1}(\cos(\propto_2 - \propto_1) + i\sin(a_2 - a_1))$

$= \frac{r_2}{2r_2}(\cos(\theta + 90 - \theta + 180) + i\sin(\theta + 90 - 180))$

$= \frac{1}{2}(\cos 270 + i\sin 270) = \frac{1}{2}(\cos -90 + i\sin -90)$

Q5

Solution (c)

$|z_1| = |z_1 z_2| = |z_1||z_2| \rightarrow |z_2| = 1$

$z_1 \rightarrow \theta \, , z_1 z_2 \rightarrow \theta + 90$

$z_2 = \frac{z_1 z_2}{z_1} = \theta + 90 - \theta = 90$

$z_2 = 1(\cos 90 + i\sin 90) = i$

Q6

Solution (b)

$z_1 \rightarrow z \rightarrow 90 - \theta$

$z_4 \rightarrow \bar{z} \rightarrow \theta - 90$

$z_2 \rightarrow - \bar{z} \rightarrow \theta + 90$

$z_3 \rightarrow -\bar{z} \rightarrow -90-\theta$

$z_1z_2z_3z_4 = arg_1 + arg._2 + arg._3 + arg._4$

$= 90-\theta+\theta+90-90-\theta+\theta-90 = zero$

Q7

Solution (a)

$z = 1+\sqrt{3}i$

$-z = -1-\sqrt{3}i$

Q8

Solution (b)

$\arg(z_1z_2) = \frac{5\pi}{18} = \theta_1+\theta_2 = \frac{5\pi}{18} \rightarrow equation\ 1$

$\arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{9} \rightarrow \theta_1-\theta_2 = \frac{\pi}{9} \rightarrow equation\ 2$

$from\ 1,2 \rightarrow \theta_1 = \frac{7\pi}{36}, \theta_2 = \frac{\pi}{12}$

Q9

Solution (a)

$\theta_1+\theta_2 = \frac{\pi}{6} \rightarrow \quad equ.1$

$\theta_1+\theta_3 = \frac{2\pi}{9} \rightarrow \quad equ.2$

$\theta_2+\theta_3 = \frac{5\pi}{8} \rightarrow \quad equ.3$

$from\ 1,2,3 \rightarrow \therefore \theta_1 = 10, \theta_2 = 20, \theta_3 = 30$

$\arg(z_1z_2z_3) = \theta_1+\theta_2+\theta_3 = 10+20+30 = 60 = \frac{\pi}{3}$

Q10

Solution (c)

$let\ z = x + yi$

$x + yi - 2 \to \tan 90 = \frac{y}{x-2} \to x - 2 = 0 \to x = 2$

$x + yi - 4 \to \tan 135 = \frac{y}{x-4} =- 1 \to y = 4 - x = 4 - 2 = 2$

$z = x + yi = 2 + 2i \to \theta = \tan^{-1}\frac{2}{2} = \frac{\pi}{4}$

Q11

Solution (b)

$z_1 = 3 + 3\sqrt{3}i \quad , z_2 =- 4 - 4\sqrt{3}i$

$z_1 + z_2 =- 1 - \sqrt{3}i$

$\therefore r = 2 , \theta =- 120$

Q12

Solution (b)

$z = 1 + \sqrt{3}i$

$z \to r = 2 , \theta = 60$

$z = 2(\cos 60 + i \sin 60)$

$z^8 = 2^8(\cos(60 \times 8) + i \sin(60 \times 8))$

$\theta = 60 \times 8 = 480 \to - 360$

$= 120 = \frac{2\pi}{3}$

Q13

Solution (d)

$z = (1 + \sqrt{3})^n \ , |z| = 8$

$|z| = 2^n \to n = 3$

$z = (1 + \sqrt{3})^3 \to r = 2^n = 2^3 = 8$

$\to \theta = 60n = 60 \times 3 = 180 = \pi$

Q14

Solution (c)

$z_1 = 2i \ , z_2 = -1 + 3i$

$z_1 - z_2 = 2i - (-1 + 3i) = 2i + 1 - 3i = 1 - i$

$r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\theta = \tan^{-1}\frac{-1}{1} = -45 \, , 135 \to refused$

Q15

Solution (b)

$z_1 z_2 = r_1 r_2(\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)) \, , \theta_1 + \theta_2 = 180$

$z_1 z_2 = r_1 r_2(\cos(180) + i \sin(180)) = r_1 r_2(-1 + 0) = -r_1 r_2$

Q16

Solution (b)

