Binomial theorem $(x^a \pm y^b)^n,\ n \in Z^+$

Find	$\left(x+\frac{2}{x}\right)^4$	$\left(x-\frac{2}{x}\right)^5$
Number of terms	$4+1=5$ terms	$5+1=6$ terms
Expand	$T_1+T_2+T_3+T_4+T_5$	$T_1-T_2+T_3-T_4+T_5-T_6$
order of middle term	$\because$ number of terms $=5$ odd $\therefore$ order of middle term $=\frac{5+1}{2}=3$ $\therefore T_3$ is the middle term	$\because$ number of terms $=6$ even $\therefore$ order of the 2 middle terms $=\frac{6}{2}$ and $\frac{6}{2}+1 \Rightarrow T_3, T_4$
T_4 Coeff of T_4 is	${}^4C_3\left(\frac{2}{x}\right)^3(x)^1=32x^{-2}$ 32	${}^5C_3\left(\frac{-2}{x}\right)^3(x)^2=-80x^{-1}$ -80
T_4 from end $=$	No of terms $-$ order from end $+1$ $=5-4+1=2 \rightarrow T_2$ from start	T_4 from end $=6-4+1=3 \rightarrow T_3$ T_4 from end $=T_3$ from the start
Sum of coeff Put $x=1$	Put $x=1$ $(1+2)^4=3^4=81$	put $x=1$ $(1-2)^5=(-1)^5=-1$
$\frac{T_4}{T_3}$	$*\ \frac{T_4}{T_3}=\frac{4-3+1}{3}\times\frac{2/x}{x}=\frac{4}{3}x^{-2}$	$*\ \frac{T_4}{T_3}=\frac{5-3+1}{3}\times\frac{-2/x}{x}=-2x^{-2}$
find x^k x^3: $k=3$ x^{-4}: $k=-4$ free of x: $k=0$	$x^c(x^a \pm x^b)^n$ $r=\frac{an-k+c}{a-b}$ $*\ (x^a \pm x^b)^n \rightarrow r=\frac{an-k}{a-b}$	$*\ (T_1, T_2, T_3) \rightarrow$ G.S $\therefore \frac{T_2}{T_1}=\frac{T_3}{T_2}$ $*\ (T_1, T_2, T_3) \rightarrow$ A.S $\therefore T_1+T_3=2T_2 \rightarrow \div T_2$ $\therefore \frac{T_1}{T_2}+\frac{T_3}{T_2}=2$

To find the greatest coeff of $(ax+b)^n$ (first a, 2nd b) $\rightarrow$

Put $\frac{T_{r+1}}{T_r} \geq 1 \Rightarrow r \leq \frac{n+1}{\left|\frac{a}{b}\right|+1}$

$*$ If $r \leq 5$ integer $\Rightarrow r=5,\ r=4 \Rightarrow T_6, T_5$

$*$ If $r \leq 3.5 \notin Z^+ \Rightarrow r=3 \rightarrow T_4$

If $r \in$ .... المطلوب $\rightarrow \frac{n+1}{\left|\frac{a}{b}\right|+1}-1 \leq r \leq \frac{n+1}{\left|\frac{a}{b}\right|+1}$

Special case In $(1+x)^n$, ${}^nC_{\frac{n}{2}}$

① If n is even $\rightarrow$ the middle is the greatest

② If n is odd $\rightarrow$ the 2 middle terms ${}^nC_{\frac{n-1}{2}}$ or ${}^nC_{\frac{n+1}{2}}$ are the greatest and they are equal.

Notices

(1) $1+8x+{}^nC_2x^2+\dots x^8=(1+x)^8$

(2) $(x+a)^n+(x-a)^n=2[T_1+T_3+T_5+\dots]$

(3) $(x+a)^n-(x-a)^n=2[T_2+T_4+T_6+\dots]$

(4) No of terms of:

$(x+3)^{10}+(x-3)^{10}=6$ terms $\rightarrow$ (5 even, 6 odd)

$(x+3)^{10}-(x-3)^{10}=5$ terms $\rightarrow$ (5 even, 6 odd)

$(x+3)^{11}+(x-3)^{11}=6$ terms $\rightarrow$ (6 even, 6 odd)

$(x+3)^{11}-(x-3)^{11}=6$ terms $\rightarrow$ (6 even, 6 odd)

(5) In $(1+x)^n$

1) $1+{}^nC_1x^1+{}^nC_2x^2+{}^nC_3x^3+\dots {}^nC_nx^n$

2) Coeff of x^k is ${}^nC_k$

3) Coeff of $T_m={}^nC_{m-1}$

4) Put $x=1 \rightarrow 2^n={}^nC_0+{}^nC_1+{}^nC_2+\dots {}^nC_n$

Put $x=-1 \rightarrow 0={}^nC_0-{}^nC_1+{}^nC_2+{}^nC_3+\dots+(-1)^n\,{}^nC_n$

5) ${}^nC_0+2\,{}^nC_1+2^2\,{}^nC_2+2^3\,{}^nC_3+\dots+2^n\,{}^nC_n=(1+2)^n=3^n$

6) $n=10,\ x=-1$

$\therefore {}^{10}C_0+{}^{10}C_2+{}^{10}C_4+\dots {}^{10}C_{10}={}^{10}C_1+{}^{10}C_3+\dots {}^{10}C_9$

$=\frac{2^{10}}{2}=2^9$

${}^nC_0+{}^nC_2+{}^nC_4+\dots+{}^nC_n={}^nC_1+{}^nC_3+\dots {}^nC_{n-1}=2^{n-1}$

(6) To find the greatest term of $(ax+by)^n$ $\Leftarrow$ or

$\frac{T_{r+1}}{T_r} \geq 1 \rightarrow \frac{n-r_{small}+1}{r_{small}} \times \frac{by}{ax} \geq 1$

$\frac{T_{r+1}}{T_{r+2}} \geq 1 \rightarrow \frac{r+1}{n-(r+1)+1} \times \frac{ax}{by} \geq 1$