Secondary 3

Algebra

Sheet 1

Basics

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Factorial of a number

$\underline{|5} = 5\times4\times3\times2\times1$ (Five factorial)

$\underline{|4} = 4\times3\times2\times1$ (Four factorial)

$\underline{|n} = n\,(n-1)\,(n-2)\ \ldots\ldots\ 3\times2\times1$ (n factorial)

Remember

1) $\underline{|n} = n\,(n-1)\,(n-2)\,(n-3)\ \ldots\ldots\ 3\times2\times1$

$= n\,(n-1)\,\underline{|n-2}$

$= n\,\underline{|n-1}$

2) $\underline{|0} = 1$

PROOF

$\underline{|n} = n\,\underline{|n-1}$

Put $n = 1 \rightarrow \underline{|1} = 1\ \underline{|1-1}$

$1 = \underline{|0}$

3) $\underline{|n} = 1$ **THEN** $n = 1$ or 0

eg: $\underline{|n-3} = 1$ **THEN** $n \in \ldots\ldots\ldots$

SOLUTION

$n-3 = 0$ or $n-3 = 1$

$n = 3$ $n = 4$

$n \in \{3, 4\}$

4) $\dfrac{\underline{|7}}{\underline{|4}} = \dfrac{7\times6\times5\times\cancel{\underline{|4}}}{\cancel{\underline{|4}}} = 7\times6\times5 = 210$

5) $\dfrac{\underline{|n-1}}{\underline{|n+1}} = \dfrac{\cancel{\underline{|n-1}}}{(n+1)\ (n)\ \cancel{\underline{|n-1}}} = \dfrac{1}{n\ (n+1)}$

6) $n\,\underline{|2n-1} = 12$ (multiply both sides by 2)

$2n\,\underline{|2n-1} = 24$

$\underline{|2n} = \underline{|4}$

$2n = 4 \rightarrow n = 2$

Algebra

Multiple choice questions:

1) $\lfloor 4 = \ldots\ldots\ldots\ldots$

a) 42 b) 24 c) 4 d) 9

SOLUTION

$\lfloor 4 = 4 \times 3 \times 2 \times 1 = \underline{24}$

2) $\lfloor 7 = \ldots\ldots\ldots\ldots$

a) 5400 b) 2040 c) 6020 d) 5040

SOLUTION

$\lfloor 7 = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = \underline{5040}$

3) $\frac{\lfloor 9}{\lfloor 8} = \ldots\ldots\ldots\ldots$

a) 8 b) 72 c) 9 d) 1

SOLUTION

$\frac{\lfloor 9}{\lfloor 8} = \frac{9\cancel{\lfloor 8}}{\cancel{\lfloor 8}} = \underline{9}$

4) $\frac{\lfloor 99}{\lfloor 100} = \ldots\ldots\ldots\ldots$

a) 100 b) 99 c) $\frac{1}{100}$ d) $\frac{1}{99}$

SOLUTION

$\frac{\lfloor 99}{\lfloor 100} = \frac{\cancel{\lfloor 99}}{100\cancel{\lfloor 99}} = \frac{1}{100}$

5) $\dfrac{\lfloor\underline{n+2}}{\lfloor\underline{n+1}} = \ldots\ldots\ldots$

a) $n + 2$ b) $n + 1$ c) $n - 1$ d) $n - 2$

SOLUTION

$$\frac{\lfloor\underline{n+2}}{\lfloor\underline{n+1}} = \frac{(n+2)\cancel{\lfloor\underline{n+1}}}{\cancel{\lfloor\underline{n+1}}} = (n+1)$$

6) $\dfrac{\lfloor\underline{n+2}}{(2n+2)\lfloor\underline{n}} = \ldots\ldots\ldots$

a) $\dfrac{(n+1)}{2}$ b) $n + 2$ c) $\dfrac{2}{(n+2)}$ d) $\dfrac{(n+2)}{2}$

SOLUTION

$$\frac{\lfloor\underline{n+2}}{(2n+2)\lfloor\underline{n}} = \frac{\lfloor\underline{n+2}}{2(n+1)\lfloor\underline{n}} = \frac{(n+2)\cancel{\lfloor\underline{n+1}}}{2\cancel{\lfloor\underline{n+1}}} = \frac{(n+2)}{2}$$