$z \to \theta$

$-z \to \theta - 180$

$\theta - 180 - \theta = -180 = -\pi$

Q17

Solution (d)

$z_1 \to \theta_1 \ , z_2 \to \theta_2$

$\theta_1 + \theta_2 = 180$

$\theta_1 = 180 - \theta_2$

$z_1 =- \overline{z_2}$

Q18

Solution (a)

$amp(z_1) + amp(z_2) = 0 \ , r_1 = r_2$

$\theta_1 + \theta_2 = 0$

$\theta_1 =- \theta_2$

Q19

Solution (c)

$\frac{1+z}{1+\bar{z}} \times \frac{z}{z} = \frac{z(1+z)}{z(1+\frac{1}{z})} = \frac{z(1+z)}{z(1+\frac{1}{z})} = \frac{z(1+z)}{z+1} = z$

$z \to \theta \, , \frac{1+z}{1+\bar{z}} \to \theta$

Q20

Solution (a)

$let\ z = x + yi \, , |z| = \sqrt{x^2 + y^2}$

$z-2=x-2+yi\,,\,|z-2|=\sqrt{(x-2)^2+y^2}$

$|z|=|z-2|$ *by squaring*

$|z|^2=|z-2|^2$

$x^2+y^2=(x-2)^2+y^2$

$x^2=x^2-4x+4$

$x=1$

Q21

Solution (b)

$let\ z=x+yi\ \ ,|z|^2=x^2+y^2$

$z-3=x+yi-3\,,|z-3|^2=(x-3)^2+y^2$

$|z-3|^2=|z|^2$

$(x-3)^2+y^2=x^2+y^2$

$x^2-6x+9=x^2$

$6x=9\rightarrow x=\frac{3}{2}$

Q22

Solution (a)

$\left|\frac{z-3i}{z+3i}\right|=1$

$\frac{|z-3i|}{|z+3i|}=1$

$|z-3i|=|z+3i|$ $\ \ ,let\ z=x+yi$

$|x+yi-3i|^{\,2}=|x+yi+3i|^{\,2}$ *by squaring*

$x^2+(y-3)^2=x^2+(y+3)^2$

$y^2 - 6y + y = y^2 + 6y + 9$

$-6y = 6y \rightarrow y = 0 \rightarrow lies\ on\ the\ x - axis$

Q23

Solution (d)

$z = \frac{i}{\cos 75 + i \sin 75} \times \frac{\cos 75 - i \sin 75}{\cos 75 - i \sin 75}$

$= \frac{i \cos 75 + \sin 75}{(\cos 75)^2 + (\sin 75)^2} = \frac{i \cos 75 + \sin 75}{1}$

$z = i \cos 75 + \sin 75 = \sin 75 + i \cos 75 \rightarrow \ 90 - \theta$

$z = \cos 15 + i \sin 15$

$z^6 = \cos(15 \times 6) + i \sin(15 \times 6)$

$\theta = 15 \times 6 = 90 = \frac{\pi}{2}$

Q24

Solution (c)

$\bar{z} \rightarrow \frac{-\pi}{3}$

$z \rightarrow \frac{\pi}{3}$

$z + zi\ , where\ z = \cos\theta + i \sin\theta$

$\cos\theta + i \sin\theta + (\cos\theta + i \sin\theta)i$

$= \cos\theta + i \sin\theta + i \cos\theta - \sin\theta$

$= \cos\frac{\pi}{3} + i \sin\frac{\pi}{3} + i \cos\frac{\pi}{3} - \sin\frac{\pi}{3} = \frac{1-\sqrt{3}}{2} + \frac{1+\sqrt{3}}{2} i$

$\tan\theta = \frac{1+\sqrt{3}}{1-\sqrt{3}} \rightarrow \theta = -75\ , 105 \rightarrow \frac{7\pi}{12}$

Q25

Solution (b)

$z^4 \to 4\theta$

$5z \to \theta \to three\ times \to 3\theta$

$4\theta = 3\theta + \frac{\pi}{3}$

$z \to \theta = \frac{\pi}{3}$

$\bar{z} \to -\theta = \frac{-\pi}{3}$

Q26

$z = \sin 20\,(1 + i\cot 20\,)$

$z = \sin 20 + i\cos 20$

$z = \cos\theta + i\sin\theta$

$z = \cos 70 + i\sin 70$

$first : r = \sqrt{x^2 + y^2} = \sqrt{(\cos 70)^2 + (\sin 70)^2} = 1$ $solution\ (a)$

