Exercise 3

(1) $\left(\frac{2a}{3} + \frac{b}{a^2}\right)^{8n}$, $8n$ is even number

middle term $= \frac{n}{2} + 1 = 9$

$= \frac{8n}{2} = 8$

$4n = 8 \longrightarrow \underline{\underline{n = 2}}$ (b)

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(2) $\because T_{12}$ is the only middle term in $\left(x^2 - \frac{1}{x}\right)^{n+5}$

$\therefore$ no. of terms is odd $\longrightarrow$ "n is even"

$\frac{n+5}{2} + 1 = 12$

$\frac{n+5}{2} = 11$ → سؤال قديم في كتاب 2023

$n + 5 = 22 \longrightarrow n = 17$ (d)

Remark

Middle term

no. of terms is even
↓
"n" is $\boxed{\text{odd}}$
↓
$\frac{n+1}{2} + 1$, $\frac{n-1}{2} + 1$

no. of terms is odd
↓
"n" is $\boxed{\text{even}}$
↓
$\frac{n}{2} + 1$

(2) $(x + \frac{1}{x})^4$

$T_{r+1} = {}^nC_r \left(\frac{1}{x}\right)^r (x)^{n-r}$

$T_4 = {}^4C_3 \left(\frac{1}{x}\right)^3 (x)^1$

$T_4 = 4 \frac{1}{x^2}$ (d)

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(3) $[(a+4b)^3(a-4b)^3]^2$

$= [(a^2-16b^2)^3]^2 = (a^2-16b^2)^6$

· no. of terms $= n+1$

$= 6+1 = 7$ (b)

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(4) $(x+y)^{2n-1}$, no. of terms $= \underline{12}$

$(2n-1)+1 = 12$

$2n = 12 \longrightarrow n = 6$ (b)

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(5) $(a+b)^{2n} + (a-b)^{2n} = 11$ terms

$\longrightarrow$ odd terms only

no. of terms

$= 2n+1$ $\rightarrow$ $n+1$ odd

$\rightarrow$ n even

$n+1 = 11 \longrightarrow n = 10$ (c)

(6) $(1+bx)^9$

Coff. of $T_6 = {}^9C_5 \, b^5$ (c)

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(7) $(3x - \frac{1}{6})^{10}$

$\because$ n is even

$\therefore$ middle term $= \frac{n}{2} + 1 = \frac{10}{2} + 1 = 6$

Coff. of $T_6 = {}^{10}C_5 \left(\frac{-1}{6}\right)^5 (3)^5$

$= \frac{-63}{8}$ (a)

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(8) $\left(\frac{2}{x} + \frac{x^2}{2}\right)^{25}$

$\because$ n is odd

$\therefore$ Two middle terms $\rightarrow \frac{25+1}{2} + 1 = 14$

$\rightarrow \frac{25-1}{2} + 1 = 13$

* Coff. of $T_{14} = {}^{25}C_{13} \left(\frac{1}{2}\right)^{13} (2)^{12} = \frac{1}{2} \, {}^{25}C_{13}$
* Coff. of $T_{13} = {}^{25}C_{12} \left(\frac{1}{2}\right)^{12} (2)^{13} = 2 \, {}^{25}C_{12}$
* $\frac{1}{2} \, {}^{25}C_{13} = \frac{1}{2} \, {}^{25}C_{12}$

${}^nC_r = {}^nC_{n-r}$

* Sum of Coff. of two M.T

$= 2 \times {}^{25}C_{12} + \frac{1}{2} \, {}^{25}C_{12}$

$= {}^{25}C_{12} \left(2 + \frac{1}{2}\right) = \frac{5}{2} \, {}^{25}C_{12}$ (c)

(9) $\left(x^2+ax+\frac{a^2}{4}\right)^4$

$=\left[\left(x+\frac{a}{2}\right)^2\right]^4$

$=\left(x+\frac{a}{2}\right)^8$

Coff. of $T_3 = {}^8C_2\left(\frac{a}{2}\right)^2$

$=\frac{28}{4}a^2=7a^2$ (a)

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(10) $(1+x-3x^2)^{2021}$

Let $x=1$

"بيحط اي رمز بـ "1
عشان هو عاوز Coff

$(1+(1)-3(1)^2)^{2021}$

$=(-1)^{2021}$ $=-1$ (a)

