Algebra

Answers of choose **by steps**

Exercise (4)

Finding the term containing x^r in the expansion of binomial.

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Q1

Solution (b)

$(1+x)^{10}$

$T_{r+1} = {}^{10}C_r(x)^r = kx^3 \qquad \therefore r = 3$

$k = {}^{10}C_3$

Q2

Solution (a)

The term free of "x" means "x^0" that is equal to one

$\left(x^2+\frac{1}{x^2}\right)^8$

$T_{r+1} = {}^{8}C_r\left(\frac{1}{x^2}\right)^r(x^2)^{8-r}$

$\left(\frac{1}{x^2}\right)^r(x^2)^{8-r} = x^0$

$x^{-2r}.\,x^{16-2r} = x^0$

$x^{16-4r} = x^0$, $16 - 4r = 0$, $\therefore r = 4$

${}^{8}C_r = {}^{8}C_4 = 70$

Q3

Solution (c)

$\left(x - \frac{1}{x}\right)^{10}$

$T_{r+1} = {}^{10}C_r\left(\frac{1}{x}\right)^r(x)^{10-r}$

$(x)^{-r}(x)^{10-r} = x^0$

$(x)^{10-2r} = x^0$, $10 - 2r = 0$, $\therefore r = 5$

$T_{r+1} = T_{5+1} = T_6$

Q4

Solution (b)

$\left(x^2 + \frac{a}{x}\right)^5$

$T_{r+1} = {}^5C_r\left(\frac{a}{x}\right)^r (x^2)^{5-r}$

$\left(\frac{1}{x}\right)^r (x^2)^{5-r} = x$

$x^{-r} . x^{10-2r} = x$

$10 - 3r = 1$

$coff. of\ T_{r+1} = T_4 = {}^5C_3 . a^3 = 10\ a^3$

Q5

Solution (c)

$\left(2x - \frac{1}{x^2}\right)^n$

$T_5 = {}^nC_4\left(\frac{-1}{x^2}\right)^4 (2x)^{n-4}$

$\left(\frac{1}{x^2}\right)^4 (x)^{n-4} = x^0$

$x^{-8} . x^{n-4} = x^0$

$n - 12 = 0\ ,\ \therefore n = 12$

Q6

Solution (b)

$\left(2x^2 + \frac{1}{2x}\right)^{3n}$

$T_9 = {}^{3n}C_8\left(\frac{1}{2x}\right)^8 (2x^2)^{3n-8}$

$\left(\frac{1}{x}\right)^8 \left(x^2\right)^{3n-8} = x^0$

$x^{-8}.x^{6n-16} = x^0 \ \ , -8 + 6n - 16 = 0 \ \ , \therefore n = 4$

$\left(2x^2 + \frac{1}{2x}\right)^{12}$

$T_{r+1} = {}^{12}C_r\left(\frac{1}{2x}\right)^r (2x^2)^{12-r}$

$\left(\frac{1}{x}\right)^r \left(x^2\right)^{12-r} = x^3$

$x^{-r}.x^{24-2r} = x^3, 24 - 3r = 3 \ \ , \therefore r = 7$

$T_{r+1} = T_8 = {}^{12}C_7\left(\frac{1}{2x}\right)^7 (2x^2)^5$

Coff. of $T_8 = {}^{12}C_7 \left(\frac{1}{2}\right)^7 (2)^5 = 198$

Q7

Solution (b)

$\left(\frac{a}{x^2} + x\right)^9$

$T_{r+1} = {}^9C_r \, . \, x^r . \left(\frac{a}{x^2}\right)^{9-r}$

$x^r . x^{-18+2r} = x^0 , r + 2r - 18 = 0 , \quad \therefore r = 6$

$Coff. of \ T_{r+1} = T_7 = {}^9C_6 \, . \, a^3 = 672 \quad , \quad \therefore a = 2$

Q8

Solution (c)

$\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$, *Absolute term* $\longrightarrow x^0$

$T_{r+1} = {}^{10}C_r\left(\frac{-k}{x^2}\right)^r\left(\sqrt{x}\right)^{10-r}$

$\left(\frac{1}{x^2}\right)^r\left(\sqrt{x}\right)^{10-r} = x^0$

$x^{-2r} \,.\, x^{5-\frac{1}{2}r} = x^0$

$-2r + 5 - \frac{1}{2}r = 0$, $\therefore r = 2$

$T_{r+1} = T_3 = {}^{10}C_2(-k)^2 = 405$

$k^2 = 9$, $k = \pm 3$

Q9

Solution (d)

$x^3(1+x)^7 \longrightarrow x^4 \quad (\div x^3)$

$(1+x)^7 \longrightarrow x$

$T_{r+1} = {}^7C_r \,.\, x^r$

$x^r = x$, $\therefore r = 1$

$Coff.\ of\ T_{r+1} = T = {}^7C_1$

Q10

Solution (c)

