Algebra

Answers of choose **by steps**

Exercise (5)

Ratio between two consecutive terms in

The binomial expansion.

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Q1

$$\frac{T_{r+1}}{T_r} = \frac{n-r+1}{r} \times \frac{2^{nd}}{1^{sr}}$$

Solution (a)

$(x+y)^{10}$

$$\frac{T_9}{T_8} = \frac{10-8+1}{8} \times \frac{y}{x} = \frac{3}{8} \times \frac{y}{x} = \frac{3y}{8x}$$

Q2

Solution (c)

$(1-x)^{12}$

$$\frac{coff.\,of\,T_6}{coff.\,of\,T_5} = \frac{n-r+1}{r} \times \frac{-1}{1} = \frac{12-5+1}{5} \times -1 = \frac{-8}{5}$$

Q3

Solution (c)

$(3+ax)^9$, $coff.\ of\ T_4 = coff.\ of\ T_3$

$$\frac{coff.\,of\,T_4}{coff.\,of\,T_3} = 1 = \frac{9-3+1}{3} \times \frac{a}{3} = \frac{7a}{9} \ , \ \therefore a = \frac{9}{7}$$

Q4

Solution (a)

$(2+x)^{50}$, $T_{17} = T_{18}$

$$\frac{T_{18}}{T_{17}} = 1 = \frac{50-17+1}{17} \times \frac{x}{2} = \frac{34x}{34} = x = 1$$

Q5

Solution (d)

$(x+y)^8$

$$\frac{T_6}{T_4} = \frac{T_6}{T_5} \times \frac{T_5}{T_4} = \left(\frac{8-5+1}{5} \times \frac{y}{x}\right) \times \left(\frac{8-4+1}{4} \times \frac{y}{x}\right) = \frac{4y}{5x} \times \frac{5y}{4x} = \frac{y^2}{x^2}$$

$y^2 \; : x^2$

Q6

Solution (c)

$(3a - 2b)^{11}$

$\because$ n is odd

$$\therefore middle\ term = \frac{n+1}{2} + 1 = \frac{11+1}{2} + 1 = 7$$

$$= \frac{n-1}{2} + 1 = \frac{11-1}{2} + 1 = 6$$

$$\frac{T_6}{T_7} = \frac{-3}{2} = \frac{6}{11-6+1} \times \frac{3a}{-2b} = \frac{18a}{-12b} = \frac{-3}{2}$$

$$\therefore \frac{a}{b} = 1$$

Q7

Solution (b)

$(a - b)^n \;, T_5 = -T_6 \;\rightarrow$ Additive inverse

$$\frac{T_6}{T_5} = -1 = \frac{n-5+1}{5} \times \frac{-b}{a}$$

$$\therefore \frac{a}{b} = \frac{n-4}{5}$$

Q8

Solution (a)

$\left(x + \frac{1}{x}\right)^{2n-1}$

$\because 2n - 1\ is\ an\ odd\ number$

$$\therefore middle\ term = \frac{2n-1+1}{2} + 1 = n + 1$$

$$= \frac{2n-1-1}{2} + 1 = n$$

$$\frac{T_n}{T_{n+1}} = \frac{n}{2n-1-n+1} \times \frac{x}{\frac{1}{x}} = \frac{n}{2n-n} \times x^2 = \frac{n}{n} \times x^2 = x^2$$

Q9

Solution (b)

$$\left(\frac{1}{x} - x^2\right)^9 \ , T_5 = 12T_4$$

$$\frac{T_5}{T_4} = 12 = \frac{9-4+1}{4} \times \frac{-x^2}{\frac{1}{x}} = \frac{6}{4} \times - x^3 = 12$$

$$- x^3 = 8 \ , \ \therefore x = -2$$

Q10

Solution (b)

$$x^n\left(\frac{1}{x} - 2\right)^n = (1 - 2x)^n$$

$$T_5 = {}^nC_4(-2x)^4 = 16\,{}^nC_4 x^4 = 2016x^4$$

$${}^nC_4 = 126 \ , \ \therefore n = 9$$

$$\frac{T_3}{T_4} = \frac{3}{9-3+1} \times \frac{1}{-2} = \frac{3}{-14} = 3 : -14 = \ -3 : 14$$

Q11

Solution (b)

$$(3 + 2x)^{74}$$

$$\frac{T_{r+1}}{T_r} = 1 = \frac{74-r+1}{r} \times \frac{2}{3} = 1$$

$$r = 30\ , \ \therefore the\ terms\ equal\ in\ coff.are\ T_{30}\ , T_{31}$$

Q12

Solution (c)

$(1+x)^{2n}$

$\frac{T_2}{T_3}+\frac{T_4}{T_3}=2 \rightarrow$ from arithmatic sequence .