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(11) $(4x+3y-5z)^n=64$

Let $x=1$

Let $y=1$

Let $z=1$

$(4(1)+3(1)-5(1))^n=64$

$=2^n=64$

$2^n=2^6 \longrightarrow n=6$

(12) $(a^2x^2 - 2ax + 1)^9$

$= \left[(ax - 1)^2\right]^9$

$= (ax - 1)^{18}$, let $x = 1$

$= (a - 1)^{18} = 0 \longrightarrow a = 1$ c

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(13) $\left(2x - \frac{3}{x^2}\right)^{19}$

let $x = 1$

$(2 - 3)^{19} = (-1)^{19} = -1$

a

نفس فكرة السؤال اللى
قبله بس بيتكلم كتير
و بيحكيلك قصة حياته

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(14) $(1 - 3x + 10x^2)^n$ $\qquad$ $(1 + x^2)^n$

let $x = 1$

$a = (1 - 3 + 10)^n$ $\qquad$ $b = (1 + 1)^n$

$a = 8^n$ $\qquad$ $\boxed{b = 2^n}$

$a = 2^{3n} = (2^n)^3$

$= b^3$

d

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(15) $7 \rightarrow (+)$ $\qquad$ $6 \rightarrow (-)$

$7 + 6 = 13$ terms $\rightarrow n + 1 = 13 \rightarrow \underline{\underline{n = 12}}$

$(a - b)^{12}$

a

(16) $(2-x)^5\,(2+x)^5$

$= [(2-x)(2+x)]^5$

$= (4-x^2)^5$

Last term $= (-x^2)^5 = -x^{10}$ (c)

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(17) $(1+ax)^7$

coff. $T_{r+1} = {}^{n}C_r\,(a)^r$

coff. $T_5 = {}^{7}C_4\,a^4 = 560$

$35a^4 = 560$

$a^4 = 16 \longrightarrow a = \pm 2$ (c)

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(18) $\left(a\sqrt{x} - \frac{1}{a\sqrt{x}}\right)^{12}$

$T_{r+1} = {}^{12}C_r\left(\frac{-1}{a}\right)^r (a)^{12-r}$

$T_9 = {}^{12}C_8\left(\frac{-1}{a}\right)^8 (a)^4 = 7920$

$495\,\frac{a^4}{a^8} = 7920$

$\frac{1}{a^4} = 16$

$a^4 = \frac{1}{16}$

$a = \pm\frac{1}{2}$ (a)

(19) $(x+a)^n$

T_x from the end $=$
$=$ no. of terms $-$ order $+1$
$= n+1 - x + 1$
$= n - x + 2$

- T_{15} from the end $= n - 15 + 2$
$= n - 13$

- $T_4 = T_{n-13}$

$4 = n - 13 \longrightarrow n = 17$ (b)

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(20) $(x+y)^n$

* Coff. of $T_6 = {}^nC_5$
* Coff. of $T_{16} = {}^nC_{15}$

${}^nC_5 = {}^nC_{15}$

$n = 5 + 15 = 20$ (b)

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(21) $\left(3x^2 + \frac{2}{3x}\right)^8$

$\because n = 8 \longrightarrow$ even

$\therefore$ middle term $= \frac{n}{2} + 1 = \frac{8}{2} + 1 = \underline{\underline{5}}$

$T_5 = {}^8C_4 \left(\frac{2}{3x}\right)^4 (3x^2)^4 = 17920$

$= 70 \times \left(\frac{2}{3}\right)^4 (3)^4 \left(\frac{1}{x}\right)^4 (x^2)^4 = 17920$

$\frac{x^8}{x^4} = 16$

$x^4 = 16 \longrightarrow x = \pm 2$ (a)

(22) $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$

* T_7 from the end $= n - 7 + 2$
$= n - 5$

- $T_7 = {}^nC_6 \left(\frac{1}{\sqrt[3]{3}}\right)^6 \left(\sqrt[3]{2}\right)^{n-6}$

- $T_{n-5} = {}^nC_{n-6} \left(\frac{1}{\sqrt[3]{3}}\right)^{n-6} \left(\sqrt[3]{2}\right)^6$

$$\frac{T_7}{T_{n-5}} = \frac{{}^nC_6 \times \left(\frac{1}{\sqrt[3]{3}}\right)^6 \left(\sqrt[3]{2}\right)^{n-6}}{{}^nC_{n-6} \left(\sqrt[3]{2}\right)^6 \left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}} = \frac{1}{6}$$

$$= \frac{{}^nC_6}{{}^nC_{n-6}} \times \left(\frac{1}{\sqrt[3]{3} \times \sqrt[3]{2}}\right)^6 \times \left(\sqrt[3]{2} \times \sqrt[3]{3}\right)^{n-6} = \frac{1}{6}$$

$$1 \times \frac{1}{36} \times \frac{\left(\sqrt[3]{6}\right)^n}{36} = \frac{1}{6}$$