7) If $\lfloor\underline{2n-1} = 5040$, then $n = \ldots\ldots\ldots$

a) 5 b) 2 c) 6 d) 4

SOLUTION

$\lfloor\underline{2n-1} = 5040 = \lfloor\underline{2n-1}$

$2n - 1 = 7 \quad \rightarrow \quad n = 4$

8) If $30\lfloor\underline{n} = \lfloor\underline{n+2}$, then $n = \ldots\ldots\ldots$

a) 4 b) 20 c) 2 d) 8

SOLUTION

$30\lfloor\underline{n} = (n+2)(n+1)\lfloor\underline{n}$

$30 = 6 \times 5$

$\therefore n = 4$

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9) If $\frac{\lfloor 8}{\lfloor 8-n} \div \frac{\lfloor 9}{\lfloor 9-n} = \frac{2}{3}$, then n = ……..

a) 9 b) 2 c) 18 d) 3

SOLUTION

$$\frac{\lfloor 8}{\lfloor 8-n} \times \frac{\lfloor 9-n}{\lfloor 9} = \frac{2}{3}$$

$$\frac{\cancel{\lfloor 8}}{\cancel{\lfloor 8-n}} \times \frac{(9-n)\cancel{\lfloor 8-n}}{9\cancel{\lfloor 8}} = \frac{2}{3}$$

$$\frac{(9-n)}{9} = \frac{2}{3} \quad \rightarrow \quad 18 = 3\,(9-n) \quad \rightarrow \quad n = 3$$

10) If $\frac{\lfloor n-3}{\lfloor 3} = \frac{\lfloor 5}{n-2}$, then n = ……..

a) 5 b) 2 c) 6 d) 8

SOLUTION

$$(n-2)\lfloor n-3 = \lfloor 3 \times \lfloor 5$$

$$\lfloor n-2 = 720 = \lfloor 6 \quad \rightarrow \quad n = 8$$

Combinations

Write all the 2 elements subsets contained in the set X = {A, B, C, D}

SOLUTION

the required subsets are:

{A, B} , {A, C} , {A, D} , {B, C} , {B, D} , {C, D}

∴ the number of these subsets equals 6

We select 2 out of 4 → So we write ${}^4C_2 = 6$

Rules:

1) ${}^nC_r = \frac{{}^nP_r}{\underline{|r}}$ $\quad n \geq r \quad , \; n \in z^+ , r \in N$

2) ${}^nC_r = \frac{\underline{|n}}{\underline{|r}\;\underline{|n-r}}$

3) ${}^nC_0 = \frac{{}^nP_0}{\underline{|0}} = \frac{1}{1} = 1$

4) ${}^nC_n = \frac{{}^nP_n}{\underline{|n}} = \frac{\underline{|n}}{\underline{|n}} = 1$

5) ${}^nC_1 = \frac{{}^nP_1}{\underline{|n}} = \frac{\underline{|n}}{1} = n$

6) ${}^nC_r = {}^nC_{n-r}$ (Radiation rule)

7) ${}^nC_x = {}^nC_y$ then $x = y$ or $x + y = n$

8) $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$ (Ratio rule)

9) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ (Addition rule)

10) ${}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 2^5 = 32$

11) ${}^nC_r = \frac{n}{r} \times {}^{n-1}C_{r-1}$

Multiple choice questions:

1) $^{8}C_{2} = \dots\dots\dots$

a) 28 b) 24 c) 52 d) 42

SOLUTION

$$^{8}C_{2} = \frac{^{8}P_{2}}{\lfloor 2} = \frac{8\times7}{2\times1} = 28$$

2) $^{50}C_{0} = \dots\dots\dots$

a) 2 b) 1 c) 50 d) 4

SOLUTION

$$^{50}C_{0} = \frac{^{50}P_{0}}{\lfloor 0} = \frac{1}{1} = 1$$

3) $^{70}C_{70} = \dots\dots\dots$

a) 1 b) 4 c) 5 d) 2

SOLUTION

$$^{70}C_{70} = \frac{^{70}P_{70}}{\lfloor 70} = \frac{\lfloor 70}{\lfloor 70} = 1$$

4) $^{100}C_{1} = \dots\dots\dots$

a) 1 b) 20 c) 2 d) 100

SOLUTION

$$^{100}C_{1} = \frac{^{100}P_{1}}{\lfloor 1} = 100$$

5) $^{20}C_{18} = \dots\dots\dots$

a) 280 b) 400 c) 190 d) 420

SOLUTION

$$^{20}C_{18} = {}^{20}C_{2} = \frac{^{20}P_{2}}{\lfloor 2} = \frac{20\times19}{2\times1} = 190$$

(Radiation rule)

6) ${}^{30}C_{27} =$ ....

a) 2080 b) 460 c) 30 d) 2700

SOLUTION

$${}^{30}C_{27} = {}^{30}C_3 = \frac{{}^{30}P_3}{\lfloor 3} = \frac{30\times29\times28}{3\times2\times1} = 4060$$

(Radiation rule)

On rule 6

7) If ${}^{n}C_2 = 45$, then n = ...

a) 20 b) 4 c) 10 d) 27

SOLUTION

$${}^{n}C_2 = 45 \rightarrow \frac{{}^{n}P_2}{\lfloor 2} = 45 \rightarrow {}^{n}P_2 = 45\lfloor 2 \rightarrow {}^{n}P_2 = 90$$

$${}^{10}P_2 = 90 \rightarrow \therefore \ n = 10$$

On rule 6

8) If ${}^{n}C_{n-2} = 45$, then n = ...

a) 20 b) 6 c) 30 d) 10

SOLUTION

$${}^{n}C_{n-2} = {}^{n}C_2 = 45 \rightarrow \frac{{}^{n}P_2}{\lfloor 2} = 45 \rightarrow {}^{n}P_2 = 45\lfloor 2 \rightarrow {}^{n}P_2 = 90$$

$${}^{10}P_2 = 90 \rightarrow \therefore \ n = 10$$

On rule 6

9) If ${}^{n}C_{n-4} = 35$, then n = ...

a) 7 b) 6 c) 3 d) 8

SOLUTION

$${}^{n}C_{n-4} = {}^{n}C_4 = 35 \rightarrow \frac{{}^{n}P_4}{\lfloor 4} = 35 \rightarrow {}^{n}P_4 = 35\lfloor 4 \rightarrow {}^{n}P_4 = 840$$

$${}^{7}P_4 = 840 \rightarrow \therefore \ n = 7$$

On rule 6

10) If $^{n+1}C_{n-1} = 66$, then n = ...

a) 10 b) 11 c) 12 d) 8

SOLUTION

$$^{n+1}C_{n-1} = {}^{n+1}C_2 = 66 \rightarrow \frac{^{n+1}P_2}{\underline{|2}} = 66 \rightarrow {}^{n+1}P_2 = 66\underline{|2}$$

$$^{n+1}P_2 = 132 = {}^{12}P_2 \rightarrow \therefore n+1 = 12 \rightarrow n = 11$$

Or (Directly)

$$^{n+1}C_2 = 66 = {}^{12}C_2 \rightarrow \therefore n+1 = 12 \rightarrow n = 11$$

On rule 1

11) If $^nC_r = 4$, $^nP_r = 24$, then n + r = ...