$(1+x)^n$

$T_{10} = {}^nC_4 \,.\, x^4 \longrightarrow Coff.of\ T_{10} = {}^nC_4$

$T_{r+1} = {}^nC_r \,.\, x^r$

$\therefore x^r = x^5$, $r = 5$

$T_6 = {}^{n}C_5 \, . \, x^5 \longrightarrow Coff. of\ T_6 = {}^{n}C_6$

${}^{n}C_4 = {}^{n}C_{10} \quad \therefore n = 4 + 10 = 14$

Q11

Solution (d)

$\left(\frac{1}{x} + kx^2\right)^8$

$T_{r+1} = {}^{8}C_r (kx^2)^r \left(\frac{1}{x}\right)^{8-r}$

$x^{2r} . \left(\frac{1}{x}\right)^{8-r} = x^7$

$x^{2r} . x^{-8+r} = x^7$

$x^{3r-8} = x^7$

$3r - 8 = 7 \quad , \therefore r = 5$

$Coff. of\ T_{r+1} = T_6 = {}^{8}C_5 \, . k^5 = 56$

$k^5 = 1 \quad , \therefore k = 1$

Q12

Solution (b)

$\left(ax + \frac{1}{bx}\right)^{10}$

$T_7 = {}^{10}C_6 \left(\frac{1}{bx}\right)^6 (ax)^4$

$Coff. of\ T_7 = {}^{10}C_6 \left(\frac{1}{b}\right)^6 a^4 = {}^{10}C_6 . b^{-6} . a^4$

$T_{r+1} = {}^{10}C_r \left(\frac{1}{bx}\right)^r (ax)^{10-r}$

$\left(\frac{1}{x}\right)^r . x^{10-r} = x^{-r} . x^{10-r} = x^0$

$10 - 2r = 0\ ,\ \therefore r = 5$

$T_6 = {}^{10}C_5 . b^{-5} . a^5$

$Coff. of\ T_6 = {}^{10}C_5 . b^{-5} . a^5$

$T_6 = T_7$

${}^{10}C_6 . b^{-6} . a^4 = {}^{10}C_5 . b^{-5} . a^5 \qquad \times b^6 . a^{-4}$

$210 = 252\ ab\ \ , ab = \frac{5}{6}$

Q13

Solution (c)

$\left(ax^2 + \frac{1}{ax}\right)^9$

$T_{r+1} = {}^9C_r \left(\frac{1}{ax}\right)^r \left(ax^2\right)^{9-r}$

$\left(\frac{1}{x}\right)^r \left(x^2\right)^{9-r} = x^{-r} . x^{18-2r} = x^{18-3r}$

$x^{18-3r} = x^3 \qquad x^{18-3r} = x^6$

$18 - 3r = 3 \qquad 18 - 3r = 6$

$r = 5 \Rightarrow T_6 \qquad r = 4 \Rightarrow T_5$

$Coff. of\ T_6 = Coff. of\ T_5$

${}^9C_5 . \frac{1}{a^5} . a^4 = {}^9C_4 . \frac{1}{a^4} . a^5$

$126\ a^{-1} = 126\ a$

$\frac{1}{a} = a\ \ , a^2 = 1\ , a = \pm 1$

Q14

Solution (b)

$$\left(2+\frac{x}{3}\right)^n$$

$$T_{r+1} = {}^nC_r\left(\frac{x}{3}\right)^r 2^{n-r}$$

$x^r = x^7 \Rightarrow r = 7 \Rightarrow T_8$

$x^r = x^8 \Rightarrow r = 8 \Rightarrow T_9$

$Coff.\,of\,T_8 = Coff.\,of\,T_9$

$${}^nC_7\left(\frac{1}{3}\right)^7 2^{n-7} = {}^nC_8\left(\frac{1}{3}\right)^8 2^{n-8}$$

$${}^nC_7 \,.\left(\frac{1}{3}\right)^7 \quad \frac{2^n}{2^7} = {}^nC_8 \,.\left(\frac{1}{3}\right)^8 .\ \frac{2^n}{2^8}$$

$${}^nC_7 .\, 2 = {}^nC_8 \,.\,\frac{1}{3}$$

$$\frac{{}^nC_7}{{}^nC_8} = 6 = \frac{n-8+1}{8} \quad , n-7 = 48\,,\ n = 55$$

Q15

Solution (b)

$$\left(\sqrt[6]{x}-\frac{1}{\sqrt[3]{x}}\right)^9$$

$$T_{r+1} = {}^9C_r\left(\frac{-1}{\sqrt[3]{x}}\right)^r (\sqrt[6]{x})^{9-r}$$

$$\left(\frac{1}{x^{\frac{1}{3}}}\right)^r \left(x^{\frac{1}{6}}\right)^{9-r} = x^0$$

$$x^{\frac{-1}{3}} .\, x^{\frac{1}{6}(9-r)} = x^0$$

$$\frac{-1}{3}r + \frac{3}{2} - \frac{1}{6}r = 0 \quad , \quad \therefore r = 3$$