${}^nC_r = {}^nC_{n-r}$

$$\left(\sqrt[3]{6}\right)^n = 216$$

$$6^n = 216^3 \rightarrow \underline{\underline{n = 9}}$$ c

(23) $\left(\frac{a}{x} + \frac{x}{a}\right)^{2n}$

$$* \; T_4 = {}^{2n}C_3 \left(\frac{x}{a}\right)^3 \left(\frac{a}{x}\right)^{2n-3}$$

$$T_4 = {}^{2n}C_3 \; x^3 a^{-3} \; a^{2n} a^{-3} \; x^{-2n} x^{-3}$$

$*$ T_4 from the end $= 2n - 4 + 2$ $= 2n - 2$

$n - x + 2$

$$T_{2n-2} = {}^{2n}C_{2n-3} \left(\frac{x}{a}\right)^{2n-3} \left(\frac{a}{x}\right)^3$$

$$= {}^{2n}C_{2n-3} \; x^{2n} x^{-3} a^{-2n} a^3 \; a^3 x^{-3}$$

$\to 1$

$$* \; \frac{T_4}{T_{2n-2}} = \frac{{}^{2n}C_3 \; x^3 a^{-3} a^{2n} a^{-3} x^{-2n} x^{-3}}{{}^{2n}C_{2n-3} \; x^{2n} x^{-3} a^{-2n} a^3 a^3 x^{-3}} = \frac{a^4}{x^4}$$

${}^nC_x = {}^nC_{n-x}$

$$= \frac{a^{-3} \cdot a^{2n} \cdot a^{-3} \cdot a^{2n} \; a^{-3} \; a^{-3}}{x^{2n} \cdot x^{-3} \cdot x^{-3} \cdot x^{-3} \cdot x^{-3} \cdot x^{2n}} = \frac{a^4}{x^4}$$

$$= \frac{a^{4n-12}}{x^{4n-12}} = \frac{a^4}{x^4}$$

$$4n - 12 = 4$$

$$4n = 16$$

$$\underline{\underline{n = 4}}$$

السؤال دا أنا
معرفتش أحله
في الفاينال

(24) $(ax+b)^{2n+1}$

At $x=2 \rightarrow (2a+b)^{2n+1}$

$\because$ two middle terms are equal

$\therefore$ $2a=b$ (b)

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(25) $(1+x)^{17}$

$* \; T_{r+4} = {}^{17}C_{r+3}\,(x)^{r+3} \longrightarrow$ Coff $= {}^{17}C_{r+3}$

$* \; T_{2r+3} = {}^{17}C_{2r+2}\,(x)^{2r+2} \longrightarrow$ Coff $= {}^{17}C_{2r+3}$

${}^{17}C_{r+3} = {}^{17}C_{2r+2}$

- $r+3 = 2r+2$

  $r=1$

- $r+3+2r+2 = 17$

  $3r+5 = 17$

  $3r = 12 \rightarrow r=4$ (b)

${}^{n}C_{x} = {}^{n}C_{y}$

$\hookrightarrow x=y$

$\hookrightarrow x+y=n$

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(26) $(1+x)^{n}$

$* \; T_{b} = {}^{n}C_{b-1}$

$* \; T_{c} = {}^{n}C_{c-1}$

$b-1+c-1=n$

$b+c-2=n$

$b+c=n+2$ (b)

(27) $(1+x)^{99}(1-x+x^2)^{99}$

$= (1+x^3)^{99}$

$= (1+x^3)^{99}$

* M.T $\longrightarrow \frac{n+1}{2}+1 = 51$

$\longrightarrow \frac{n-1}{2}+1 = 50$ $\quad T_{50} = T_{51}$

* $T_{50} = T_{51}$

$^{99}C_{49}(x^3)^{49} = {}^{99}C_{50}(x^3)^{50}$

$x^{147} = x^{300} \longrightarrow \times \frac{1}{x^{147}}$ $\quad {}^{99}C_{49} = {}^{99}C_{50}$

$1 = x^3$

$\longrightarrow x = 1$

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(28) * $(1+mx)^4 \longrightarrow$ M.T $= \frac{n}{2}+1 = \frac{4}{2}+1 = 3$