a) 5 b) 3 c) 4 d) 7

SOLUTION

$$^nC_r = \frac{^nP_r}{\underline{|r}}$$

$$4 = \frac{24}{\underline{|r}} \rightarrow \underline{|r} = 6 \rightarrow r = 3$$

$$^nP_3 = 24 \rightarrow {}^4P_3 = 24 \rightarrow n = 4$$

$$\therefore n + r = 4 + 3 = 7$$

On rule 1

12) If $^{n-1}C_{r+1} = 120$, $^{n-1}P_{r+1} = 720$, then ^nC_r = ...

a) 120 b) 165 c) 220 d) 55

SOLUTION

$$^{n-1}C_{r+1} = \frac{^{n-1}P_{r+1}}{\underline{|r+1}}$$

$$120 = \frac{720}{\underline{|r+1}} \rightarrow \underline{|r+1} = 720 \rightarrow r+1 = 3 \rightarrow r = 2$$

$$^{n-1}P_3 = 720 = {}^{10}P_3 \rightarrow n - 1 = 10 \rightarrow n = 11$$

$$\therefore {}^{11}C_3 = 165$$

On rule 1

13) If $^{n}C_3 = 4\ ^{n}P_2$, then $n = \dots\dots\dots\dots$

a) 12 b) 16 c) 22 d) 26

SOLUTION

$$\frac{^{n}P_3}{\lfloor 3} = 4\ ^{n}P_2$$

$$\frac{n(n-1)(n-2)}{6} = 4\, n\, (n-1)$$

$$\frac{(n-2)}{6} = 4 \rightarrow n = 26$$

On rule 1

14) If $^{n}P_r = 30\ ^{n}P_{r-2}$, $28\ ^{n}C_r = 15 \times {}^{n}C_{r-2}$, then $n - r = \dots\dots$

a) 12 b) 8 c) 4 d) 20

SOLUTION

$$28\ \frac{^{n}P_r}{\lfloor r} = 15\ \frac{^{n}P_{r-2}}{\lfloor r-2}$$

$$28\ \frac{30\ \cancel{^{n}P_{r-2}}}{r\,(r-1)\cancel{\lfloor r-2}} = 15\ \frac{\cancel{^{n}P_{r-2}}}{\cancel{\lfloor r-2}}$$

$$\frac{28 \times 30}{r\,(r-1)} = 15 \rightarrow r = 8$$

$$^{n}P_8 = 30\ ^{n}P_6$$

$$\frac{\lfloor n}{\lfloor n-8} = 30\,\frac{\lfloor n}{\lfloor n-6}$$

$$\frac{\cancel{\lfloor n}}{\cancel{\lfloor n-8}} = 30\ \frac{\cancel{\lfloor n}}{(n-6)\,(n-7)\cancel{\lfloor n-8}}$$

$$(n-6)\,(n-7) = 30$$

$$6 \times 5 = 30 \rightarrow n - 6 = 6 \rightarrow n = 12$$

$$\therefore n - r = 12 - 8 = 4$$

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Example 1:

On rule 1

If ${}^{n}C_r = \frac{1}{120}\ {}^{n}P_r$, then Find r , n

SOLUTION

$\frac{{}^{n}P_r}{\lfloor r} = \frac{1}{120}\ {}^{n}P_r$

$\frac{1}{\lfloor r} = \frac{1}{120} \rightarrow \lfloor r = 120 \rightarrow r = 5$

$n \geq r$

$n \geq 5 \rightarrow n \in \{5, 6, 7, \dots\dots\}$

On rule 7

15) If ${}^{n}C_5 = {}^{n}C_{10}$, then $n = \dots\dots$

a) 15 b) 10 c) 5 d) 20

SOLUTION

$n = 10 + 5 = 15$

On rule 7

16) If ${}^{12}C_{r+1} = {}^{12}C_{3r-1}$, then $r = \dots\dots$

a) 1 or 2 b) 1 or 3 c) 4 or 2 d) 2 or 3

SOLUTION

$r + 1 = 3r - 1$ or $r + 1 + 3r - 1 = 12$

$2r = 2$ | $4r = 12$

$r = 1$ | $r = 3$

On rule 7

17) If ${}^{25}C_{2n-14} = {}^{25}C_{n-1}$, then $n = \dots\dots$

a) 13 or 40 b) 14 c) 13 d) 13 or 3

SOLUTION

$2n - 14 = n - 1$ or $2n - 14 + n - 1 = 25$

$n = 13$ | $3n = 40 \rightarrow n = \frac{40}{3} \notin N$ (refused)