$\frac{2}{2n-2+1}+\frac{2n-3+1}{3}=2$

$by\ solving \rightarrow 2n^2-9n=-7$

Q13

Solution (b)

$(1+x)^{24}$

$\frac{T_r}{T_{r+1}}=4=\frac{r}{24-r+1}=\frac{r}{25-r}=4$

$r=20\ ,\ \therefore\ the\ two\ terms\ are\ T_{20}\ ,T_{21}$

Q14

Solution (c)

$(1+x)^{10}$

$\frac{T_{r+1}}{T_r}\geq 1$, $\frac{T_{r+1}}{T_{r+2}}\geq 1$

$\frac{10-r+1}{r}\geq 1$, $\frac{r+1}{10-(r+1)+1}\geq 1$

$11-r\geq 1$, $r+1\geq 10-r$

$11\geq 2r$, $2r\geq 9$

$r\leq 5.5$, $r\geq 4.5$

$\therefore r=5$

$T_{r+1}=T_6$

Q15

Solution (d)

$(2x+7y)^3$

$T_1 \to {}^3C_0 \times 7^0 \times 2^3$

$T_1 = 8 \to \ smallest\ term$

$T_2 \to {}^3C_1 \times 7^1 \times 2^2$

$T_2 = 84$

$T_3 \to {}^3C_2 \times 7^2 \times 2^1$

$T_3 = 294$

$T_4 \to {}^3C_3 \times 7^3 \times 2^0$

$T_4 = 343 \to greatest\ term$

Q16

Solution (a)

$(x+y)^n \quad , T_7\ is\ the\ greatest\ term$

$\frac{T_7}{T_6} > 1 \quad , \quad \frac{T_7}{T_8} > 1$

$\frac{n-6+1}{6} > 1 \quad , \quad \frac{7}{n-7+1} > 1$

$n-5 > 6 \quad , \quad 7 > n-6$

$n > 11 \quad , \quad 13 > n$

$\therefore 11 < n < 13 \quad , n = 12$

Q17

Solution (c)

$(2x+y)^8$

$\frac{T_r}{T_{r+1}} \geq 1 \quad , \quad \frac{T_{r+1}}{T_{r+2}} \geq 1$

$\frac{8-r+1}{r} \times \frac{1}{2} \geq 1 \quad , \quad \frac{r+1}{8-(r+1)-1} \times \frac{2}{1} \geq 1$

$\frac{9-r}{2r} \geq 1 \quad , \quad \frac{2r+2}{8-r} \geq 1$

$9 - r \geq 2r \quad , \quad 2r + 2 \geq 8 - r$

$9 \geq 3r \quad , \quad 3r \geq 6$

$3 \geq r \quad , \quad r \geq 2$

$2 \leq r \leq 3 \;\rightarrow\; \therefore r = 2 \; or \; 3$

${}^{8}C_2 \times 2^6 = {}^{8}C_3 \times 2^5 = 1792$

Q18

Solution (c)

$(1+2x)^n \;\rightarrow sum\; of\; coff. = 6561$

$let\; x = 1 \rightarrow (1+2)^n = 3^n = 6561 = 3^8 \quad , \therefore n = 8$

$(1+2x)^8$

$\frac{T_{r+1}}{T_r} \geq 1 \quad , \quad \frac{T_{r+2}}{T_{r+1}} \geq 1$

$\frac{8-r+1}{r} \times \frac{2}{1} \geq 1 \quad , \quad \frac{r+1}{8-(r+1)-1} \times \frac{1}{2} \geq 1$

$\frac{9-r}{r} \times 2 \geq 1 \quad , \quad \frac{r+1}{8-r} \times \frac{1}{2} \geq 1$

$18 - 2r \geq r \quad , \quad r + 1 \geq 16 - 2r$

$18 \geq 3r \quad , \quad 3r \geq 15$

$6 \geq r \quad , \quad r \geq 5$

$5 \leq r \leq 6 \;\rightarrow\; \therefore r = 5 \; or \; 6$

${}^{8}C_5 \times 2^5 = {}^{8}C_6 \times 2^6 = 1792$

Q19

Solution (a)

$(1+4x)^8 \rightarrow at\ x=\frac{1}{3} \rightarrow \left(1+\frac{4}{3}\right)^8$

$\frac{T_r}{T_{r+1}} \geq 1$

$\frac{8-r+1}{r} \times \frac{4}{3} \geq 1$

$\frac{9-r}{3r} \times 4 \geq 1$

$36-4r \geq 3r$

$36 \geq 7r$

$5.1 \geq r \rightarrow r=5$

${}^8C_5\left(\frac{4}{3}\right)^5$

Q20

Solution (a)

$(a+x)^{20}$

$\frac{T_{11}}{T_{10}}$, $\frac{T_{11}}{T_{12}}$

$\frac{20-10+1}{10} \times \frac{1}{a} \geq 1$, $\frac{11}{20-11+1} \times a \geq 1$

$\frac{11}{10} \geq a$, $a \geq \frac{10}{11}$

$\frac{10}{11} \leq a \leq \frac{11}{10}$