· $T_3 = {}^4C_2\, m^2$

* $(1-mx)^6 \longrightarrow$ M.T $= \frac{n}{2}+1 = 4$

· $T_4 = {}^6C_3(-m)^3$

$^4C_2\, m^2 = -{}^6C_3\, m^3$

$6 = -20m \longrightarrow m = \frac{-3}{10}$ ⓐ

(29) $\left(ax + \frac{1}{bx}\right)^{10}$

$T_6 = {}^{10}\cancel{C}_5 \left(\frac{1}{b}\right)^5 (a)^5 = {}^{10}\cancel{C}_5$

$\frac{a^5}{b^5} = 1$

$\frac{a}{b} = 1$ (b)

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(30) $(ax+b)^n$

$\because$ two middle terms are equal

$\therefore$ $ax = b$, $x = \frac{b}{a}$ (a)

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(31) $(1+x)^n = 1 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$

$a_1 \to {}^nC_1$, $a_2 \to {}^nC_2$, $a_3 \to {}^nC_3$

$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$

* $\frac{a_2 + a_3}{a_2}$

$= 1 + \frac{a_3}{a_2} = 1 + \frac{{}^nC_3}{{}^nC_2} = 3$

$= 1 + \frac{n-3+1}{3} = 3$

$= \frac{3}{3} + \frac{n-2}{3} = 3$

$= \frac{3+n-2}{3} = 3$

$= n+1 = 9$

$n = 8$ (c)

(32) $1 + \frac{5}{2}x + \frac{5\times4}{4\underline{|2}} + \frac{5\times4\times3}{8\underline{|3}}x^3 + \dots + \frac{1}{32}x^5$

$\left(1+\frac{1}{2}x\right)^5 = 1024$

$\left(1+\frac{1}{2}x\right)^5 = 4^5$

$1+\frac{1}{2}x = 4$

$\frac{1}{2}x = 3$

$x = 6$ (c)

إن شاء الله هتاخد الشكل
دا في إعدادي هندسة
اسمها "الاشتقاق الذوقي"

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(33) $1 - 6x + \frac{6\times5}{2\times1} - \dots + x^6$

$(1-x)^6 = 64$

$1-x = \pm 2$

$1-x = 2$

$x = -1$

$1-x = -2$

$x = 3$

$S.S = \{-1, 3\}$ (c)

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(34) $(1+\sqrt{3})^6 \longrightarrow \underline{\underline{7}}$ terms

${}^nC_r(\sqrt{3})^r \longrightarrow$ "r" should be even to get an integer number

1. ${}^nC_0$ "T_1"
2. ${}^nC_2$ "T_3"
3. ${}^nC_4$ "T_5"
4. ${}^nC_6$ "T_7"

$\underline{\underline{4 \text{ Terms}}}$

4 (b)

(35) $(\sqrt{3} + \frac{1}{\sqrt{3}})^7$

$$= \underbrace{(\sqrt{3})^7}_{\text{not integer}} + \underbrace{{}^7C_1(\frac{1}{\sqrt{3}})(\sqrt{3})^6}_{\text{not integer}} + \underbrace{{}^7C_2(\frac{1}{\sqrt{3}})^2(\sqrt{3})^5}_{\text{not integer}} + \cdots + \underbrace{(\frac{1}{\sqrt{3}})^7}_{\text{not integer}}$$

zero ⓐ

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(36) $(x+y)^{2000} + (x-y)^{2000} \rightarrow$ odd terms only

$\because 2000 + 1 = 2001$

$\therefore$ no. of odd terms > no. of even terms

* odd terms = 1001 ⓓ

* even terms = 1000 → 500 − 500 = Zero

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(37) $(x+y)^{1000} - (x-y)^{1000} \rightarrow$ even terms only

$\because 1000 + 1 = 1001$

$\therefore$ * odd terms = 501

* even terms = 500 ⓑ

Remark

Summation

* $(x+a)^n + (x-a)^n = 2[T_1 + T_3 + T_5 + \cdots]$

* $(x+a)^n - (x-a)^n = 2[T_2 + T_4 + T_6 + \cdots]$

(38) $\underbrace{(x+y)^{16}}_{\downarrow} + \underbrace{(x-y)^{14}}_{\downarrow}$

17 terms + 15 terms

= 32 terms (d)

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(39) $(x+y)^n + (x-y)^n \rightarrow$ odd terms

no. of odd terms = 16

*Case 1 → no. of odd terms > no. of even terms

16 > 15

$\because$ 16 + 15 = 31 terms

$\therefore$ no. of terms = n+1

31 = n+1 → $\underline{\underline{n = 30}}$

* Case 2 → no. of odd terms = no. of even terms

16 = 16

$\because$ 16 + 16 = 32 terms

$\therefore$ no. of terms = n+1

32 = n+1 → $\underline{\underline{n = 31}}$

30 or 31

(c)