$second : amplitude\ is\ 70$ $solution\ (d)$

Q27

Solution (d)

$z = 1 + i\tan 15 = x + yi$

$r = \sqrt{1^2 + (\tan 15)^2} = \sqrt{(\sec 15)^2} = \sec 15$

Q28

Solution (c)

$z+|z|=8+12i$

$x+yi+\sqrt{x^2+y^2}=8+12i$

$yi=12i\rightarrow y=12$

$x+\sqrt{x^2+12^2}=8$

$x=-5$

$|z|^2=x^2+y^2=5^2+12^2=169$

Q29

Solution (a)

$\frac{\cos\theta-i\sin\theta}{\cos 3\theta-i\sin 3\theta}=\frac{\cos(-\theta)+i\sin(-\theta)}{\cos(-3\theta)-i\sin(-3\theta)}=\frac{z^{-1}}{z^{-3}}=\frac{z^3}{z^1}=z^2$

$\cos 2\theta+i\sin 2\theta=i$

$\cos 2\theta=0\rightarrow 2\theta=90\rightarrow\theta=45=\frac{\pi}{4}$

Q30

Solution (a)

$\frac{x+yi}{y-xi}=i\quad,\ \frac{x-yi}{y+xi}=-i$

n$\frac{a+bi}{a-bi}=x+yi\rightarrow\ \therefore x^2+y^2=1$

Q31

Solution (b)

$\frac{a^3+i}{a^2+ai-1}=\frac{a^3-i^3}{a^2+ai-1}=\frac{(a-i)(a^2+ai-1)}{a^2+ai-1}=a-i$

$z=a-i\rightarrow\bar{z}\rightarrow a+i$

Q32

Solution (b)

$z = \left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)^n \to r = 1\,,\theta = 60$

$z = (1(\cos 60 + i \sin 60\,))^{\,n} = 1^n(\cos 60n + i \sin 60n) = 1$

$\cos 60n = 1 \qquad\qquad \sin 60n = 0$

$n = 6\,,0 \qquad\qquad n = 6\,,0$

$\therefore n = 6$

$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\cos\theta = 2\left(\cos\frac{\theta}{2}\right)^2 - 1 = 1 - 2\left(\sin\frac{\theta}{2}\right)^2$

Q33

Solution (b)

$1 - \cos\theta + i\sin\theta$

$2\left(\sin\frac{\theta}{2}\right)^2 + 2i\sin\frac{\theta}{2} \times \cos\frac{\theta}{2}$

$2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2} + i\cos\frac{\theta}{2}\right) = 2\sin\frac{\theta}{2}\left(\cos\left(90 - \frac{\theta}{2}\right) + i\sin\left(90 - \frac{\theta}{2}\right)\right)$

$r \to 2\sin\frac{\theta}{2}\,,\, amplitude \to 90 - \frac{\theta}{2} = \frac{\pi}{2} - \frac{\theta}{2}$

Q34

Solution (a)

$1 + \cos 2\theta + i\sin 2\theta$

$1 + 2(\cos\theta)^2 - 1 + i\,.\,2\sin\theta\cos\theta = 2(\cos\theta)^2 + 2i\sin\theta\cos\theta$

$= 2\cos\theta\,(\cos\theta + i\sin\theta\,)$

$\therefore r = 2\cos\theta = |z|$

Q35

Solution (b)

$$1+\cos 40+i\sin 40=1+2(\cos 20)^2-1+i.2\sin 20\cos 20$$

$$=2(\cos 20)^2+2i\sin 20\cos 20=2\cos 20\,(\cos 20+i\sin 20\,)$$

$$\theta=20=\frac{\pi}{9}$$

Q36

$$z_1=\cos 70+i\sin 70$$

$$z_2=\cos 20+i\sin 20$$

$$z_1+z_2=\cos 70+\cos 20+i(\sin 70+\sin 20)=1.2817+1.2817i$$

$$first:\theta=\tan^{-1}\frac{y}{x}=\tan^{-1}1=45 \qquad solution(c)$$

$$second:r=\sqrt{2\times(1.2817)^2}=1.8126 \qquad solution(d)$$

Q37

Solution (c)

$z_1 = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\ \ ,\theta_1 \to \frac{2\pi}{3}$

$z_2 = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\ \ ,\theta_2 \to \frac{\pi}{6}$

$\frac{z_1}{z_2} = \theta_1 - \theta_2 = \frac{2\pi}{3} - \frac{\pi}{6} = \frac{\pi}{2}$