On rule 7

18) If ${}^{10}C_{x^2+1} = {}^{10}C_{3x-1}$, then $x = ...$

a) 1 or 2 b) 2 or 5 c) 4 or 1 d) 2 or 4

SOLUTION

$x^2 + 1 = 3x - 1$ or $x^2 + 1 + 3x - 1 = 10$

$x^2 - 3x + 2 = 0$ | $x^2 + 3x - 10 = 0$

$(x - 2)(x - 1) = 0$ | $(x + 5)(x - 2) = 0$

$x = 2$ or $x = 1$ | $x = 2$ or $x = -5$ (refused)

On rule 8

19) ${}^{13}C_5 : {}^{13}C_4 = ...$

a) 1 : 2 b) 2 : 1 c) 5 : 9 d) 9 : 5

SOLUTION

$${}^{13}C_5 : {}^{13}C_4 = \frac{13-5+1}{5} = \frac{9}{5}$$

$$\frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{n-r+1}{r}$$

On rule 8

20) ${}^{27}C_4 : {}^{27}C_3 = ...$

a) 1 : 6 b) 7 : 1 c) 6 : 1 d) 1 : 7

SOLUTION

$${}^{27}C_4 : {}^{27}C_3 = \frac{27-4+1}{4} = 6$$

On rule 8

21) ${}^{17}C_6 : {}^{17}C_7 = ...$

a) 11 : 7 b) 7 : 11 c) 8 : 11 d) 11 : 8

SOLUTION

$${}^{17}C_6 : {}^{17}C_7 = \frac{7}{17-7+1} = \frac{7}{11}$$

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22) If ${}^{n}C_3 + 2\,{}^{n}C_2 + {}^{n}C_1 = 120$, then n = ...

a) 2 b) 4 c) 13 d) 8

SOLUTION

$${}^{n}C_3 + {}^{n}C_2 + {}^{n}C_2 + {}^{n}C_1 = 120$$

$${}^{n+1}C_3 + {}^{n+1}C_2 = 120$$

$${}^{n+2}C_3 = {}^{10}C_3$$

$n + 2 = 10 \rightarrow n = 8$

23) $\dfrac{{}^{17}C_6 + {}^{17}C_5}{{}^{18}C_5} = \text{.........}$

a) $\dfrac{19}{6}$ b) $\dfrac{13}{6}$ c) $\dfrac{11}{6}$ d) $\dfrac{13}{5}$

SOLUTION

$$\frac{{}^{17}C_6 + {}^{17}C_5}{{}^{18}C_5} = \frac{{}^{18}C_6}{{}^{18}C_5} = \frac{18-6+1}{6} = \frac{13}{6}$$

24) If ${}^{n+1}P_{r+1} : {}^{n+1}C_{r+1} = 720 : 1$, ${}^{n}C_{r-2} + {}^{n}C_{r-3} = 56$

then n + r = ...

a) 12 b) 14 c) 13 d) 18

SOLUTION

$${}^{n+1}P_{r+1} \div \frac{{}^{n+1}P_{r+1}}{\lfloor r+1} = 720$$

$$\cancel{{}^{n+1}P_{r+1}} \div \frac{\lfloor r+1}{\cancel{{}^{n+1}P_{r+1}}} = 720$$

$$\lfloor r+1 = \lfloor 6 \rightarrow r = 5$$

$${}^{n}C_{r-2} + {}^{n}C_{r-3} = 56$$

$${}^{n+1}C_{r-2} = 56$$

$${}^{n+1}C_3 = 56 = {}^{8}C_3 \rightarrow n = 7$$

$$\therefore n + r = 5 + 7 = 12$$