(40) $(x+\frac{1}{x})^{2n}$

$2n \rightarrow$ even $\rightarrow$ M.T $= \frac{2n}{2} + 1 = n+1$

$\cdot\, T_{n+1} = {}^{2n}C_n = \frac{{}^{2n}P_n}{\lfloor n} = \frac{1 \times 3 \times 5 \times \cdots \times (2n-1) 2^n}{\lfloor n}$

a ✓✓ b ✓✓ d ✓✓

(c)

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(41) $\sum_{r=0}^{20} {}^{20}C_r = {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_{19} + {}^{20}C_{20}$

$\because$ summation of $(x+y)^{20}$, let $x=1, y=1$

$\therefore (1+1)^{20} = 2^{20}$ (a)

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(42) $\sum_{n=0}^{13} {}^{27}C_r = \underbrace{{}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots {}^{20}C_{13}}_{\downarrow \text{ 14 terms}}$

$\underbrace{(x+y)^{27}}_{\downarrow \text{ 28 terms}}$

$\because 14 = \frac{1}{2} \times 28$

Sum. of 14 terms $= \frac{1}{2}$ sum. of 28 terms

$= \frac{1}{2} \times 2^{27}$

$= 2^{26}$ (c)

(43) $(1+ax)^n = 1 + 8x + 24x^2 + \cdots + a^n x^n$

$= {}^nC_0 + {}^nC_1 ax + {}^nC_2 a^2x^2 + \cdots + a^n x^n$

* ${}^nC_1\, a x = 8x$

$\frac{\lfloor n}{\lfloor 1 \lfloor n-1} \times a = 8 \quad = \quad \frac{n \cancel{\lfloor n-1}}{\cancel{\lfloor n-1}} \times a = 8$

$na = 8 \longrightarrow a = \frac{8}{n}$

* ${}^nC_2\, a^2 x^2 = 24x^2$

$\frac{\lfloor n}{\lfloor 2 \lfloor n-2} a^2 = 24 \quad = \quad \frac{n(n-1)\cancel{\lfloor n-2}}{2 \cancel{\lfloor n-2}} \times a^2 = 24$

$n(n-1) \times a^2 = 48$

$= (n^2 - n) \times \left(\frac{8}{n}\right)^2 = 48$

$= 64 - \frac{64}{n} = 48$

$64\left(1 - \frac{1}{n}\right) = 48$

$1 - \frac{1}{n} = \frac{48}{64}$

$\frac{1}{n} = \quad - \frac{3}{4} = \frac{1}{4}$

$\underline{\underline{n = 4}}$

$, \; na = 8$

$4a = 8 \longrightarrow \underline{\underline{a = 2}}$

$\frac{a-n}{a+n} = \frac{2-4}{2+4} = \frac{-2}{6} = \frac{-1}{3}$ (c)

(44) $1 + {}^{20}C_1\, i + {}^{20}C_2\, i^2 + {}^{20}C_3\, i^3 + \dots + {}^{20}C_{20}\, i^{20}$

$$= (1+i)^{20}$$

$$= \left[(1+i)^2\right]^{10} = (2i)^{10} = 1024\, i^{10}$$

$$= -1024 \quad c$$

أول ثانوي

$*\ (1+i)^2 = 2i$

$*\ (1-i)^2 = -2i$

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(45) ${}^{n}C_0 - {}^{n}C_1 + {}^{n}C_2 - {}^{n}C_3 + \dots (-1)^n\ {}^{n}C_n$

$$= (1-1)^n = \text{zero}^n = 0 \quad c$$

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(46) ${}^{n}C_0 + {}^{n}C_1 \left(\frac{1}{3}\right) + {}^{n}C_2 \left(\frac{1}{3}\right)^2 + \dots + {}^{n}C_n \left(\frac{1}{3}\right)^n$

$$= \left(1 + \frac{1}{3}\right)^n = \left(\frac{4}{3}\right)^n \quad c$$

Remark

$$(1+a)^n = 1 + {}^{n}C_1\, a + {}^{n}C_2\, a^2 + {}^{n}C_3\, a^3 + {}^{n}C_4\, a^4 + \dots + a^n$$

(47)

$$[(x+y)^{10} + {}^{10}C_1 (x-y)(x+y)^9 + {}^{10}C_2 (x-y)^2 (x+y) + \cdots + (x-y)^{10}]$$

$$[(x+y) + (x-y)]^{10}$$

$$= [x + \cancel{y} + x - \cancel{y}]^{10} = (2x)^{10} = \underline{1024\, x^{10}}$$

1 term (a)