$$T_{r+1} = T_4 = {}^{9}C_3(-1)^3 = -84$$

Q16

Solution (b)

$$\left(x^5 - \frac{k}{x^2}\right)^{7n}$$

$$T_{r+1} = {}^{7n}C_r\left(\frac{-k}{x^2}\right)^r (x^5)^{7n-r}$$

$$\left(\frac{1}{x^2}\right)^r (x^5)^{7n-r} = x^0$$

$$x^{-2r}.\,x^{35n-5r} = x^0$$

$$35n - 7r = 0\ ,\ r = 5n$$

$$T_{r+1} = T_{5n+1}$$

Q17

Solution (a)

$$\left(x^2 + 2 + \frac{1}{x^2}\right)^6 = \left(\left(x + \frac{1}{x}\right)^2\right)^6 = \left(x + \frac{1}{x}\right)^{12}$$

$$T_{r+1} = {}^{12}C_r\left(\frac{1}{x}\right)^r x^{12-r}$$

$$\left(\frac{1}{x}\right)^r (x)^{12-r} = x^2$$

$$x^{-r}\,.\,x^{12-r} = x^2$$

$$12 - 2r = 5\ ,\ r = 5$$

$$T_6 = {}^{12}C_5$$

Q18

Solution (a)

$$\left(ax^2+\frac{b}{x}\right)^{12}$$

$$T_7 = {}^{12}C_6\left(\frac{-b}{x}\right)^6 (ax^2)^6 =-\ 924\, b^6 a^6 x^6$$

the term containing x^6

Q19

Solution (a)

$$(1+x)^{n+m}$$

$$T_{r+1} = {}^{n+m}C_r \,.\, x^r$$

$x^r = x^n \Rightarrow r = n$

$x^r = x^m \Rightarrow r = m$

$${}^{n+m}C_m = {}^{n+m}C_n$$

$$m+n=n+m$$

$\therefore The\ two\ coff.\ are\ equal$

Q20

Solution (a)

$$1+(1+x)+(1+x)^2+\ \ldots\ldots(1+x)^n$$

$G.S \longrightarrow a = 1\ , r = (1+x)\ ,\ l = (1+x)^n$

$$S_n = \frac{lr-a}{r-1} = \frac{(1+x)^n(1+x)-1}{(1+x)-1} = \frac{(1+x)^{n+1}-1}{x}$$

$$S_n = \frac{(1+x)^{n+1}}{x} - \frac{1}{x} = \frac{1}{x}(1+x)^{n+1} - \frac{1}{x}$$

$\therefore a = 0 , b = 1 , n = n + 1 , k = k + 1$

$r = \frac{an-k}{a-b} = \frac{0-(k+1)}{-1} = k + 1$

$T_{r+1} = T_{k+2} = {}^{n+1}C_{k+1}$

Q21

Solution (a)

$\left(x + \frac{1}{x}\right)^{2n} + \left(x - \frac{1}{x}\right)^{2n} \quad , \quad let\ n = 3$

$\left(x + \frac{1}{x}\right)^{6} + \left(x - \frac{1}{x}\right)^{6} \longrightarrow odd\ terms\ only\ T_1 , T_3 , T_5$

$T_{r+1} = {}^{6}C_r \left(\frac{-1}{x}\right)^r (x)^{6-r}$

$x^{-r} . x^{6-r} = x^0$

$6 - 2r = 0 \quad , \therefore r = 3$

The term contain x^0 $is\ T_4 , \therefore value = zero$

Q22

Solution (a)

$\left(1 + 3x + 3x^2 + x^3\right)^{15}$

$\left((1 + x)^3\right)^{15} = (1 + x)^{45}$

$T_{r+1} = {}^{45}C_r . x^r$

$x^r = x^9 \quad , r = 9$

$the\ coff.\ of\ x^9\ is\ {}^{45}C_9$

Q22

Solution (c)

$$\left(1+x+x^3+x^4\right)^{10} = \left(1+x+x^3(1+x)\right)^{10} = \left((1+x)(1+x^3)\right)^{10}$$

$$= (1+x)^{10}\left(1+x^3\right)^{10}$$

$we\ can\ get\ x^4 from\ 2\ ways$

1) $from\ (1+x)^{10}\ only\ ,\ a=0\ \ b=1\ \ n=10\ k=4$

$r = \frac{an-k}{a-b} = \frac{0-4}{0-1} = 4$

$T_5\ has\ x^4\ \ ,\ {}^{10}C_5 = 210$

2) $(1+x)^{10}\ (1+x^3)^{10}$

$\downarrow \qquad \downarrow$

$x^1 \qquad x^3$

$\downarrow \qquad \downarrow$

$T_2 \qquad T_2$

${}^{10}C_1 \times {}^{10}C_1 = 100$

$\therefore\ the\ coff.\ of\ x^4 = 210 + 100 = 310$