$\left(\frac{z_1}{z_2}\right)^7 = 7 \times \frac{\pi}{2} = \frac{7\pi}{2} \to - 2\pi$

$= \frac{3\pi}{2} \to - 2\pi$

$= \frac{-\pi}{2}$

Q38

Solution (c)

$\frac{1+i\tan 18}{1-i\tan 18} \to \times \frac{\cos 18}{\cos 18}$

$= \frac{\cos 18+i\sin 18}{\cos 18-i\sin 18} = \frac{\cos 18+i\sin 18}{\cos(-18)+i\sin(-18)}\ ,\ z_1 \to \theta_1 = 18\ \ ,z_2 \to \theta_2 =- 18$

$\theta_1 - \theta_2 = 18 - (-18) = 36 \to \frac{\pi}{5}$

Q39

Solution (a)

$x - \frac{1}{x} = i$

$x^2 + \frac{1}{x^2} = i^2 + 2 = 1$

$x^4 + \frac{1}{x^4} = 1^2 - 2 =- 1$

$x^8 + \frac{1}{x^8} = (-1)^2 - 2 =- 1$

$x^{16} + \frac{1}{x^{16}} = (-1)^2 - 2 =- 1$

$x^{32} + \frac{1}{x^{32}} = (-1)^2 - 2 =- 1$

Q40

Solution (d)

$|z_1 + z_2| < |z_1| + |z_2|$

$if\ |z_1 + z_2| = |z_1| + |z_2|$

$\therefore z_1\ and\ z_2\ have\ the\ same\ \theta$

$\theta_1 - \theta_2 = zero$

Q41

Solution (c)

$A = \cos\theta + i\sin\theta \quad , B = \cos\alpha + i\sin\alpha$

$AB = \cos(\theta+\alpha) + i\sin(\theta+\alpha)$

$\frac{1}{AB} = \frac{1}{\cos(\theta+\alpha)+i\sin(\theta+\alpha)} \times \frac{\cos(\theta+\alpha)-i\sin(\theta+\alpha)}{\cos(\theta+\alpha)-i\sin(\theta+\alpha)}$

$\frac{1}{AB} = \frac{\cos(\theta+\alpha)-i\sin(\theta+\alpha)}{\cos(\theta+\alpha)^{\ 2}+i\sin(\theta+\alpha)^{\ 2}} = \cos(\theta+\alpha) - i\sin(\theta+\alpha)$

$\frac{1}{2}\left(AB + \frac{1}{AB}\right) = \frac{1}{2}(\cos(\theta+\alpha) + i\sin(\theta+\alpha) + \cos(\theta+\alpha) - i\sin(\theta+\alpha))$

$= \frac{1}{2}(2\cos(\theta + a)) = \cos(\theta + a)$

Q42

Solution (a)

$\frac{(\cos\theta+i\sin\theta)^4}{(\sin\theta+i\cos\theta)^5} = \frac{(\cos\theta+i\sin\theta)^4}{(\sin(90-\theta)+i\cos(90-\theta))^5}$

$arg \rightarrow 4\theta - 5(90-\theta) = 4\theta - 5\times 90 + 5\theta = 9\theta - 450 = 9\theta - 90$

$\cos(9\theta - 90) + i\sin(9\theta - 90) = \sin 9\theta - i\cos 9\theta$

Q43

Solution (c)

$z - 1 + i \quad , |z| = 2$

$let\ z = x + yi\ , |z| = \sqrt{x^2+y^2} = 2 \rightarrow |z|^2 = x^2 + y^2 = 4 \rightarrow equ.1$

$x + yi - 1 + i = x - 1 + (y+1)i$

$\tan 180 = \frac{y+1}{x-1} \rightarrow y + 1 = 0 \rightarrow y = -1$

$by\ substitution\ in\ equ.1 \rightarrow x^2 + y^2 = 4$

$x^2 + (-1)^2 = 4$

$x^2 = 3 \rightarrow x = \pm\sqrt{3}$

$at\ \theta = 180 \rightarrow 2^{nd}\ quad. \rightarrow x\ negative$

$\therefore z = x + yi = -\sqrt{3} - 1$

Q44

Solution (c)

$z_2 = -\overline{z_1}$

Q45

Solution (b)

$z_1 = x + yi \rightarrow \overline{z_1} = x - yi$

$z_2 = x` + y`i \rightarrow \overline{z_2} = x` - y`i$

$\overline{z_1} + i\overline{z_2} = (x - yi) + i(x` - y`i) = 0 + 0i$

$x - yi + x` + y`i = 0 + 0i$

$x + y` = 0$ , $-y + x` = 0$

$x = -y`$ , $x` = y$

$z_1 z_2 = (x + yi)(x` - y`i) = (x + yi)(y - xi) = xy - x^2 i + y^2 i + yx$

$= 2xy + (y^2 - x^2)i$

$\tan 180 = \frac{y^2 - x^2}{2xy} = 0 \rightarrow y^2 - x^2 = 0$

$(y - x)(y + x) = 0 \rightarrow y - x = 0 , y + x = 0 \rightarrow y = \pm x$

$at\ y = -x$

$z_1 \rightarrow \theta = \tan^{-1}\left(\frac{y}{x} = \frac{-x}{x} = -1\right) = 135 = \frac{3\pi}{4}$

Q46

Solution (a)

$its\ center\ is\ the\ origin , r = 3$

Q47

Solution (c)

$let\ z = x + yi$

$z + \bar{z} = 2x$

$z\bar{z} = r^2 = x^2 + y^2$

$|z|^2 + 8(z + \bar{z}) + z\bar{z} = 8$

$x^2 + y^2 + 8 \times 2x + x^2 + y^2 = 8$

$2x^2 + 2y^2 + 16x = 8$

$x^2 + y^2 + 8x = 4$

$center = (-4 , 0) , r^2 = (-4)^2 + 0 + 4 = 20$

$area\ of\ circle = \pi r^2 = 20\pi$

Q48

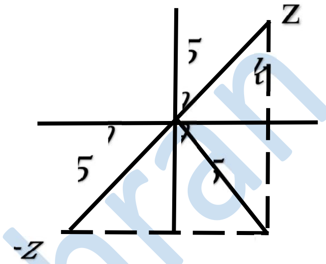
Solution (c)

$z = 5(\cos\theta + i\sin\theta)\,, r = 5$

$\sin\theta = \frac{3}{5} = \frac{k}{5} \rightarrow k = 3$

$\cos\theta = \frac{4}{5} = \frac{l}{5} \rightarrow l = 4$

$A = \frac{1}{2} \times 2l \times 2k = \frac{1}{2} \times 2 \times 4 \times 2 \times 3 = 24$



Q49

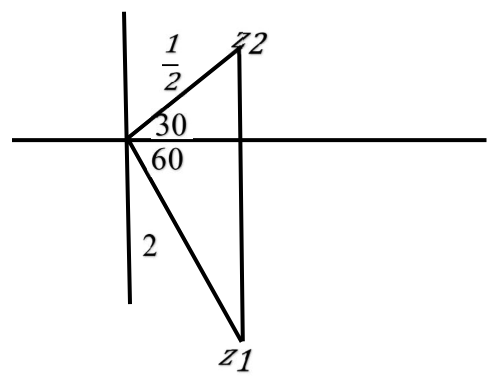
Solution (b)

$z_1 = 1 - \sqrt{3}i \rightarrow r = 2\,, \theta =- 60$

$\theta_2 - \theta_1 = 90$

$\theta_2 + 60 = 90 \rightarrow \theta_2 = 30$

$|z_1 z_2| = |z_1||z_2| = 1 \rightarrow 2|z_2| = 1 \rightarrow |z_2| = \frac{1}{2}$



Q50

Solution (c)

$z = x + yi \rightarrow r_1 = \sqrt{x^2 + y^2}\,, \theta_1 = \tan^{-1}\frac{y}{x}$

$zi = (x + yi)i = xi - y = y - xi \rightarrow r_2 = \sqrt{x^2 + y^2}\,, \theta_2 = \frac{-x}{y}$

$r_1 = r_2$

$let\ z = 1 - \sqrt{3}i \rightarrow x = 1\,, y =- \sqrt{3}$

$\tan^{-1}\frac{y}{x} =- 60 \;\;, \tan^{-1}\frac{-x}{y} = 30$

$m(AOB) = 90 = \frac{\pi}{2}$

Q51

Solution (b)

$z_1 = 3(\cos A + i \sin A)$

$z_2 = 2(\cos B + i \sin B)$

$z_3 = 4(\cos C + i \sin C)$

$z_4 = \cos D + i \sin D$

$z_1 z_2 z_3 z_4 = 3 \times 2 \times 4(\cos(A + B + C + D) + i \sin(A + B + C + D))$

$ABCD\ quad. \rightarrow A + B + C + D = 360$

$z_1 z_2 z_3 z_4 = 24(\cos 360 + i \sin 360) = 24$